

# Robot Manipulation Control Using Jacobian Estimation.<sup>\*</sup>

Josué Gómez<sup>\*</sup> Chidentree Treesatayapun<sup>\*</sup> América Morales<sup>\*</sup>

<sup>\*</sup> *Robotics and Advanced Manufacturing Programm  
CINVESTAV-Salttillo*

*Ramos Arizpe, 25903, Mexico (Josuegomezcasa@gmail.com)*

**Abstract:** The high performance of robots nowadays is a key point to develop position tasks with greater precision. For this reason manipulators are going beyond to a simple routine in daily industrial life. However, sometimes is difficult to know the exact dynamic system and commonly the uncertainties are not included in the complete mathematical form of the robot model, therefore the use of estimation available information like control signal and robot tip position allows to know factors which can be considered difficult to extract from the model. In this paper, an adaptive Kalman filter is computed to estimate Jacobian based on available information of the robot to deal with the robot control in the task space. Although the entries to Jacobian estimate may vary in a non-linear way, the adaptive Kalman filter get a linear and reasonable way of the Jacobian entries to task space control. The results obtained by the simulation show that the adaptive Kalman filter expose higher quality performance than conventional Kalman filter by tuning the covariance matrix and a fading factor.

**Keywords:** robot manipulator, Jacobian estimate, adaptive Kalman filter, task space control

## 1. INTRODUCTION

Position controller for robot automation are widely used in industrial process. Therefore, robot manipulators have increased rapidly in multidisciplinary applications such as: painting, polishing, drilling, among others. They are more refined due to advances such as: fast response time, high precision, high speed motion and high level of performance as is mentioned by M. Al Mashagbh and M. Behrad, (2015). However, position control is not a simple task, during decades different controllers have been used kinematic and dynamic control and one of the main challenges in control robotic manipulator is to design controllers to achieve specified task through the robot tip. Conventionally, there are two frameworks proposed to perform robot manipulators, including model based methods and model free ones. Generally speaking, the industrial manipulators can be either torque or velocity controller and the majority of them are programmed to reach a specific position. On the other hand, limited for the accuracy of the kinematic and dynamic model, the model

based methods are not sufficient, so far. Furthermore, robotic manipulators are nonlinear systems with unknown and changeable parameters, therefore mathematical models of such systems are complex, and quite often exclude some physical phenomena, such as manipulators tip vibration or deformation which present a power transmission system problem, as was exposed by G. Piotr (2013). In the case of inaccurate unknown systems control, acceptable tools are artificial intelligence methods such as artificial neural networks and fuzzy systems, which are applied for manipulators nonlinearities compensation, see the work by M. Magdalena and G. Piotr, (2016). In such method the mathematical model of the manipulator is unnecessary and the incorporation of fuzzy logic and artificial neural network, known as fuzzy neural network (FNN) have caught the attention of several researchers as Lee *et al.*, (2000). The application of intelligent controllers in robotic systems allow to compensate the uncertain nonlinear dynamics. Recently, researches have presented the use of online adaptive FNN for robots, as Tresatayapun (2015) where a parallel structure for data input-output adaptive controllers

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was developed to control a 7 degree of freedom (DOF) robotic arm, the proposed controller is composed by an adaptive fuzzy rules emulated network (FREN), regarding the human knowledge of robotic arm. The use of intelligent controllers reduce the dependency of the kinematic and dynamic model. Nevertheless, the problem of achieving better performance of the robots can be divided into two parts, a modelling problem and an estimation problem. While modelling problem concerns with developing robust and accurate models, the estimation problem is regarded to a better proper use of the available process and measurement information use in controllers design. Conceptually, a good a priori knowledge of the process and measurement information depends on factors such as the type of application and the process dynamics, which are difficult to obtain. Also, the estimation environment in the case of robot kinematic applications is not always fixed but is subject to change, aforementioned by A. H. Mohamed *et al.*, (1999). The aim of this work is to implement a position control in the task space, for  $n$  DOF robot manipulator, with a Jacobian estimate based on adaptive Kalman filter, to accomplish desired position of robot end effector, using conventional proportional controller to porbe it. The information contained in analytical Jacobian is an approximation of the rate of change between joint position and robotic workspace. The proposal of Jacobian estimate is established for this paper takes advantage of the kinematic model and the principle idea is to utilize the Jacobian estimate coming through an adaptive Kalman filter for control design due to adaptive covariance matrix can capture the instantaneous change of the Jacobian and further improve the response of STKF. This filter only requires to be fed with tip robot velocity and joints velocity to make the computation of the Jacobian information. Moreover, the presented approach is able to relate the input signals of the actuators to the position and velocity of the robot tip directly. Thus, the actuation control is achieved together with the robotic control. The adaptive Kalman filter is characterized to enhance the robustness against uncertainties and work within the controller to compensate nonlinearities. The Jacobian estimate and controller were simulated for an omnidirectional robot manipulator, where 5 DOF are from the robotic arm and 3 DOF for the omnidirectional mobile platform. In Section 2 is presented the Jacobian estimation methodology. Section 3 provides the control design. In Section 4 simulation results are depicted. Finally, Section 5 closes with some conclusions.

