AMCA

Memorias del Congreso Nacional de Control Automático
ISSN: 2594-2492

Robust output feedback control for robot manipulators

Mauro López * **Javier Pliego** ** **Marco Arteaga** *
Emmanuel Nuño *** **Amalinalli Ortíz** *

* *Departamento de Control y Robótica, División de Ingeniería Eléctrica de la Facultad de Ingeniería. UNAM. Apdo. Postal 70–256, México, CDMX, 04510, México. Email: marteagp@unam.mx.*
** *Departamento de Electrónica y Telecomunicaciones, División de Física Aplicada, CICESE*
*** *Departamento de Ciencias Computacionales, Centro Universitario de Ciencias Exactas e Ingenierías, Universidad de Guadalajara. Guadalajara, Mexico. Email: emmanuel.nuno@cucei.udg.mx*

Abstract: Most robust control schemes for rigid robots assume velocities measurements to be available. A solution to this problem is by using tachometers, however, this can increase the cost and the velocities signals obtained can be contaminated with noise. These facts motivates us to design a control and observer scheme to solve the tracking position problem without measurement of velocities joints of a manipulator robot.
This work presents a robust control scheme designed in conjunction with an observer for rigid robots. Additionally, the control scheme does not need to know the dynamic model of the manipulator. Particular emphasis is placed on the experimental results, which validate the proposed control algorithm.

Keywords: Observer, robust control,

1. INTRODUCTION

It is well know that robot manipulators are generally used in repetitive tasks with high precision and accuracy. Among the main problems which have to solved, two of the most important are the lack of the velocity measurements and of an exact robot model. In (Nicosia and Tomei, 1990) is presented one of the earliest schemes designed to work only with joint measurements. In that paper, a nonlinear observer is presented which can be used to achieve tracking control. A disadvantage of their approach is that an exact knowledge of the robot model is required. On the other hand, to handle an inaccurate model some adaptive and robust schemes have been proposed (Slotine and Li, 1987), (Spong, 1992).

Thus, some robust control schemes have been proposed, which need only position measurements for control and do not require an accurate robot model. In (Canudas de Wit and Fixot, 1991), a sliding observer is developed, in (Berghuis and Nijmeijer, 1994) a quite simple control-observer scheme is proposed which notably does not need any knowledge of the robot model parameters, nor structure, ti achieve uniform ultimate boundedness of the tracking and observations errors. In the last decades, several researches have been developed adaptive and robust control schemes for trajectory tracking of robot manipulators that achieve asymptotic convergence or ultimate boundedness of the tracking errors (Zhang et al., 2000; Arteaga-Pérez, 2003; Galicki, 2008).

An adaptive algorithm may be much more complex and is aimed at estimating on line the unknown parameters to get exact tracking. An alternative to use of observers consists in employing linear filters to overcome the lack of joint velocities. More recently, (Parra-Vega et al., 2003) proposed a sliding PID control, which is able to achieve an exact tracking without any knowledge of the model for implementation. Sliding mode control approaches guarantee fast convergence of the tracking errors in the presence of model uncertainties and external disturbances. Such robustness properties makes the sliding mode control suitable in several robotic tasks. First-order sliding schemes are discontinuous controllers which generates high-frequency oscillations (*chattering*) Fridman (2011). Such oscillations are undesirable because they can damage the actuators or produce dangerous system vibrations. In order to reduce chattering several methods have been proposed in the last decades. Among them, the most popular are the second and higher-order sliding mode controls (Bartolini et al., 2003; Levant, 2005; Moreno and Osorio, 2008, 2012).

In this paper, a robust tracking control scheme, together with a nonlinear observer, is proposed that guarantee uniform ultimate boundedness of the tracking and observation errors. The rest of the paper is organized as follows. The robot, as well as some properties are given in Section 2. The tracking controller with observer scheme is proposed in Section 3. Section 4 presents some experimental results. The paper conclusions are stated in Section 5.

