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A Fourier series based tracking controller for robot manipulators ⋆

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Abstract: In this document, we propose a non-linear PD tracking controller with adaptive Fourier series compensation. Asymptotic convergence of the position and velocity errors is proven. The proposed controller is validated on a 2-DOF robot manipulator. Experimental validation confirmed the results that were obtained theoretically.

Keywords: Fourier analysis, Robotic Manipulator, Adaptive Control.

1. INTRODUCTION

Feedfoward control is a conceptually simple strategy that uses the inverse dynamics of a given system evaluated along the reference trajectories as a control law (Kelly et al. (2005)). Nevertheless, to apply this kind of control strategy, one must have access to a precise model of the plant and all of its parameters. An improvement on this control strategy is given by the PD control plus feedforward scheme.

The dynamics of robot manipulators are usually difficult to model due to the existence of non-linearities and parametric uncertainties. Consequently, model based controllers do not always perform adequately. To circumvent this issue, a wide variety of techniques, such as neural networks and fuzzy systems, have been explored. Robust controllers are designed to work despite having a limited amount of information on the plant. Model-free controllers offer a robust framework to solve the tracking and regulation problem.

Regressor-based adaptive control, as it is studied in chapters 14, 15 and 16 of (Kelly et al. (2005)), is a very effective technique for achieving the control objective when parametric uncertainties are present in the plant. Nevertheless, a certain degree of knowledge regarding the structure of the mathematical model is required to implement this kind of controllers. On the other hand, regressor-free adaptive controllers do not need that degree of knowledge of the system. Approximation techniques, like Fourier series, allow us to implement regressor-free control and attain robustness even without complete knowledge of the system's dynamics.

A kind of Fourier series based controllers have been studied by several authors recently. For example, in Khorashadizadeh and Majidi (2017) Fourier series were used to solve the chaotic synchronization problem applied to communications. The Fourier controller was compared with a fuzzy system and it was determined that both of them exhibit a satisfactory performance. Nevertheless, it was stated that Fourier series are superior in terms of computational efficiency and simplicity. In Tsai and Huang (2008) a function approximation based controller was used to control a highly non-linear pneumatic servo system in the presence of a time-varying payload. It was found that the Fourier controller gave the system a high level of robustness. In Khorashadizadeh and Fateh (2013) Fourier series were used to estimate the uncertainty of a given model for an electrically driven robot manipulator. The voltage control strategy that was developed on the paper was shown to function adequately depending on the number of Fourier terms used. It was also found that every possible frequency of the uncertain signal should be covered by the Fourier terms for optimal performance.

In this document, we solve the tracking control problem for robot manipulators with unknown parameters and dynamics that make it difficult to implement the feedforward part of the controller. This is done by using Fourier series to approximate the unknown dynamics. We propose a non-linear PD tracking controller with adaptive Fourier series compensation. Asymptotic convergence of the position and velocity errors is proven and the control scheme is validated experimentally.

Regressor-free adaptive control based on Fourier series offers a simple, and computationally efficient, alternative to the more common model-free controllers that use neural networks like the one described in Puga-Guzmán et al. (2014).

2. PRELIMINARIES

The maximum and minimum eigenvalues of the matrix M are denoted by $\lambda_{\text{Max}}\{M\}$ and $\lambda_{\min}\{M\}$ respectively. The norm of $\boldsymbol{x} \in \mathbb{R}^n$ is denoted by $\|\boldsymbol{x}\|$.

The hyperbolic tangent function is defined as

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}. \tag{1}$$

Throughout this document the following notation is used to denote the hyperbolic tangent of every element of a

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vector $\boldsymbol{x} \in \mathbb{R}^n$.

$$\mathbf{tanh}(\boldsymbol{x}) = \begin{bmatrix} \tanh(x_1) \\ \tanh(x_2) \\ \vdots \\ \tanh(x_n) \end{bmatrix} \tag{2}$$

The sign function is defined as

$$\text{sign}(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases} . \tag{3}$$

As in the definition of $\mathbf{tanh}(\boldsymbol{x})$, $\mathbf{sign}(\boldsymbol{x})$ is used to denote

$$\mathbf{sign}(\boldsymbol{x}) = \begin{bmatrix} \text{sign}(x_1) \\ \text{sign}(x_2) \\ \vdots \\ \text{sign}(x_n) \end{bmatrix} . \tag{4}$$

2.1 Robot dynamics

The dynamics of a n-link robot manipulator, in joint space, considering the presence of viscous friction at the robot joints can be written as (Kelly et al. (2005)):

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + C(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) + F_v\dot{\boldsymbol{q}} = \boldsymbol{\tau} \tag{5}$$

where $\boldsymbol{q} \in \mathbb{R}^n$ is the vector of joint positions, $M(\boldsymbol{q})$ is the symmetric positive definite inertia matrix $C(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} \in \mathbb{R}^n$ is the vector of centripetal and Coriolis torques, $\boldsymbol{g}(\boldsymbol{q}) \in \mathbb{R}^n$ is the vector of gravitational torques, $F_v \in \mathbb{R}^{n \times n}$ is a diagonal positive definite matrix containing the viscous friction coefficients of the joints, and $\boldsymbol{\tau} \in \mathbb{R}^n$ is the vector of torque inputs.

