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Preliminar ideas on a real-time optimization strategy based on the super-twisting algorithm

Ixbalank Torres * **Alejandro Vargas** ** **Fernando López-Caamal** *** **Héctor Hernández-Escoto** ***

* *C. A. Telemática, Departamento de Ingeniería Electrónica, División de Ingenierías del campus Irapuato-Salamanca, Universidad de Guanajuato. Carretera Salamanca - Valle de Santiago Km. 3.5 + 1.8, Comunidad de Palo Blanco, Salamanca, Mexico (email: ixbalank@ugto.mx)*
** *Laboratorio de Investigación en Procesos Avanzados de Tratamiento de Aguas, Unidad Académica Juriquilla, Instituto de Ingeniería - Universidad Nacional Autónoma de México. Blvd. Juriquilla No. 3001, 76230 Querétaro, Mexico (email: avargasc@ii.unam.mx)*
*** *Departamento de Ingeniería Química, Universidad de Guanajuato, Noria Alta s/n, C.P. 36050, Guanajuato, Mexico (email: fernando.lopez@ugto.mx, hhee@me.com)*

Abstract: Real-time optimization is a control method that aims for operating a process in optimal conditions along its operation. Within this class of methods, Extremum Seeking Control is a model-free real-time optimization strategy, which uses only output measurements to compute the optimal controlled input of a process with a convex input-output map. However, its convergence time is long. In this article, a real-time optimization strategy based on the super-twisting algorithm is introduced. As shown by our simulations results, its convergence time may be shorter with respect to the gradient-based optimization algorithm used by classical extremum seeking control. The feasibility of the real-time optimization strategy proposed is demonstrated in simulations for a biohydrogen production process.

Keywords: Real-time optimization, gradient-based optimization, super-twisting algorithm, biohydrogen production.

1. INTRODUCTION

Optimization problems are everywhere, people try to make the best choices all along their life; we try to select the best candidate in an election (or at least the less worse); manufacturers aim for maximum efficiency in the operation of their production processes, energetic industries desire to maximize the energy produced by their processes, etc. Optimization is an important method in decision science and in the analysis of physical systems. To make use of this method, some objective must be firstly identified, that is a quantitative measure of the performance of the process of interest. This objective could be profit, time, potential energy, or any quantity or combination of quantities that can be represented by a single number. The objective depends on certain characteristics of the process, called optimization variables or unknowns. Thus, the objective of the optimization is to find the optimal values of the variables that maximize or minimize the objective. However, often the variables are restricted, or constrained, in some way. The strategy followed must therefore consider these constraints when the optimization problem is solved (Nocedal and Wright, 2000).

Real-time optimization (RTO) encompasses a family of optimization methods that incorporate process measurements in the optimization framework to drive a real process (or plant) to optimal performance, while meeting operation constraints. RTO has emerged over the past forty years to overcome the drawbacks associated with plant-model mismatch. Uncertainty can have three main sources, namely, (i) parametric uncertainty when the values of the model parameters do not correspond to the reality of the process at hand; (ii) structural plant-model mismatch when the structure of the model is not perfect; (iii) process disturbances. Of course these three sources are not mutually exclusive. RTO incorporates process measurements in the optimization framework to combat the detrimental effect of uncertainties (Marchetti et al., 2016).

In recent years several RTO strategies have been proposed, some of them are model-based while others are model-free. Extremum seeking control is a model-free approach used in control applications where there is a nonlinearity in the control problem and the nonlinearity has a local minimum or a local maximum (Ariyur and Krstic, 2003). In this article some preliminar ideas about using the super-twisting algorithm as an extremum seeking controller are presented. Indeed, the use of second-order sliding modes to solve real-time optimization problems has been recently explored (Angulo, 2015). Besides, proportional-integral extremum seeking controllers similar to the one proposed in this work has been recently promoted (Guay and Dochain, 2017).

The article is organized as follows: in Section 2 the optimization problem to solve is presented. In Section 3 the super-twisting algorithm and the sliding variable considered to solve the optimization problem are proposed. In Section 4 the feasibility of the real-time optimization strategy is demonstrated through closed-loop simulations on a biohydrogen production process. Finally, in Section 5 some conclusions and perspectives about the proposed strategy are discussed.