2. DYNAMIC MODEL OF A MANIPULATOR ROBOT

The dynamic behavior of a fully actuated n-degree-of-freedom (DOF) robot manipulators can be derived from the Euler-Lagrange equations of motion resulting in

$$\boldsymbol{H}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau} \tag{1}$$

where $\boldsymbol{q} \in \Re^n$ is the joint position vector, $\boldsymbol{H}(\boldsymbol{q}) \in \Re^{n\times n}$ is the inertia matrix, $\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} \in \Re^n$ represents the centrifugal and Coriolis forces/torques, $\boldsymbol{g}(\boldsymbol{q}) \in \Re^n$ is the gravity vector, $\boldsymbol{\tau} \in \Re^n$ is the vector input torque .

Model (1) have the following properties (Kelly et al., 2005):

Property 1. The inertia matrix $\boldsymbol{H}(\boldsymbol{q})$ is symmetric and positive definite, *i.e.* $\boldsymbol{y}^{\mathrm{T}}\boldsymbol{H}(\boldsymbol{q})\boldsymbol{y} > 0$, for all $\boldsymbol{q}, \boldsymbol{y} \in \Re^n$. □

Property 2. The inertia matrix $\boldsymbol{H}(\boldsymbol{q})$ satisfies

$$\lambda_{\mathrm{h}}\|\boldsymbol{y}\|^2 \leq \boldsymbol{y}^{\mathrm{T}}\boldsymbol{H}(\boldsymbol{q})\boldsymbol{y} \leq \lambda_{\mathrm{H}}\|\boldsymbol{y}\|^2,$$

where

$$\lambda_{\mathrm{h}} = \lambda_{\min_{\forall}\boldsymbol{q}}\{\boldsymbol{H}(\boldsymbol{q})\}, \quad \lambda_{\mathrm{H}} = \lambda_{\max_{\forall}\boldsymbol{q}}\{\boldsymbol{H}(\boldsymbol{q})\}$$

for all $\boldsymbol{q}, \boldsymbol{y}, \in \Re^n$ □

Property 3. The matrix $\dot{\boldsymbol{H}}(\boldsymbol{q}) - 2\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})$ is skew-symmetric, *i.e.*,

$$\boldsymbol{y}^{\mathrm{T}}\left(\dot{\boldsymbol{H}}(\boldsymbol{q}) - 2\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\right)\boldsymbol{y} = 0,$$

for all $\boldsymbol{q}$, $\dot{\boldsymbol{q}}$ and $\boldsymbol{y} \in \Re^n$. □

3. PROPOSED CONTROL–OBSERVER SCHEME

In this section, the tracking control problem of rigid robot manipulators without velocity measurements is studied. Continuous robust control for trajectory tracking without velocities measurements of rigid robots is presented. First, it is defined the following vector functions

$$\lceil \boldsymbol{y} \rfloor^{\alpha} = \begin{bmatrix} |y_1|^{\alpha}\mathrm{sign}(y_1) \\ \vdots \\ |y_n|^{\alpha}\mathrm{sign}(y_n) \end{bmatrix} \tag{2}$$

$$\tanh(\boldsymbol{y}) = \begin{bmatrix} \tanh(y_1) \\ \vdots \\ \tanh(y_n) \end{bmatrix}, \quad \forall \boldsymbol{y} \in \Re^n. \tag{3}$$

where $\alpha \in \Re$ is a positive constant. The function $|y|^{\alpha}\mathrm{sign}(y)$ can be written as

$$|y|^{\alpha}\mathrm{sign}(y) = \begin{cases} y^{\alpha}, & y > 0 \\ 0, & y = 0 \\ -(-y)^{\alpha}, & y < 0 \end{cases} \tag{4}$$

Notice that $|y|^{\alpha}\mathrm{sign}(y)$ is well-defined at $y = 0$ and

$$\lim_{y\to 0^+} |y|^{\alpha}\mathrm{sign}(y) = 0, \quad \lim_{y\to 0^-} |y|^{\alpha}\mathrm{sign}(y) = 0.$$

Then, $\lceil \boldsymbol{y} \rfloor^{\alpha}$ in (2) is a continuous vector function for all $\boldsymbol{y}$ and its derivative is given by

$$\frac{\mathrm{d}}{\mathrm{d}t}\lceil \boldsymbol{y} \rfloor^{\alpha} = \alpha\boldsymbol{\Gamma}(\lceil \boldsymbol{y} \rfloor^{\alpha-1})\dot{\boldsymbol{y}} \tag{5}$$

where $\boldsymbol{\Gamma}(\lceil \boldsymbol{y} \rfloor^{\alpha-1}) = \mathrm{diag}\left\{\lceil \boldsymbol{y}_1 \rfloor^{\alpha-1}, \ldots, \lceil \boldsymbol{y}_n \rfloor^{\alpha-1}\right\} \in \Re^{n\times n}$.