The following properties hold for robot manipulators with revolute joints (Kelly et al. (2005)).

Property 1. The inertia matrix $M(\boldsymbol{q}) \in \mathbb{R}^{n \times n}$ is a symmetric and positive definite matrix. Therefore, because of the Rayleigh-Ritz theorem, it holds that

$$\lambda_{\min}\{M(\boldsymbol{q})\}\|\boldsymbol{x}\|^2 \le \boldsymbol{x}^T M(\boldsymbol{q})\boldsymbol{x} \le \lambda_{\text{Max}}\{M(\boldsymbol{q})\}\|\boldsymbol{x}\|^2 \tag{6}$$

for all $\boldsymbol{x} \in \mathbb{R}^n$

Property 2. The vector of centripetal and Coriolis torques satisfies

$$\|C(\boldsymbol{q}, \boldsymbol{x})\boldsymbol{y}\| \le k_{C_1}\|\boldsymbol{x}\|\|\boldsymbol{y}\| \tag{7}$$

for all $\boldsymbol{q}, \boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^n$ where $k_{C_1} > 0$ is a positive constant.

Property 3. The matrix

$$\frac{1}{2}\dot{M}(\boldsymbol{q}) - C(\boldsymbol{q}, \dot{\boldsymbol{q}}) \tag{8}$$

is skew-symmetric. It is also true that

$$\dot{M}(\boldsymbol{q}) = C(\boldsymbol{q}, \dot{\boldsymbol{q}}) + C(\boldsymbol{q}, \dot{\boldsymbol{q}})^T. \tag{9}$$

Property 4. The residual dynamics term $\boldsymbol{h}(t, \tilde{\boldsymbol{q}}, \dot{\tilde{\boldsymbol{q}}})$ is defined as

$$\begin{aligned} \boldsymbol{h}(t, \tilde{\boldsymbol{q}}, \dot{\tilde{\boldsymbol{q}}}) &= [M(\boldsymbol{q}_d) - M(\boldsymbol{q}_d - \tilde{\boldsymbol{q}})]\ddot{\boldsymbol{q}}_d \\ &+ [C(\boldsymbol{q}_d, \dot{\boldsymbol{q}}_d) - C(\boldsymbol{q}_d - \tilde{\boldsymbol{q}}, \dot{\boldsymbol{q}}_d - \dot{\tilde{\boldsymbol{q}}}_d)]\dot{\boldsymbol{q}}_d \\ &+ \boldsymbol{g}(\boldsymbol{q}_d) - \boldsymbol{g}(\boldsymbol{q}_d - \tilde{\boldsymbol{q}}) \end{aligned} \tag{10}$$

where

$$\tilde{\boldsymbol{q}} = \boldsymbol{q}_d - \boldsymbol{q}. \tag{11}$$

There exist constants $k_{h_1} \ge 0$ and $k_{h_2} \ge 0$ such that

$$\|\boldsymbol{h}(t, \tilde{\boldsymbol{q}}, \dot{\tilde{\boldsymbol{q}}})\| \le k_{h_1}\|\dot{\tilde{\boldsymbol{q}}}\| + k_{h_2}\|\mathbf{tanh}(\tilde{\boldsymbol{q}})\|, \tag{12}$$

for all $\tilde{\boldsymbol{q}}, \dot{\tilde{\boldsymbol{q}}} \in \mathbb{R}^n$.

2.2 Fourier series

A trigonometric polynomial is a sum that has the form:

$$f(x) = a_0 + \sum_{m=1}^{l}(a_m \cos mx + b_m \sin mx). \tag{13}$$

If the sum is infinite, it is called a trigonometric series. If $f(x)$ is an integrable function on $[-\pi, \pi]$, the sum is called a Fourier series.

Trigonometric polynomials have interesting properties related to the approximation of functions. This is due to the fact that the space of real valued trigonometric polynomials is a unital subalgebra of the space of continuous functions that separates points. As a consequence of the Stone-Weierstrass theorem, the set of all trigonometric polynomials is dense in the space of continuous functions defined on a compact set (see Rudin (1976)).