2. PROBLEM FORMULATION

Let us consider a dynamic system described by the following state space model:

$$\begin{aligned} \dot{x}(t) &= f(x,u,w) \\ y(t) &= h(x) \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}$ is the controlled input, $w \in \mathbb{R}$ is the uncontrolled input and $y \in \mathbb{R}$ is the measured output.

In addition, let us consider an unknown function $y = l(u,w)$, with $l : \mathbb{R}^2 \to \mathbb{R}$, which maps the inputs to the output in steady state. The optimization problem to solve is stated as

$$\begin{aligned} &\max_u l(u,w) \\ &\text{such that:} \\ &\dot{x}(t) = f(x,u,w) \\ &y(t) = h(x) \\ &u_{min} \leq u \leq u_{max}. \end{aligned} \tag{2}$$

The objective function $l(u,w)$ to maximize satisfies:

Assumption 1. The function $l(u,w)$ is twice continuously differentiable with respect to u and has an unique maximizer u^* of l in an open neighborhood $\mathcal{N}$ for each value of w (Nocedal and Wright, 2000). Moreover,

$$\begin{aligned} \nabla_u l(u^*,w) &= \frac{\partial l}{\partial u}|_{(u^*,w)} = 0, \\ \nabla_u^2 l(u^*,w) &= \frac{\partial^2 l}{\partial u^2}|_{(u^*,w)} < 0. \end{aligned}$$

Since we are interested in computing a finite maximum value of the output y, the uncontrolled input satisfies:

Assumption 2. The uncontrolled input $w(t)$ is a bounded function of time, i. e. $|w(t)| \leq c$.

Hence, the problem is to propose an algorithm to find the optimal controlled input u^* in the neighborhood $\mathcal{N}$, such that the output y is maximized for each value of the uncontrolled input w.

3. REAL-TIME OPTIMIZATION BASED ON THE SUPER-TWISTING ALGORITHM

In the last years a powerful collection of algorithms for optimization of smooth functions has been developed. All algorithms require the user to supply initial guess, usually denoted by $u(0)$. Beginning at $u(0)$, optimization algorithms generate a sequence of iterates $\{u(k)\}_{k=0}^{\infty}$ that terminate when either no more progress can be made or when it seems that a solution has been approximated with sufficient accuracy. In deciding how to move from one iterate $u(k)$ to the next, the algorithms use information about the function l at $u(k)$, and possibly also information from earlier iterates $u(0)$, $u(1), \ldots,$ $u(k-1)$. They use this information to find a new iterate $u(k+1)$ with a higher function value l than $u(k)$ (Nocedal and Wright, 2000).

The optimization problem (2) can be solved by line search methods, in which the optimization algorithm chooses a direction p_k and searches along this direction from the current iterate $u(k)$ for a new iterate with a higher function value $l(u)$. The direction along which the function l increases most rapidly is the gradient $\nabla_u l$ (Nocedal and Wright, 2000). In the gradient-based line search method each iteration computes the gradient $\nabla_u l(u(k), w(k))$ and then decides how far to move along that direction. The iteration is given by

$$u(k+1) = u(k) + \gamma \nabla_u l(u(k), w(k)),$$

where the scalar $\gamma > 0$ is called the step length. To select γ, a tradeoff between a substantial increase of l and the shortest convergence time to u^* must be considered (Nocedal and Wright, 2000).

Let us define $\gamma := \alpha \Delta t$, with $\Delta t = t(k+1) - t(k)$, and $\alpha > 0$. Replacing it in (3) we have

$$u(k+1) = u(k) + \alpha \Delta t \nabla_u l(u(k), w(k))$$

or

$$\frac{\Delta u}{\Delta t} = \alpha \nabla_u l(u(k), w(k)).$$

By applying the limit when $\Delta t \to 0$ we have

$$\dot{u}(t) = \alpha \nabla_u l(u, w). \qquad (3)$$

Notice that the optimization algorithm (3) is the optimization algorithm used by classical extremum seeking controllers (Ariyur and Krstic, 2003).

Since the optimization algorithm (3) uses the gradient $\nabla_u l(u, w)$ but $l(u, w)$ is an unknown function, in real applications either, an approximation or an estimation of $\nabla_u l(u, w)$ must be considered.