3.1 Observer design

It is desired to design a position tracking control law while velocity measurements are not available. Then, the corresponding observation and tracking errors are defined as

$$\boldsymbol{z} \triangleq \boldsymbol{q} - \hat{\boldsymbol{q}}, \tag{6}$$

and

$$\Delta\boldsymbol{q} \triangleq \boldsymbol{q} - \boldsymbol{q}_{\mathrm{d}}, \tag{7}$$

respectively.

Based on (Arteaga-Pérez et al., 2006), we propose the following velocity observer

$$\dot{\boldsymbol{\xi}} = \boldsymbol{z} \tag{8}$$

$$\dot{\hat{\boldsymbol{q}}}_{\mathrm{o}} = \dot{\boldsymbol{q}}_{\mathrm{d}} - \boldsymbol{\Lambda}_{\mathrm{x}}\Delta\boldsymbol{q} + \boldsymbol{K}_{\mathrm{d}}\boldsymbol{\Lambda}_{\mathrm{z}}\boldsymbol{\xi} \tag{9}$$

$$\dot{\hat{\boldsymbol{q}}} = \dot{\hat{\boldsymbol{q}}}_{\mathrm{o}} + \boldsymbol{\Lambda}_{\mathrm{z}}\boldsymbol{z} + \boldsymbol{K}_{\mathrm{d}}\boldsymbol{z} + \eta \tag{10}$$

$$\dot{\eta} = \mathrm{k}_1 \int \mathrm{Sign}(\boldsymbol{z}) + \mathrm{k}_2\lceil \boldsymbol{z} \rfloor^{\alpha} \tag{11}$$

where $\boldsymbol{\Lambda}_{\mathrm{z}}, \boldsymbol{\Lambda}_{\mathrm{x}}, \boldsymbol{K}_{\mathrm{d}}, \mathrm{k}_1, \mathrm{k}_2 \in \mathbb{R}^{n\times n}$ are positive definite diagonal matrices.

3.2 Controller design

The next step consists in designing a tracking controller by using the estimated velocities. Based on (Arteaga-Pérez et al., 2006) we define

$$\boldsymbol{s} = \dot{\hat{\boldsymbol{q}}} - \dot{\boldsymbol{q}}_{\mathrm{d}} + \boldsymbol{\Lambda}_{\mathrm{x}}\lceil \boldsymbol{\Delta q} \rfloor^{\alpha} \tag{12}$$

$$\dot{\boldsymbol{\sigma}} = \boldsymbol{K}_{\beta}\boldsymbol{s} + \mathrm{sign}(\boldsymbol{s}), \tag{13}$$

where $\boldsymbol{K}_{\beta i} \in \mathbb{R}^{n\times n}$ is a positive definite diagonal matrix, $\mathrm{sign}(\boldsymbol{s}) = [\mathrm{sign}(s_{i1}), \ldots, \mathrm{sign}(s_{in})]^{\top}$ with s_{ij} element of $\boldsymbol{s}$ for $j = 1, \ldots, n$, $\boldsymbol{q}_{\mathrm{d}}$, $\dot{\boldsymbol{q}}_{\mathrm{d}}$ is the position and velocity desired and $\alpha \in \mathbb{R}$.

Consider now the following variables

$$\dot{\boldsymbol{q}}_{\mathrm{o}} = \dot{\hat{\boldsymbol{q}}} - \boldsymbol{\Lambda}_{\mathrm{z}}\lceil \boldsymbol{z} \rfloor^{\alpha} \tag{14}$$

$$\dot{\boldsymbol{q}}_{\mathrm{r}} = \dot{\boldsymbol{q}}_{\mathrm{d}i} - \boldsymbol{\Lambda}_{\mathrm{x}}\lceil \boldsymbol{\Delta q} \rfloor^{\alpha} - \boldsymbol{K}_{\gamma}\boldsymbol{\sigma} \tag{15}$$

$$\boldsymbol{s}_{\mathrm{o}} \triangleq \dot{\boldsymbol{q}}_{\mathrm{o}} - \dot{\boldsymbol{q}}_{\mathrm{r}}, \tag{16}$$

where $\boldsymbol{K}_{\gamma} \in \mathbb{R}^{n\times n}$ is a positive definite diagonal matrix. Based on the previous definitions, the proposed control law is given by

$$\boldsymbol{\tau} = -\boldsymbol{K}\tanh(\boldsymbol{s}_{\mathrm{o}}) \tag{17}$$

where $\boldsymbol{K} \in \Re^{n\times n}$ is a positive definite diagonal matrix. At this moment we are working on the theoretical test.