An interesting property of trigonometric polynomials is given by the following theorem (Wilcox and Myers (2009)).

Let f be a square integrable funtion on the interval $[-\pi, \pi]$ and s_n be the n^{th} partial sum of the Fourier series for f. Then s_n converges to f in the $\mathcal{L}_2$ norm. That is, given any $\epsilon > 0$, there exists an N such that $n > N$ implies $\|s_n - f\|_2 < \epsilon$.

This theorem highlights the fact that when using trigonometric polynomials to approximate a function, the approximation error can be made arbitrarily small by adding terms to the polynomial.

3. PROPOSED CONTROLLER

First, we define

$$\boldsymbol{z}(t) = M(\boldsymbol{q}_d)\ddot{\boldsymbol{q}}_d + C(\boldsymbol{q}_d, \dot{\boldsymbol{q}}_d)\dot{\boldsymbol{q}}_d + \boldsymbol{g}(\boldsymbol{q}_d) + F_v\dot{\boldsymbol{q}}_d, \tag{14}$$

that is, the dynamics of the robot manipulator evaluated on the desired trajectories.

The so called feedforward control law $\boldsymbol{\tau}_{ff}$ has a relatively simple structure,

$$\boldsymbol{\tau}_{ff} = \boldsymbol{z}(t). \tag{15}$$

Nevertheless, this control law requires the exact knowledge of the plant parameters plus a PD action to function adequately (see chapter 12 of Kelly et al. (2005)). In order for us to find a way around this problem, we first substract (5) from (14) which gives us:

$$M(\boldsymbol{q})\ddot{\tilde{\boldsymbol{q}}} + C(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\tilde{\boldsymbol{q}}} + \boldsymbol{h}(t, \tilde{\boldsymbol{q}}, \dot{\tilde{\boldsymbol{q}}}) + F_v\dot{\tilde{\boldsymbol{q}}} = \boldsymbol{z}(t) - \boldsymbol{\tau}, \tag{16}$$

where we have defined the tracking error as

$$\tilde{\boldsymbol{q}} = \boldsymbol{q}_d - \boldsymbol{q}. \tag{17}$$

The function $\boldsymbol{z}(t)$ can be approximated by a finite number of terms from the Fourier series. Thus, we may write

$$\boldsymbol{z}(t) = W^T\boldsymbol{\phi}(t) + \boldsymbol{\epsilon}, \tag{18}$$

in which $W \in \mathbb{R}^{N \times n}$ is the coefficient matrix for the Fourier terms, $\boldsymbol{\phi} \in \mathbb{R}^N$ is a vector containing the first N terms (where N is odd) of the Fourier series, and

$\boldsymbol{\epsilon} \in \mathbb{R}^n$ is the approximation error of the series, that is, $\boldsymbol{\epsilon} = [\epsilon_1 \quad \epsilon_2 \quad \dots \quad \epsilon_n]^T$.

For the general case, the structure of $\boldsymbol{\phi}(t)$ and W may be written as

$$\boldsymbol{\phi}(t) = \begin{bmatrix} 1 \\ \sin(\omega t) \\ \cos(\omega t) \\ \sin(2\omega t) \\ \cos(2\omega t) \\ \vdots \\ \sin\left(\frac{N-1}{2}\omega t\right) \\ \cos\left(\frac{N-1}{2}\omega t\right) \end{bmatrix} \in \mathbb{R}^N,$$

where $\omega \in \mathbb{R}$ is a constant value that can be used as a tuning parameter (Khorashadizadeh and Majidi (2017)), and

$$W^T = \begin{bmatrix} a_{01} & a_{11} & b_{11} & a_{21} & b_{21} & \dots & a_{\frac{N-1}{2}1} & b_{\frac{N-1}{2}1} \\ a_{02} & a_{12} & b_{12} & a_{22} & b_{22} & \dots & a_{\frac{N-1}{2}2} & b_{\frac{N-1}{2}2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ a_{0n} & a_{1n} & b_{1n} & a_{2n} & b_{2n} & \dots & a_{\frac{N-1}{2}n} & b_{\frac{N-1}{2}n} \end{bmatrix} \in \mathbb{R}^{n\times N}$$

The proposed control law has the form

$$\boldsymbol{\tau} = \widehat{W}^T\boldsymbol{\phi}(t) + K_p\mathbf{tanh}(\gamma\tilde{\boldsymbol{q}}) + K_d\dot{\tilde{\boldsymbol{q}}} + \Delta\mathbf{sign}(\boldsymbol{r}), \quad (19)$$

with

$$\boldsymbol{r} = \dot{\tilde{\boldsymbol{q}}} + \beta\mathbf{tanh}(\gamma\tilde{\boldsymbol{q}}), \quad (20)$$

where $K_p,\ K_d,\ \Delta \in \mathbb{R}^{n\times n}$ are diagonal positive definite matrices, $\gamma, \beta \in \mathbb{R}^+$ are positive constants, and $\widehat{W}$ is the matrix of estimated Fourier coefficients.