In order to bypass the uncertainty related to the unknown function $l(u, w)$, let us consider not the gradient but the sign of the gradient in algorithm (3), which, indeed, contains the direction information. A robust optimization algorithm can then be proposed as

$$\dot{u}(t) = \alpha \,\mathrm{sign}(\nabla_u l(u, w)).$$

The equation above is a first order sliding mode, with the gradient $\sigma = \nabla_u l(u, w)$ as the sliding variable, and has the form of the integral part of the super-twisting algorithm with sliding variable defined as $\sigma = \nabla_u l(u, w)$.

Let us now suppose that σ satisfies the following assumption,

Assumption 3. The dynamics of the sliding variable can be described by

$$\dot{\sigma}(t) = f'(t, x) + g'(t, x)u(t)\,,$$

where f' and g' are unknown smooth functions (Shtessel et al., 2014).

This way, the anti-windup super-twisting controller

$$\begin{aligned} u(t) &= \lambda|\sigma|^{1/2}\mathrm{sign}(\sigma) + u_1(t) \\ \dot{u}_1(t) &= \begin{cases} G_{aw}(u_{min} - u(t)); & u < u_{min} \\ G_{aw}(u_{max} - u(t)); & u > u_{max} \\ \alpha\mathrm{sign}(\sigma); & u_{min} \le u \le u_{max} \end{cases}, \end{aligned} \qquad (4)$$

where $\lambda > 0$ is the proportional gain, $\alpha > 0$ is the integral gain, G_{aw} is the anti-windup gain and $\sigma = \nabla_u l(u, w)$ is the sliding variable, guarantees the appearance of a 2-sliding mode $\sigma = \dot{\sigma} = 0$, which attracts the trajectories in finite time. The control $u(t)$ enters in finite time the segment $[u_{min}, u_{max}]$ and stays there. It never leaves the segment, if the initial value is inside at the beginning (Shtessel et al., 2014).

Thus, the main result of this work is stated as:

Conjecture 1. If $\alpha > 0$ and $\sigma = 0$ in finite time, due to Assumptions 1, 2 and 3, the controlled input $u(t) \in \mathcal{N}$, computed by the super-twisting controller (4), is a maximizer of the output $y(t)$ for each value of $w(t)$.

4. RESULTS

In order to verify the feasibility of the super-twisting controller (4) as real-time optimization strategy, let us consider the biohydrogen production process modeled by the following set of ordinary differential equations (ODE) (Torres Zúniga et al., 2015),

$$\begin{bmatrix} \dot{Glu} \\ \dot{Ace} \\ \dot{Pro} \\ \dot{Bu} \\ \dot{EtOH} \\ \dot{X} \\ \dot{CO_2} \\ \dot{H_2} \end{bmatrix} = Kr - D \begin{bmatrix} Glu - Glu_{in} \\ Ace \\ Pro \\ Bu \\ EtOH \\ X \\ CO_2 \\ H_2 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \rho_{CO_2} \\ \rho_{H_2} \end{bmatrix}, \qquad (5)$$

where Glu, Ace, Pro, Bu, $EtOH$, X, CO_2 and H_2 represent the concentrations (in gL^{-1}) of glucose, acetate, propionate, butyrate, ethanol, biomass, carbon dioxide and hydrogen, respectively, in the liquid phase. The vector r describes the kinetics of the involved biological reactions (in $gL^{-1}d^{-1}$), $D = Q_{in}/V$ is the dilution rate (in d^{-1}) and ρ_{CO_2} and ρ_{H_2} the gas flow rates of carbon dioxide and hydrogen (in $gL^{-1}d^{-1}$), respectively. Finally, $K \in \mathbb{R}^{8\times 2}$ represents the matrix of pseudo-stoichiometric coefficients.

The reaction pathway is described by two Monod-type reactions occurring in parallel. Thus, the vector r is composed of the specific glucose uptake rate multiplied by the biomass concentration in the reactor:

$$r = \begin{bmatrix} \dfrac{\mu_{max,1} Glu}{K_{Glu,1} + Glu} \\ \dfrac{\mu_{max,2} Glu}{K_{Glu,2} + Glu} \end{bmatrix} X,$$

where $\mu_{max,l}$ is the maximum specific growth rate of the microorganisms (in $g[Glu]g[X]^{-1}d^{-1}$) and $K_{Glu,l}$ is the half-saturation constant (in gL^{-1}) for $l = 1, 2$.