We claim that the n-link robot manipulator in closed loop with the control law (17) and the observer (8)-(10) has the following properties:

i) Any closed-loop variable is bounded.
ii) The position tracking errors converge to zero.
iii) The position tracking errors converge to zero.

It remains as future research to provide an analytical proof and formalize the result in the form of a theorem.

4. EXPERIMENTAL RESULTS

In this section experimental results are presented. The experimental setup is composed of a fully actuated Geomagic Touch device 3-DOF (see Fig .1). This device runs in a computer with Windows 7. The robot has six joints but only the first three joints are actuated by DC motors. Therefore last three joints were mechanically braked during the experiments.

The controller was implemented on a PC computer and programmed using Microsoft Visual Studio with a sample time of 1[ms] in C++ language.

The Gain tuning has been done heuristically for the scheme, because some gains depend on uncertain terms. The control gains were set to $\boldsymbol{\Lambda}_{\mathrm{x}} = 15.0, 16.0, 15.0$, $\boldsymbol{K}_{\boldsymbol{\gamma}} = 0.5, 0.5, 0.5$, $\boldsymbol{K}_{\boldsymbol{\beta}} = 0.05, 0.05, 0.05$, $\boldsymbol{K} = 0.16, 0.28, 0.26$, $\boldsymbol{\Delta}_z = 1.0, 1.0, 1.0$, $\boldsymbol{K}_{di} = 15.0, 15.0, 15.0$, $\boldsymbol{K}_1 = 0.1, 0.1, 0.1$, $\boldsymbol{K}_2 = 0.016, 0.018, 0.016$, and $\alpha = 9/11$.

In each of the joints a reference signal of different frequency was chosen. It is important to mention that the amplitudes of the desired trajectories for the joints are different, therefore the scales are also different. The desired trajectory is given by

$$\boldsymbol{q}_{\mathrm{d}}(t) = \frac{180}{\pi} \begin{bmatrix} -0.4 + 0.4\cos(3t) \\ 1.2 + 0.2\cos(t) \\ -1.56 - 0.1\cos(2t) \end{bmatrix} [^\circ]$$

The initial position of the robot is $\boldsymbol{q}(0) = [\,0\ \ 80\ \ -95\,]^{\mathrm{T}}\ [^\circ]$.

Fig. 1. Experimental platform; Geomagic touch of *3D Systems*

Figure 2 shows the desired and real trajectories. As can be appreciated, the results are pretty good. In Figure 3 shows the tracking errors. They are smaller than 1 ° during most of the motion. Furthermore, the table 1 shows the performance index. In Figure 6 observation errors are shown. Notice that they are smaller than 0.5 ° during most of the motion. Furthermore, the table 2 shows the performance index. Figure 4 shows that the control signal is bounded and does not saturate the actuator. For comparison purposes, the estimated and desired velocities are shown in 5. As expected, after the transient response the estimated velocity converges to the desired one. In Figure 7 are shown the estimated position only for comparison. Finally, in Figure 8 are shown the estimated velocity and the derivative desired position.

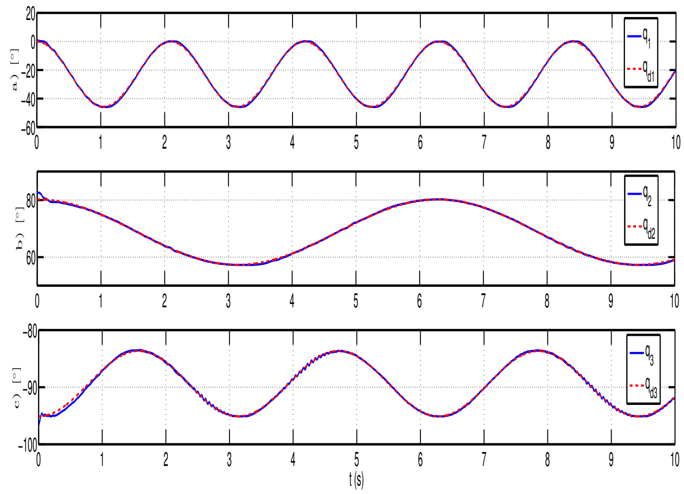

Fig. 2. Positions $\boldsymbol{q}_{\mathrm{i}=1,..,3}$ (——), $\boldsymbol{q}_{\mathrm{di}=1,..,3}$ (- - -).