## 2. ROBOTIC JACOBIAN ESTIMATOR

During this section is presented the development discrete-time Jacobian matrix for a robot and the estimation of the Jacobian matrix with an adaptive Kalman filter.

### 2.1 Discrete-time Jacobian matrix

The true state of a system is evolved from the state, the system is given by

$$\begin{aligned} x &= f_x(q_1, \dots, q_n) \\ y &= f_y(q_1, \dots, q_n) \\ z &= f_z(q_1, \dots, q_n) \end{aligned} \quad (1)$$

where  $[x, y, z]$  denotes the robot's end effector in 3-D coordinate,  $f_x$ ,  $f_y$  and  $f_z$  are geometric functions of joints position  $q_1, \dots, q_n$ . The equation (1) represents the end effector position in the task space of the robot, in terms of the joint position  $q_n$ . The movement of robot manipulator can be considered as a mapping of geometric Jacobian  $Jg(q)$ , commonly from the joint velocity  $\dot{q}(k)$  to the end effector pose  $\chi(q)$

$$\chi(q) = [p(q)] \quad (2)$$

where  $p(q)$  is the position, which denotes the robot's end effector in 3-D coordinate. In this work is just considered the position of the end effector in the equation (2).

the time derivative of end effector position reveals the velocity relationship and  $J_g(q) = J_A(q)$

$$\dot{\chi} = J_A(q)\dot{q} \quad (3)$$

in a practical control system, is usually simplified by the first order approximation within a single control interval as

$$\Delta\chi = J_A(q)\Delta q \quad (4)$$

The equation (4) can be expressed within discrete time domain

$$\frac{\chi(k+1) - \chi(k)}{Ts} = J_A(q) \left[ \frac{q(k) - q(k-1)}{Ts} \right] \quad (5)$$

where  $\Delta q = \omega(k)$  is the velocity of the joints,  $k$  is the time index and  $Ts$  is the sampling time, the equation (5) can be written as:

$$\Delta\chi = [Ts \cdot J_A(q(k))] \omega(k) \quad (6)$$

the state vector of  $x(k+1)$ ,  $y(k+1)$  and  $z(k+1)$  are formed by the entries of the robot Jacobian matrix  $J_A(q(k))$

$$[Ts \cdot J_A(q(k))] = \begin{bmatrix} \frac{\partial fx}{\partial q_1} & \dots & \dots & \frac{\partial fx}{\partial q_n} \\ \frac{\partial fy}{\partial q_1} & \dots & \dots & \frac{\partial fy}{\partial q_n} \\ \frac{\partial fz}{\partial q_1} & \dots & \dots & \frac{\partial fz}{\partial q_n} \end{bmatrix} \cdot Ts \quad (7)$$

and reorganizing the Jacobian matrix in equation (7) as a vector  $J_v(q(k))$ , since now is a vector state which is going to be estimated.

$$J_v(q(k)) = \left[ \frac{\partial fx}{\partial q_1} \dots \frac{\partial fx}{\partial q_n}, \frac{\partial fy}{\partial q_1} \dots \frac{\partial fy}{\partial q_n}, \frac{\partial fz}{\partial q_1} \dots \frac{\partial fz}{\partial q_n} \right] \quad (8)$$