Furthermore, the differential equations for the gas phase with constant gas volume are

$$\frac{dCO_{2,gas}}{dt} = -\frac{CO_{2,gas}Q_{gas}}{V_{gas}} + \rho_{CO_2}\frac{V}{V_{gas}} \qquad (6)$$

$$\frac{dH_{2,gas}}{dt} = -\frac{H_{2,gas}Q_{gas}}{V_{gas}} + \rho_{H_2}\frac{V}{V_{gas}}, \qquad (7)$$

with the total biogas flow at the reactor output given by

$$Q_{gas} = \frac{RT_{amb}}{P_{atm} - p_{vap,H_2O}} V \left(\frac{\rho_{H_2}}{M_{H_2}} + \rho_{CO_2}\right) \qquad (8)$$

$$\rho_{H_2} = k_L a_{H_2}(H_2 - M_{H_2} K_{H,H_2} p_{H_2,gas}) \tag{9}$$

$$p_{H_2,gas} = \frac{H_{2,gas} R T_{reac}}{M_{H_2}} \tag{10}$$

$$\rho_{CO_2} = k_L a_{CO_2}(CO_2 - K_{H,CO_2} p_{CO_2,gas}) \tag{11}$$

$$p_{CO_2,gas} = CO_{2,gas} R T_{reac}, \tag{12}$$

where $CO_{2,gas}$ and $H_{2,gas}$ are, respectively, the carbon dioxide concentration (in $molL^{-1}$) and the hydrogen concentration (in gL^{-1}) in the gas phase.

As shown in Equation (8), the total biogas flow rate at the reactor output (in Ld^{-1}) is the sum of the hydrogen gas flow rate plus the carbon dioxide gas flow rate ($Q_{gas} = q_{H_2,gas} + q_{CO_2,gas}$). The carbon dioxide and the hydrogen gas flow rates are calculated by considering the transfer of the gas from the liquid phase to the gas phase. The carbon dioxide and the hydrogen concentrations at the liquid-gas interface in equilibrium are calculated by considering the Henry law. The pressure of each gas component can be calculated using the ideal gas law for the two gases.

We are interested in maximizing the hydrogen production rate (in $L[H_2]L^{-1}d^{-1}$) defined as the hydrogen flow rate produced per volume unit,

$$HPR = \frac{q_{H_2,gas}}{V}. \tag{13}$$

Therefore, the biohydrogen production can be described as the model (1), with $u = Q_{in}$, $w = Glu_{in}$, $y = HPR$. Thus, the optimization problem to solve is stated as

$$\begin{aligned} &\max_{Q_{in}} l(Q_{in}, Glu_{in}) \\ &\text{such that:} \\ &Q_{in,min} \leq Q_{in} \leq Q_{in,max}. \end{aligned} \tag{14}$$

Since the optimization strategy proposed is based in the gradient ∇l_u, but the function $l(Q_{in}, Glu_{in})$ is unknown, such a gradient is approximated as

$$\nabla l_u = \frac{\Delta HPR}{\Delta Q_{in}}. \tag{15}$$

Ramírez et al. (2015) propose a static model to describe the effect of the organic loading rate (OLR) on the hydrogen production rate (HPR) in the dark fermenter of interest as

$$HPR(OLR) = -4.53 \times 10^{-5} OLR^3 + 6.95 \times 10^{-3} OLR^2 \tag{16}$$

where the OLR is defined as

$$OLR = 1.067 * DGlu_{in} = 1.067 \frac{Q_{in} Glu_{in}}{V}$$

For $Glu_{in} = 20gL^{-1}$ the optimal input flow rate obtained is $Q^*_{in} = 4.31Ld^{-1}$ (corresponding to an optimal hydraulic retention time of $5.01h$), which maximizes the HPR to $HPR_{max} = 24.24L[H_2]L^{-1}d^{-1}$.