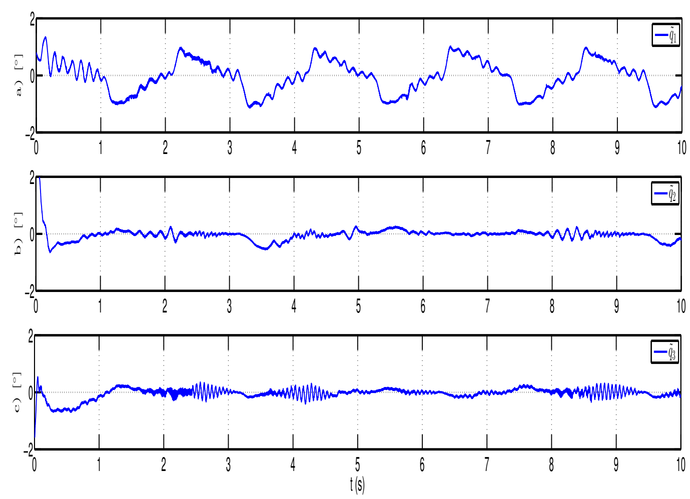

Fig. 3. Tracking errors a) $\tilde{q}_1$, b) $\tilde{q}_2$ and c) $\tilde{q}_3$.

In order to compare, in a more objective way, the behavior of the system controlled with the 3 different schemes, we have calculated the RMSE from 0 to 30 seconds (30,000 samples), of the position error as follows:

scheme	Δq_1	Δq_2	Δq_3
Scheme proposed	0.3300	0.1618	0.1720
PID	4.3778	3.7616	3.1295
Pliego-J. (Submitted for review)	0.7326	0.6856	0.5508