The a measurement matrix  $H(k)$  in the equation (9) is a block diagonal matrix defined as

$$H(k) = \begin{bmatrix} [\omega_1(k) \dots \omega_n(k)] & & 0 \\ & \ddots & \\ 0 & & [\omega_1(k) \dots \omega_n(k)] \end{bmatrix} \quad (9)$$

the output of the system  $\Upsilon(k) = \frac{\Delta x}{T_s}$  represents the approximation of the end effector time derivative position.

$$\Upsilon(k) = H(k)J_v(q(k)) \quad (10)$$

equation (10) is developed for  $n$  DOF simplifying operations for  $\Delta x(k+1)$ ,  $\Delta y(k+1)$  and  $z(k+1)$  as,

$$\begin{aligned} \Delta x(k+1) &= \sum_{i=1}^n \omega_i(k) \cdot \frac{\partial fx(k)}{q_i(k)} \\ \Delta y(k+1) &= \sum_{i=1}^n \omega_i(k) \cdot \frac{\partial fy(k)}{q_i(k)} \\ \Delta z(k+1) &= \sum_{i=1}^n \omega_i(k) \cdot \frac{\partial fz(k)}{q_i(k)} \end{aligned} \quad (11)$$

## 2.2 Kalman Filter for Jacobian estimation

In this section is presented an adaptive Kalman filter, also known as Strong Tracking Kalman filter (STKF), based on the implementation of Minhan Li *et al.*, (2017). Then a Kalman filter is used

to observe the states of the system continuously. A reasonable estimation of the Jacobian at every control interval can be obtained by the following recursive formulation

$$\begin{aligned} K(k) &= P(k)H^T(k) [H(k)P(k)H^T(k)]^{-1} \\ P(k+1) &= [I - K(k)H(k)] P(k) \\ \hat{J}(k+1) &= \hat{J}(k) + K(k) [\Upsilon(k) - H(k)\hat{J}(k)] \end{aligned} \quad (12)$$

where  $\hat{J}(k)$  is the predicted Jacobian estimate,  $P(k)$  is the predicted error covariance,  $K(k)$  is the optimal Kalman gain,  $P(k+1)$  is the updated error covariance and  $\hat{J}(k+1)$  is the updated state estimate.

Note that, conventional Kalman filter heavily depends on the exact knowledge of the process and measurement models, or by the case of end effector velocity  $\Upsilon(k)$ , in the equation (10). However, in this study, the process model is a linear approximation. To overcome conventional Kalman filter degraded is proposed the concept of Strong Tracking Kalman Filter (STKF). Besides it is important to point out, That the STKF only requires the known values  $\omega(k)$  and  $\Upsilon(k)$  from the robot. The fading factor  $\lambda_k$  can be derived from the innovation sequence  $v_k = \Upsilon(k) - H(k)\hat{J}(k)$ , which guarantees the Jacobian is observable all time.

For STKF is needed to compute the fading factor  $\lambda_k$  and the updated covariance matrix  $Q_k$  is including in the updated error covariance matrix  $P(k+1)$ . This is depicted in (13)

$$P(k+1) = [I - K(k)H(k)P(k)] \lambda_k + Q_k \quad (13)$$

the value of  $\lambda_k$  is restricted for the value of  $C_k$  as is expressed in the equation (14)

$$\lambda_k = \begin{cases} C_k, & \text{when } C_k \geq 1 \\ 1, & \text{when } C_k < 1 \end{cases} \quad (14)$$

the value of  $C_k$  is in terms of the trace value of matrices  $M_k$  and  $N_k$

$$C_k = \frac{tr[N_k]}{tr[M_k]} \quad (15)$$

$M_k$  and  $N_k$  can be computed as (16)

$$\begin{aligned} N_k &= V_k - H(k)Q_kH(k)^T \\ M_k &= H(k)P(k)H(k)^T \end{aligned} \quad (16)$$