Under control law (19), equation (16) can be written as

$$M(\boldsymbol{q})\ddot{\tilde{\boldsymbol{q}}} + C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\tilde{\boldsymbol{q}}} + \boldsymbol{h}(t,\tilde{\boldsymbol{q}},\dot{\tilde{\boldsymbol{q}}}) + F_v\dot{\tilde{\boldsymbol{q}}} = W^T\boldsymbol{\phi}(t)$$
$$+\boldsymbol{\epsilon} - \widehat{W}^T\boldsymbol{\phi}(t) - K_p\mathbf{tanh}(\gamma\tilde{\boldsymbol{q}}) - K_d\dot{\tilde{\boldsymbol{q}}} - \Delta\mathbf{sign}(\boldsymbol{r}). \quad (21)$$

The coefficients of the matrix $\widehat{W}$ are updated according to the following equation:

$$\dot{\widehat{W}} = \Gamma\boldsymbol{\phi}(t)\boldsymbol{r}^T \quad (22)$$

where $\Gamma \in \mathbb{R}^{N\times N}$ is a symmetric positive definite matrix.

The error between the ideal coefficients and the estimated ones is denoted by

$$\widetilde{W} = W - \widehat{W}. \quad (23)$$

3.1 Convergence of the error dynamics

To study the convergence of the position and velocity errors the following assumptions are made.

Assumption 1. The vector of desired trajectories $\boldsymbol{q}_d$, and the derivatives $\dot{\boldsymbol{q}}_d$ and $\ddot{\boldsymbol{q}}_d$ are continuous and their norm is bounded.

Assumption 2. There exists a constant $k_\epsilon \geq 0$ such that $k_\epsilon \geq \|\boldsymbol{\epsilon}\|$ (Khorashadizadeh and Majidi (2017)).

Now, consider the Lyapunov function

$$V(t,\tilde{\boldsymbol{q}},\dot{\tilde{\boldsymbol{q}}},\widetilde{W}) = \frac{1}{2}\dot{\tilde{\boldsymbol{q}}}^T M\dot{\tilde{\boldsymbol{q}}} + \alpha\mathbf{tanh}(\gamma\tilde{\boldsymbol{q}})^T M\dot{\tilde{\boldsymbol{q}}}$$
$$+\sum_{i=1}^{n} k_{p_i}\gamma^{-1}\ln(\cosh(\gamma\tilde{q}_i)) + \frac{1}{2}T_r(\widetilde{W}^T\Gamma^{-1}\widetilde{W}) \quad (24)$$

which is a positive definite function where k_{p_i} is the i-th element of the diagonal K_p matrix and $\tilde{q}_i$ is the i-th element of the vector $\tilde{\boldsymbol{q}}$. By using the properties of the inertia matrix we can see that

$$V(t,\tilde{\boldsymbol{q}},\dot{\tilde{\boldsymbol{q}}},\widetilde{W}) \geq \begin{bmatrix} \|\dot{\tilde{\boldsymbol{q}}}\| \\ \|\mathbf{tanh}(\gamma\tilde{\boldsymbol{q}})\| \end{bmatrix}^T P \begin{bmatrix} \|\dot{\tilde{\boldsymbol{q}}}\| \\ \|\mathbf{tanh}(\gamma\tilde{\boldsymbol{q}})\| \end{bmatrix}$$
$$+\frac{1}{2}T_r(\widetilde{W}^T\Gamma^{-1}\widetilde{W}) \quad (25)$$

with

$$P = \begin{bmatrix} \frac{1}{2}\lambda_{\min}\{M\} & \frac{\alpha}{2}\lambda_{\mathrm{Max}}\{M\} \\ \frac{\alpha}{2}\lambda_{\mathrm{Max}}\{M\} & \frac{\gamma^{-1}}{2}\lambda_{\min}\{K_p\} \end{bmatrix}. \quad (26)$$

For P to be positive definite, by Sylvester's theorem, we must be able to select α such that

$$0 < \alpha < \frac{\sqrt{\gamma^{-1}\lambda_{\min}\{K_p\}\lambda_{\min}\{M\}}}{\lambda_{\mathrm{Max}}\{M\}}. \quad (27)$$

Since the right-hand side of (27) is comprised entirely by positive constants, it is always possible to chose an α that satisfies the inequality (because of the density property of the real numbers). Notice that (24) is radially unbounded because of (25).