Table 1. RMSE of the position error

variable	Δz_1	Δz_2	Δz_3
Scheme proposed	0.5641	0.1807	0.1518

Table 2. RMSE of the observation error

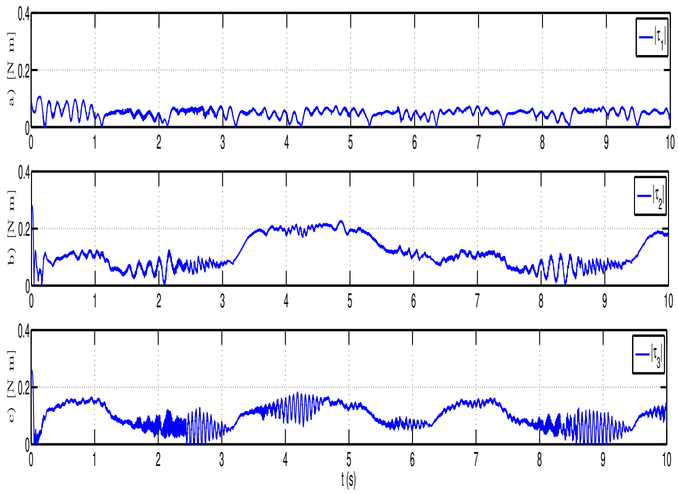

Fig. 4. Control signal, a) $|\tau_1|$, b) $|\tau_2|$ and c) $|\tau_3|$

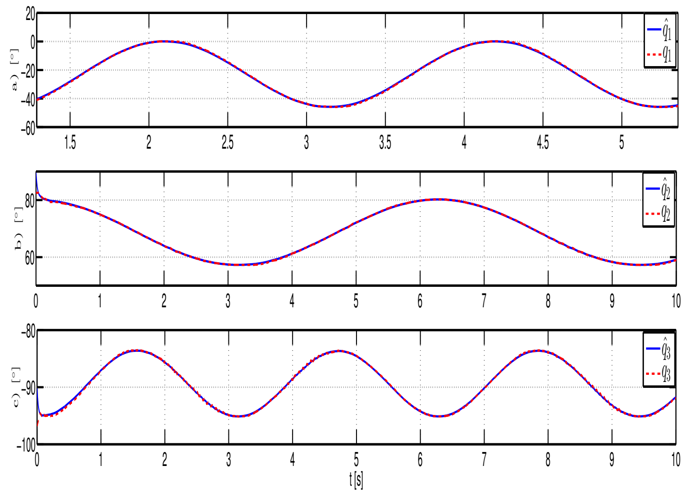

Fig. 7. Measurement-estimate position, $\boldsymbol{q}$ (- - -) v_s $\hat{\boldsymbol{q}}$ (—).

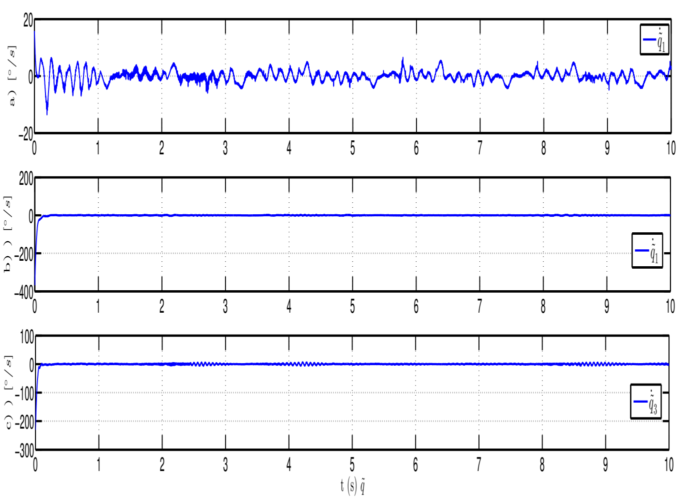

Fig. 5. Estimate-desired velocity errors, $\dot{\tilde{\boldsymbol{q}}}_{\mathrm{i}=1,..,3} \approx \dot{\hat{\boldsymbol{q}}}_{\mathrm{i}=1,..,3} - \dot{q}_{\mathrm{di}=1,..,3}$.

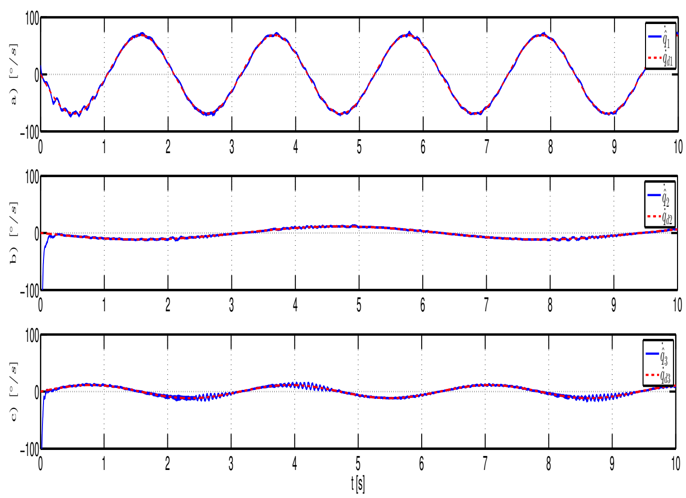

Fig. 8. Joint velocities, $\dot{\boldsymbol{q}}$ v_s (——) $\dot{\boldsymbol{q}}_d$ (- - -).

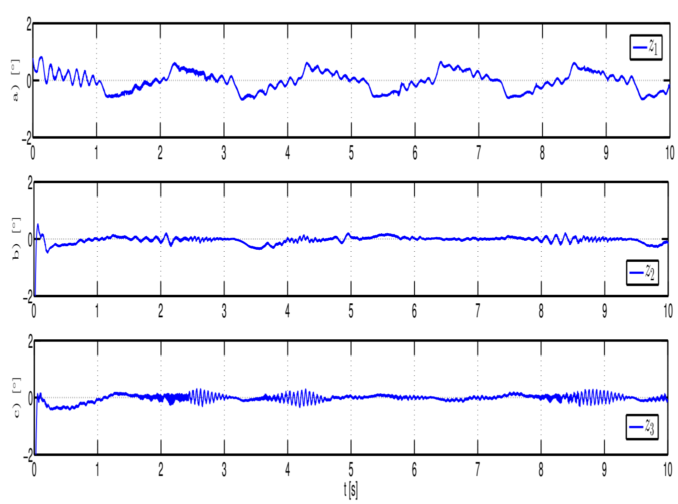

Fig. 6. Observation error $\boldsymbol{z} = \boldsymbol{q} - \hat{\boldsymbol{q}}$.