the matrix  $V_k$  is

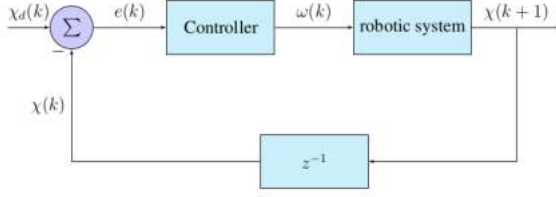


Fig. 1. Control block diagram

$$V_k = \begin{cases} v_0 v_0^T & k = 0 \\ \frac{0.95V_{k-1} + v_k v_k^T}{1.95} & k \geq 1 \end{cases} \quad (17)$$

the updated covariance matrix  $Q_k$  is

$$Q_k = K(k)\hat{C}(k)K(k) \quad (18)$$

and the estimated covariance of innovation matrix is presented as follow

$$\hat{C}(k) = \frac{1}{N} \sum_{n=k-N+1}^k v_n v_n^T \quad (19)$$

where the size of the matrices in table 1 are in agreement with the  $n$  DOF robot manipulator

Table 1. Parameters matrices for STKF

Variable	Name
$H(k)$	measurement matrix
$\hat{J}(k)$	state estimate
$P(k)$	error covariance
$K(k)$	optimal Kalman gain
$P(k+1)$	updated error covariance
$\hat{J}(k+1)$	updated state estimate
$N$	parameter matrix $Ck$
$M$	parameter matrix $Ck$
$Q$	updated covarianace matrix
$\hat{C}$	estimated covariance of innovation

### 3. CONTROLLER DESIGN

#### 3.1 Proposed controller

Fig. 1 depicts the block diagram for the controller, where  $\chi_d(k)$  is the desired position of the tip robot and  $\chi(k)$  is the current position in the task space. The proposed controller is a conventional proportional controller  $P$  and the output signal  $\omega(k)$  is introduced in the Jacobian estimate algorithm based on Kalman filter. Where the error is defined by the equation (20)

$$e(k) = \chi_d(k) - \chi(k) \quad (20)$$

and  $K_p$  is definite positive diagonal matrix that contains the control gains for the end effector position  $p(q)$ , see equation (21).

$$\dot{e}(k) = -K_p e(k) \quad (21)$$

the analytic Jacobian pseudoinverse  $J_A(q)$  is

$$J_A^+(q) = J_A^T(q)(J_A(q)J_A^T(q))^{-1} \quad (22)$$

now it is possible to calculate the signal of the controller when it is introduced the equations (21) and (22)

$$\omega(k) = J_A^+(q)\dot{e}(q) \quad (23)$$

and the updated control signal is

$$\omega(k+1) = q(k) + \omega(k) \cdot Ts \quad (24)$$

The signal in the equation (24) allows end effector update to reach the desired position.

#### 3.2 Algorithm for Jacobian estimation and controller

The complete process of the control scheme is given in Fig. 2. Within a control interval, the controller utilizes the estimated Jacobian  $\hat{J}(k)$ . By applying the control signal  $\omega(k)$  to joints robot, the robot tip will move to a new position  $\chi(k)$ . The Kalman filter will then produce the updated Jacobian  $\hat{J}(k+1)$  with respect to the measurements. By repeating the above steps, the robot will be controlled to follow a given desired  $\chi_d(k)$ . The use of STKF estimates the Jacobian vector  $\hat{J}(k+1)$  whose entries are considered the robot system states. Although the entries of the Jacobian may vary in a complex non linear way during motions, which can cause overpeaks damaging the actuators. The changes of the entries must be considered rational and linear through the computation of the STKF. The condition in the equation (25) needs to be accomplished to close the proposed algorithm

$$\hat{J}(k+1) \triangleq J_v \quad (25)$$

### 4. SIMULATION RESULTS

#### 4.1 Omnidirectional robot manipulator

Mobile manipulation, the seamless integration and synchronization of mobility and manipulation, is considered a key technology for professional service robotics as well as for future flexible manufacturing scenarios, as is referenced by R. Bischoff *et al.*, (2011). The combination and coordination of the mobility provided by a mobile