Next, we compute the time derivative of (24).

$$\dot{V}(t,\tilde{\boldsymbol{q}},\ \dot{\tilde{\boldsymbol{q}}},\widetilde{W}) = \frac{1}{2}\dot{\tilde{\boldsymbol{q}}}^T\dot{M}\dot{\tilde{\boldsymbol{q}}} + \dot{\tilde{\boldsymbol{q}}}^T M\ddot{\tilde{\boldsymbol{q}}} + \alpha\gamma\dot{\tilde{\boldsymbol{q}}}^T\mathrm{Sech}^2(\gamma\tilde{\boldsymbol{q}})M\dot{\tilde{\boldsymbol{q}}}$$
$$+\alpha\mathbf{tanh}(\gamma\tilde{\boldsymbol{q}})^T\dot{M}\dot{\tilde{\boldsymbol{q}}} + \alpha\mathbf{tanh}(\gamma\tilde{\boldsymbol{q}})^T M\ddot{\tilde{\boldsymbol{q}}} \quad (28)$$
$$+\mathbf{tanh}(\gamma\tilde{\boldsymbol{q}})^T K_p\dot{\tilde{\boldsymbol{q}}} + T_r(\widetilde{W}^T\Gamma^{-1}\dot{\widetilde{W}}).$$

The matrix $\mathrm{Sech}^2(\boldsymbol{x})$ is defined as in (Kelly et al. (2005)), that is

$$\mathrm{Sech}^2(\boldsymbol{x}) = \begin{bmatrix} \mathrm{sech}^2(x_1) & 0 & \dots & 0 \\ 0 & \mathrm{sech}^2(x_1) & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & \mathrm{sech}^2(x_n) \end{bmatrix}. \quad (29)$$

By using property 3 of the previous section and equations (20) and (21) we can write (28) as

$$\dot{V}(t,\tilde{\boldsymbol{q}},\ \dot{\tilde{\boldsymbol{q}}},\widetilde{W}) = -\dot{\tilde{\boldsymbol{q}}}^T[F_v + K_d]\dot{\tilde{\boldsymbol{q}}} - \boldsymbol{r}^T\boldsymbol{h} - \boldsymbol{r}^T\widetilde{W}^T\boldsymbol{\phi} \quad (30)$$
$$+\alpha\gamma\dot{\tilde{\boldsymbol{q}}}^T\mathrm{Sech}^2(\gamma\tilde{\boldsymbol{q}})M\dot{\tilde{\boldsymbol{q}}} + \alpha\mathbf{tanh}(\gamma\tilde{\boldsymbol{q}})^T C^T\dot{\tilde{\boldsymbol{q}}}$$
$$-\alpha\mathbf{tanh}(\gamma\tilde{\boldsymbol{q}})^T[F_v + K_d]\dot{\tilde{\boldsymbol{q}}} + \boldsymbol{r}^T[\boldsymbol{\epsilon} - \Delta\mathbf{sign}(\boldsymbol{r})]$$
$$-\alpha\mathbf{tanh}(\gamma\tilde{\boldsymbol{q}})^T K_p\mathbf{tanh}(\gamma\tilde{\boldsymbol{q}}) + T_r(\widetilde{W}^T\Gamma^{-1}\dot{\widetilde{W}}).$$

To further simplify (30) we use the properties that the trace of a matrix possesses (Ioannou and Sun (2012)),

$$T_r(A+B) = T_r(A) + T_r(B) \quad (31)$$
$$T_r(yx^T) = x^T y, \quad (32)$$

and equation (22)

$$\dot{V}(t,\tilde{\boldsymbol{q}},\dot{\tilde{\boldsymbol{q}}},\widetilde{W}) = -\dot{\tilde{\boldsymbol{q}}}^T[F_v+K_d]\dot{\tilde{\boldsymbol{q}}} - \boldsymbol{r}^T\boldsymbol{h} + \boldsymbol{r}^T[\boldsymbol{\epsilon} - \Delta\mathbf{sign}(\boldsymbol{r})] + \alpha\gamma\dot{\tilde{\boldsymbol{q}}}^T\mathrm{Sech}^2(\gamma\tilde{\boldsymbol{q}})M\dot{\tilde{\boldsymbol{q}}} + \alpha\mathbf{tanh}(\gamma\tilde{\boldsymbol{q}})^T C^T\dot{\tilde{\boldsymbol{q}}} - \alpha\mathbf{tanh}(\gamma\tilde{\boldsymbol{q}})^T[F_v+K_d]\dot{\tilde{\boldsymbol{q}}} - \alpha\mathbf{tanh}(\gamma\tilde{\boldsymbol{q}})^T K_p\mathbf{tanh}(\gamma\tilde{\boldsymbol{q}}). \quad (33)$$