5. CONCLUSIONS

In this paper we addressed the problem of trajectory tracking of robot manipulators without using velocity measurements. To overcome this particular problem a robust controller in combination with a nonlinear velocity observer are proposed. The performance of proposed control-observer scheme was assessed by experimental tests. Experimental results show that the control-observer drives the tracking and observation errors near to the origin (ultimate boundedness).

The main advantages observed from the outcomes of this particular set of experiments are the following

- the proposed controller generates smoother control signals than the well-known PID controller.
- The proposed scheme does not saturate actuators.
- The implementation of the controller is simple in the sense that the knowledge of robot dynamic model and their parameters are not needed.

It remains as future work evaluate the performance of the proposed approach in bilateral teleoperation tasks.

ACKNOWLEDGEMENTS

This work is supported by the **DGAPA-UNAM** under grant **IN114617**.

REFERENCES

Arteaga-Pérez, M. (2003). Robot control and parameter estimation with only joint position measurements. *Automatica*, 39(1), 67–73.

Arteaga-Pérez, M.A., Castillo-Sánchez, A.M., and Parra-Vega, V. (2006). Cartesian control of robots without dynamic model and observer design. *Automatica*, 42, 473–480.

Bartolini, G., Pisano, A., Punta, E., and Usai, E. (2003). A survey of applications of second-order sliding mode control to mechanical systems. *International Journal of Control*, 76(9-10), 875–892.

Berghuis, H. and Nijmeijer, H. (1994). Robust control of robots via linear estimated state feedback. *IEEE Transactions on Automatic Control*, 39(20), 2159–2162.

Canudas de Wit, C. and Fixot, N. (1991). Robot control via robust estimated state feedback. *IEEE Transactions on Automatic Control*, 36(12), 1497–1501.

Fridman, L. (2011). *Sliding Mode Enforcement after 1990: Main Results and Some Open Problems*, volume 412 of *Lecture Notes in Control and Information Sciences*. Springer-Verlang.

Galicki, M. (2008). An adaptive regulator of robotic manipulators in the task space. *IEEE Transactions on Automatic Control*, 53(4), 1058–1062.

Kelly, R., Santibáñez, V., and Loria, A. (2005). *Control of robot manipulators in joint space*. Advanced textbooks in control and signal processing. Springer–Verlag.

Levant, A. (2005). Quasi-continuous high-order sliding-mode controllers. *IEEE Transactions on Automatic Control*, 50(11), 1812–1816.

Moreno, J. and Osorio, M. (2008). A lyapunov approach to second-order sliding mode controllers and observers. *IEEE Conference on Decision and Control*, 2856–2861.

Moreno, J. and Osorio, M. (2012). Strict lyapunov functions for the super-twisting algorithm. *IEEE Transactions on Automatic Control*, 57(4), 1035–1040.

Nicosia, S. and Tomei, P. (1990). Robot control by using only loint position measurements. *IEEE Transactions on Automatic Control*, 35(9), 1058–1061.

Parra-Vega, V., Arimoto, S., Liu, Y., Hirzinger, G., and Akella, P. (2003). Dynamic sliding pid control for tracking of robot manipulators: theory and experiments. *IEEE Transactions on Robotics and Automation*, 19(6), 967–976.

Slotine, J.J.E. and Li, W. (1987). On the adaptive control of robot manipulators. *International Journal of Robotics Research*, 6(3), 49–59.

Spong, M.W. (1992). On the robust control of robot manipulators. *IEEE Transactions on Automatic Control*, 37(11), 1782–1786.

Zhang, F., Dawson, D., de Queiroz, M., and Dixon, W. (2000). Global adaptive output feedback tracking control of robot manipulators. *IEEE Transactions on Automatic Control*, 45(6), 1203–1208.