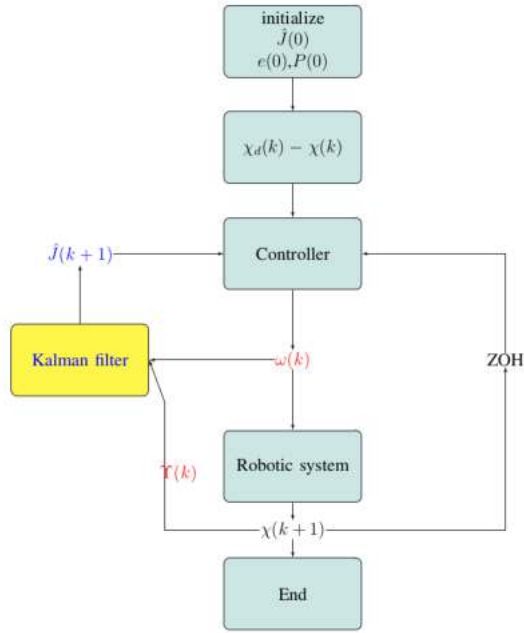


Fig. 2. The flowchart of the STKF for Jacobian estimate proposal.

platform and of the manipulation capabilities provided by a robot arm leads to complex analytical problems for research is mentioned by S. Sharma *et al.*, (2012). Our research is based on the Kuka youBot mobile manipulator, which has 3 DOF for omnidirectional mobile platform and 5 DOF for manipulator arm. The kinematic model parameters of the mobile manipulator are given according to the modified Denavit-Hartenberg convention, the coordinate frame transformation of the robot is shown in Fig. 3.

#### 4.2 Results

In this section is presented the results of the simulation of the Kuka youBot. The main idea is that the robot end effector reaches the desired position within work with the task space controller and the estimation of Jacobian by the STKF. A unified controller is presented for robot regulation and tracking. The proposed controller enable simple P control structure to guarantee the prescribed performance of end effector position error for both cases, either regulation or tracking. The first results presented are for regulation case. The Jacobian estimate was simulated for 8 DOF omnidirectional robot manipulator, controlling position of the end effector. The home position of the end effector is  $[0.1430, 0, 0.6480]$  and the desired position is  $[1, 0.75, 0.25]$ , moreover the control gain matrix  $K_p$  in equation (21), has a value of 0.7 for each position. The Fig. 4 depicts the results

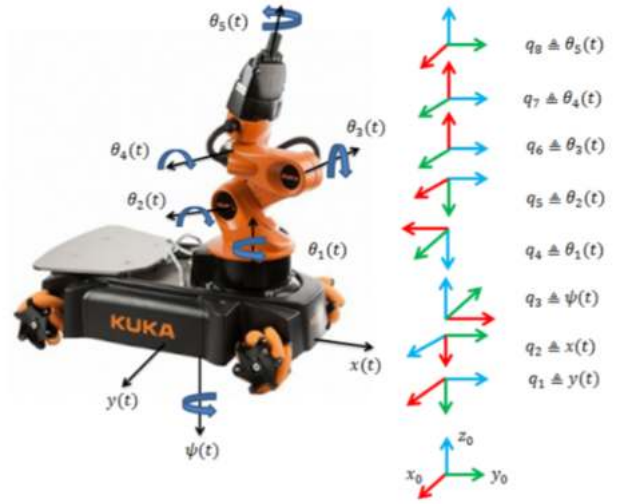


Fig. 3. Kuka youBot an omnidirectional mobile manipulator for research and education.

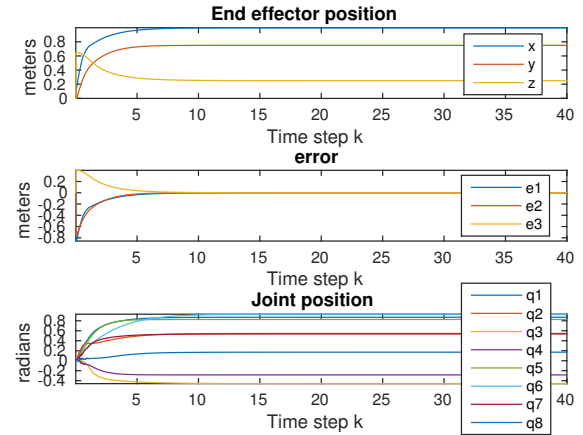


Fig. 4. Performance of desired position in the x, y and z directions during regulation process and including control error and joints position.

of the end effector reaching the desired position, the convergence of the error and the motion of the joints, while the end effector complete the control. During this simulation is used the  $\hat{J}$  and one can see the satisfactory response for the task space position control, under regulation case.