To continue with the analysis we look for an upper bound on (33). To this end, notice that from the Rayleigh-Ritz theorem we have

$$-\dot{\tilde{\boldsymbol{q}}}^T[K_d+F_v]\dot{\tilde{\boldsymbol{q}}} \le -\lambda_{\min}\{K_d+F_v\}\|\dot{\tilde{\boldsymbol{q}}}\|^2 \quad (34)$$

and

$$-\alpha\mathbf{tanh}(\gamma\tilde{\boldsymbol{q}})^T K_p\mathbf{tanh}(\gamma\tilde{\boldsymbol{q}}) \le -\alpha\lambda_{\min}\{K_p\}\|\mathbf{tanh}(\gamma\tilde{\boldsymbol{q}})\|^2. \quad (35)$$

By using the Cauchy-Schwarz theorem we may write

$$-\alpha\mathbf{tanh}(\gamma\tilde{\boldsymbol{q}})^T[F_v+K_d]\dot{\tilde{\boldsymbol{q}}} \le \alpha\lambda_{\mathrm{Max}}\{K_d+F_v\}\|\dot{\tilde{\boldsymbol{q}}}\|\|\mathbf{tanh}(\gamma\tilde{\boldsymbol{q}})\|. \quad (36)$$

From property 2 of the preliminaries section, and using the triangle inequality we can state that

$$\alpha\mathbf{tanh}(\gamma\tilde{\boldsymbol{q}})^T C^T\dot{\tilde{\boldsymbol{q}}} \le \alpha k_{C_1}\|\dot{\boldsymbol{q}}_d\|_M\|\dot{\tilde{\boldsymbol{q}}}\|\|\mathbf{tanh}(\gamma\tilde{\boldsymbol{q}})\| + \alpha k_{C_1}\sqrt{n}\|\dot{\tilde{\boldsymbol{q}}}\|^2, \quad (37)$$

in which $\|\dot{\boldsymbol{q}}_d\|_M$ is meant to denote an upper bound on the norm of the desired velocity vector.

Taking into account property 4 we can write

$$-\boldsymbol{r}^T\boldsymbol{h} \le \alpha k_{h_2}\|\mathbf{tanh}(\gamma\tilde{\boldsymbol{q}})\|^2 + [\alpha k_{h_1}+k_{h_2}]\|\mathbf{tanh}(\gamma\tilde{\boldsymbol{q}})\|\|\dot{\tilde{\boldsymbol{q}}}\| + k_{h_1}\|\dot{\tilde{\boldsymbol{q}}}\|^2. \quad (38)$$

Since the maximum value of $\mathrm{sech}(x)$ is 1, we have $\lambda_{\mathrm{Max}}\{\mathrm{Sech}^2(\gamma\tilde{\boldsymbol{q}})\} = 1$. Therefore,

$$\alpha\gamma\dot{\tilde{\boldsymbol{q}}}^T\mathrm{Sech}^2(\gamma\tilde{\boldsymbol{q}})M\dot{\tilde{\boldsymbol{q}}} \le \alpha\gamma\lambda_{\mathrm{Max}}\{M\}\|\dot{\tilde{\boldsymbol{q}}}\|^2. \quad (39)$$

As a consequence of the equivalence of norms ($\|\boldsymbol{r}\|_2 \le \|\boldsymbol{r}\|_1$), assumption 2, and the Cauchy-Schwarz inequality we have

$$\boldsymbol{r}^T[\boldsymbol{\epsilon} - \Delta\mathbf{sign}(\boldsymbol{r})] \le -[\lambda_{\min}\{\Delta\} - k_\epsilon]\sum_{i=1}^{n}|r_n|. \quad (40)$$

Finally, putting together all the previously presented inequalities and rearranging the terms in matrices as it is done by Puga-Guzmán et al. (2014), we may write

$$\dot{V}(t,\tilde{\boldsymbol{q}},\dot{\tilde{\boldsymbol{q}}},\widetilde{W}) \le -\frac{1}{2}\begin{bmatrix}\|\dot{\tilde{\boldsymbol{q}}}\| \\ \|\mathbf{tanh}(\gamma\tilde{\boldsymbol{q}})\|\end{bmatrix}^T \Omega_1 \begin{bmatrix}\|\dot{\tilde{\boldsymbol{q}}}\| \\ \|\mathbf{tanh}(\gamma\tilde{\boldsymbol{q}})\|\end{bmatrix} - [\lambda_{\min}\{\Delta\} - k_\epsilon]\sum_{i=1}^{n}|r_n| \quad (41)$$