The Fig. 5 shows the behavior of the control to reach the desired position, simultaneously it is possible to appreciate joint velocities  $\omega_n(k)$  (or control signals) which lead to zero after a while. The behavior of the Jacobian estimate as

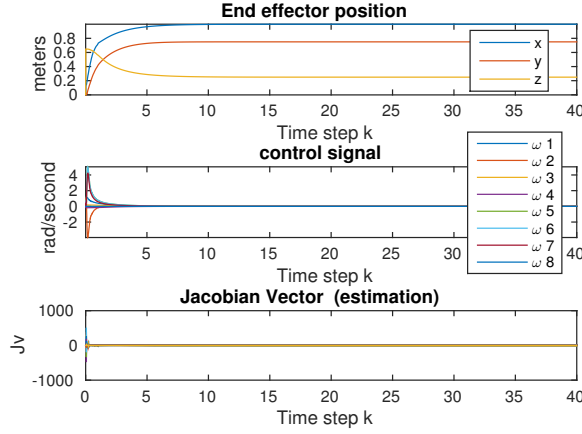


Fig. 5. Desired position in the x, y and z directions during regulation process and including control signal and Jacobian estimate.

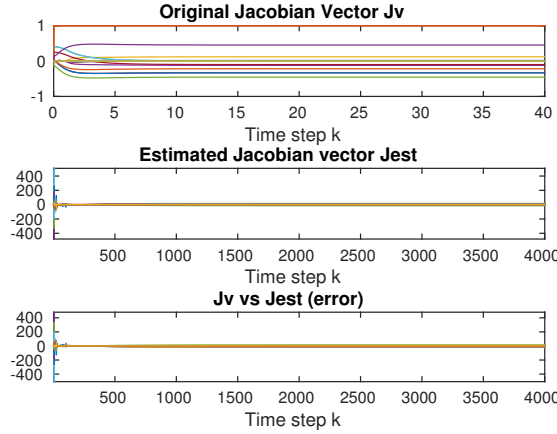


Fig. 6. Results of original Jacobian, Jacobian estimate and the convergence among both for tracking performance.

vector  $Jv$  it is assumed acceptable and working in agreement with the control.

A comparison among original Jacobian  $Jv$  and Jacobian estimate  $\hat{J}$  is portrait in Fig. 6. It is easy to observe an overshoot at the beginning of the simulation for Jacobian estimate, but the error between the real and estimate Jacobian is close to zero. Consequently, the STKF complies an acceptable estimation to control the robot.

The second results presented are for tracking case. The home position of the end effector is  $[0.1430, 0, 0.6480]$  and the desired position for tracking is given for the equation (26)

$$\begin{aligned} x_d(k) &= 0.5 \cdot \sin \left( 4 \cdot \pi \cdot \frac{k}{k_{max}} \right) \\ y_d(k) &= 0.5 \cdot \cos \left( 4 \cdot \pi \cdot \frac{k}{k_{max}} \right) \\ z_d(k) &= 0.25 \cdot \text{abs} \left( \sin \left( 4 \cdot \pi \cdot \frac{k}{k_{max}} + \frac{\pi}{4} \right) \right) \end{aligned} \quad (26)$$

Where  $Pd(k) = [x_d(k), y_d(k), z_d(k)]$  and the value of control gains is equal to 1.75. The tracking results are shown in Fig. 7 and 8. It is found that the controller is able to drive the Kuka youBot to follow a desired path and obtain satisfied results. However, the adaptation of the STKF should work in smooth way to avoid that the Jacobian changes dramatically. The convergence of the desired path is achieved by the robot end effector using the Jacobian estimate, so that the error among real Jacobian and estimated Jacobian throughout of the tracking path keep close to zero.