with

$$\Omega_1 = \begin{bmatrix}\alpha s_1 & -\frac{1}{2}[\alpha s_4 + s_5] \\ -\frac{1}{2}[\alpha s_4 + s_5] & s_2 - \alpha s_3\end{bmatrix} \quad (42)$$

and

$$\begin{aligned} s_1 &= \lambda_{\min}\{K_p\} - k_{h_2} \\ s_2 &= \lambda_{\min}\{K_d+F_v\} - k_{h_1} \\ s_3 &= \gamma\lambda_{\mathrm{Max}}\{M\} + k_{C_1}\sqrt{n} \\ s_4 &= k_{h_2} + k_{C_1}\|\dot{\boldsymbol{q}}_d\|_M + \lambda_{\mathrm{Max}}\{K_d+F_v\} \\ s_5 &= k_{h_1}. \end{aligned} \quad (43)$$

For Ω_1 to be a positive definite matrix we impose the restrictions

$$\lambda_{\min}\{K_p\} > k_{h_2} \implies s_1 > 0 \quad (44)$$

$$\lambda_{\min}\{K_d+F_v\} > k_{h_1} \implies s_2 > 0. \quad (45)$$

And by Sylvester's theorem, Ω_1 is positive definite if there exists α such that

$$\frac{2s_1s_2 - s_4s_5}{s_4^2+4s_1s_3} - 2\sqrt{\frac{s_1^2s_2^2 - s_1s_4s_5s_2 - s_1s_3s_5^2}{(s_4^2+4s_1s_3)^2}} < \alpha < \frac{2s_1s_2 - s_4s_5}{s_4^2+4s_1s_3} + 2\sqrt{\frac{s_1^2s_2^2 - s_1s_4s_5s_2 - s_1s_3s_5^2}{(s_4^2+4s_1s_3)^2}}, \quad (46)$$

which will always be true if

$$s_1 > \frac{s_4s_5}{s_2} \quad (47)$$

and

$$s_1 > \frac{s_3s_5^2 + s_2s_4s_5}{s_2^2}. \quad (48)$$

Accordingly, the derivative $\dot{V}(t,\tilde{\boldsymbol{q}},\dot{\tilde{\boldsymbol{q}}},\widetilde{W})$ will be negative semi-definite if K_p is chosen big enough and

$$\lambda_{\min}\{\Delta\} > k_\epsilon. \quad (49)$$

Now, since (24) is positive definite and radially unbounded, every level set $V(t,\tilde{\boldsymbol{q}},\dot{\tilde{\boldsymbol{q}}},\widetilde{W}) = c$ is compact. From the fact that $V(t,\tilde{\boldsymbol{q}},\dot{\tilde{\boldsymbol{q}}},\widetilde{W})$ is a decreasing function of time, we can conclude that every solution of the system of differential equations (21)-(22) is bounded on their maximal interval of definition.

Next, notice that by taking the integral of (41) we obtain

$$\int_0^t\|\dot{\tilde{\boldsymbol{q}}}\|^2dt + \int_0^t\|\mathbf{tanh}(\gamma\tilde{\boldsymbol{q}})\|^2 \le \frac{V(0,\tilde{\boldsymbol{q}}(0),\dot{\tilde{\boldsymbol{q}}}(0),\widetilde{W}(0))}{\lambda_{\min}\{\Omega_1\}}. \quad (50)$$

Therefore, $\tilde{\boldsymbol{q}},\dot{\tilde{\boldsymbol{q}}} \in \mathcal{L}_\infty^n$ and $\tilde{\boldsymbol{q}},\dot{\tilde{\boldsymbol{q}}} \in \mathcal{L}_2^n$. Since every element in $\widehat{W}$ is bounded and $\boldsymbol{\phi}(t) \in \mathcal{L}_\infty^N$ we can conclude from (21) that $\ddot{\tilde{\boldsymbol{q}}} \in \mathcal{L}_\infty^n$. As a consequence of Lemma A.5 of Kelly et al. (2005) (Barbalat's lemma for the vector case), we have

$$\lim_{t\to\infty}\|\tilde{\boldsymbol{q}}\| = 0 \quad (51)$$

$$\lim_{t\to\infty}\|\dot{\tilde{\boldsymbol{q}}}\| = 0. \quad (52)$$

4. EXPERIMENTAL VALIDATION

The experimental setup is a 2-DOF revolute joint robot arm located at Instituto Tecnlgico de La Laguna, Mexico, previously used in (Zavala-Río et al. (2015)). The robot actuators are direct-drive brushless servomotors operated

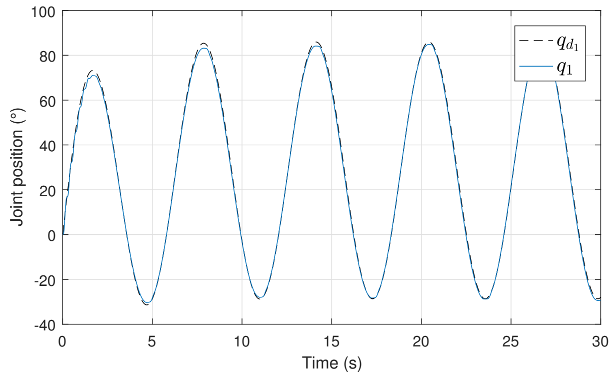

Fig. 1. Position and desired position for the first link

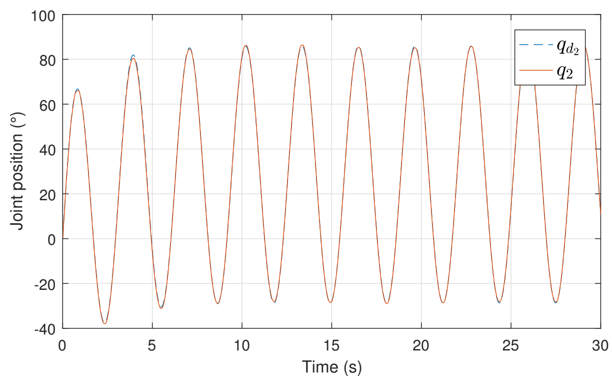

Fig. 2. Position and desired position for the second link

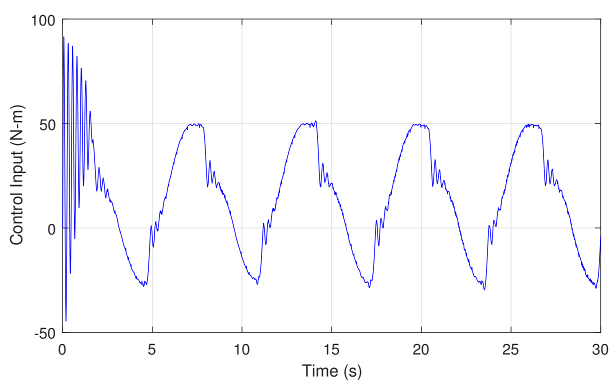

Fig. 3. Control input for the first link

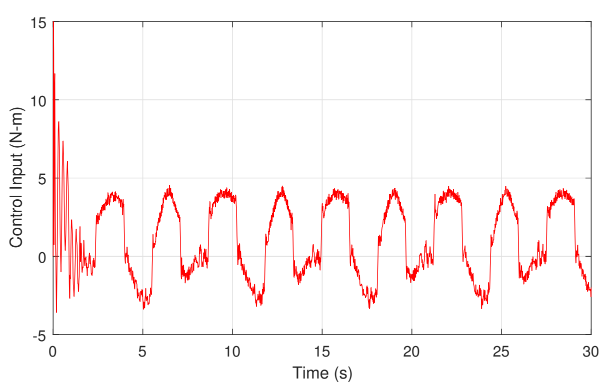

Fig. 4. Control input for the second link

Fig. 5. 2-DOF robot manipulator used in the experiments

in torque mode. That is, they act as torque sources and receive analogue voltages as a torque reference signal. Joint positions are obtained using incremental encoders which send the information to a DSP 32-bit microprocessor. The control algorithm is executed at a 2.5 millisecond sampling period on a PC-host computer.

The selected tuning gains are presented in table 1. The adaptation gain Γ was selected to be Γ =diag{200}. For each link we used the first five terms of the Fourier series.

The desired joint positions were chosen as

$$q_{d_1} = \sin(t) + 0.5(1 - e^{-0.5t})$$
$$q_{d_2} = \sin(2t) + 0.5(1 - e^{-0.5t})$$

Figures 1 and 2 show the position and desired position of the first and second links respectively. Figures 2 and 3 show the demanded control input. Figure 5 shows the experimental setup.

Experimental results show an adequate performance of the controller. The tracking errors become smaller as time progresses. The control input remains within the physically acceptable bounds ($\tau_1 \in [-150\,Nm, 150\,Nm], \tau_2 \in [-15\,Nm, 15\,Nm]$).

Table 1. Controller gains

Link	K_p	K_v	Δ
1	1000	5	0.5
2	10	5	0.1

5. CONCLUSION

A non-linear PD controller with adaptive Fourier series compensation was proposed. Asymptotic convergence of the position and velocity errors to zero was theoretically proven. Experimental results show a decent performance of the controller.

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