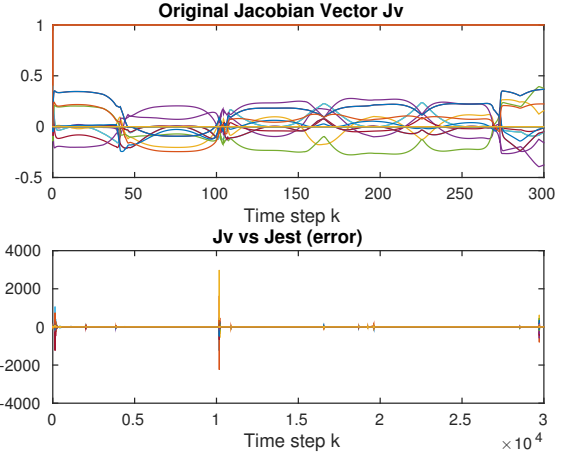


Fig. 7. Results of original Jacobian and the convergence among original Jacobian and Jacobian estimate.

The innovation sequence  $v_k$  for tracking is illustrated in Fig. 9, which is the error between the measured output  $\Upsilon(k)$  and the predicted output  $H(k)\hat{J}(k)$ . The use of adaptive covariance matrix and fading factor improve the ability of the STKF to follow the change in the Jacobian. What makes the error  $v_k$  decrease as is presented in the results.

## 5. CONCLUSION

Jacobian estimate is computed with available information of robot manipulator, the STKF only requires the control signals and measurement output of the robot to calculate a satisfactory performance of the proposed analytical Jacobian vector.

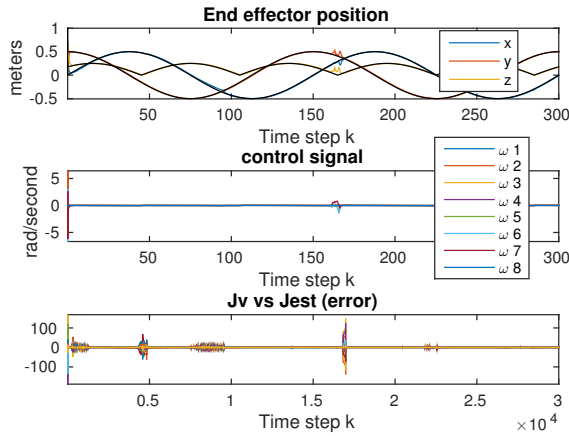


Fig. 8. Desired position in the x, y and z directions during tracking process, including control signal and Jacobian error

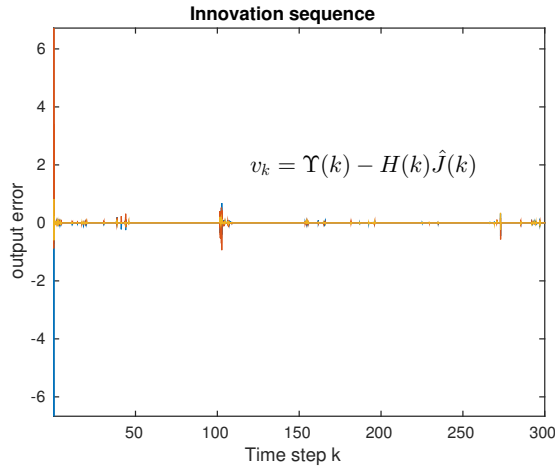


Fig. 9. Innovation sequence for tracking,  $v_k = Y(k) - H(k)\hat{J}(k)$ .

The work of this study was established within discrete time where Jacobian estimate vector performs acceptable agreement with controller, so it is possible to apply this proposal for  $n$  DOF robot manipulator. The results demonstrate that adaptive Kalman filter performs the estimation of Jacobian just tuning the use of variable covariance matrix and fading factor. The STKF has a good accuracy and convergence due to an adaptive covariance matrix can capture the instantaneous change of the Jacobian and further improve the tracking of STKF. Fading factor and covariance matrix are updated based on the difference error of the measured output and predicted output. The results presented by simulation through this paper are considered acceptable for regulation

and tracking purposes. The application of adaptive Kalman filter for the Jacobian estimation can be applied for different type of controllers. For future work is planning to analyze the stability of the STKF to guarantee the convergence of internal parameters, as well it is planned to extend the control of the entire robot's pose, including the orientation.

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