The closed-loop system, biohydrogen production process + real time optimization strategy, was simulated in Matlab, the ODEs were solved by the stiff solver *ode15s* and the parameters of the model (5)-(13) were taken from (Torres Zúniga et al., 2015).

In order to solve the optimization problem (14), let us first consider the gradient-based line search algorithm (3) with the gradient approximated by (15) and $\alpha = 1$. The influent glucose considered is $Glu_{in} = 20Ld^{-1}$, while the restriction on the input is $4h \leq HRT \leq 12h$, with $HRT = V/Q_{in}$, the hydraulic retention time of the process. Figure 1 shows the HPR obtained from model (5)-(13). As can be observed, the optimization started five days after the process beginning, then, the HPR slowly converges to its maximum value. On the other hand, Figure 2 shows the HRT computed by (3). As can be verified, the input converges to the optimal value after eighty days from the optimization beginning. This is a large convergence time because the HRT of the process ranges from 4 to 12 hours. The convergence time is therefore an important disadvantage for such algorithm (as for classical extremum seeking control). As can be regarded, the optimal HRT computed by the algorithm (3) is lower than the theoretical optimal HRT. Nevertheless, the maximum HPR are the same $HPR_{max} = 24.24L[H_2]L^{-1}d^{-1}$. This is because the models considered are different. While the online optimization algorithm (3) considers that the process is described by the dynamical model (5)-(13), the theoretical calculus was made over the static model (16).

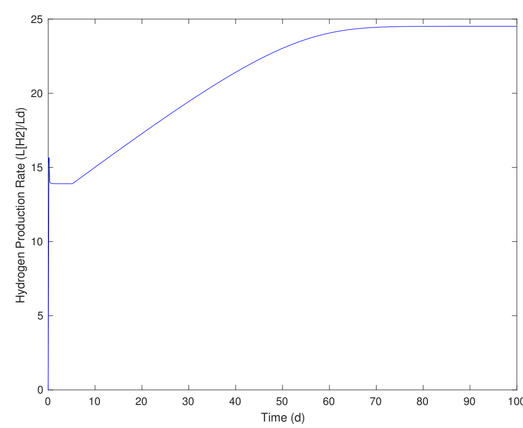

Fig. 1. HPR computed by the gradient-based optimization algorithm (3).

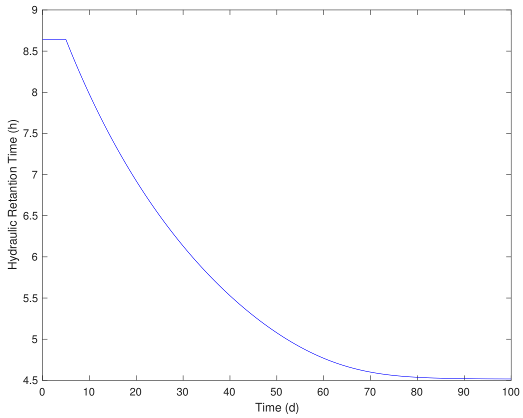

Fig. 2. HRT computed by the gradient-based optimization algorithm (3).

Let us now consider the super-twisting controller (4) with the gradient approximated by (15) and the parameters $\lambda = 0.075$, $\alpha = 1$ and $G_{aw} = 30$. The influent glucose considered is shown in Figure 3, while the restriction on the input is $4h \leq HRT \leq 12h$.

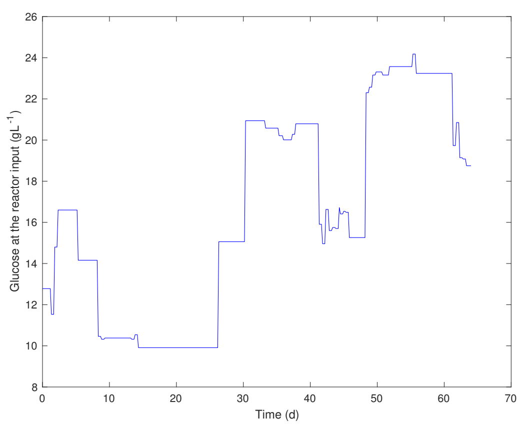

Fig. 3. Glucose at the bioreactor input.

Figure 4 shows in green the HPR obtained from model (5)-(13). As can be observed, once the optimization started five days after the process beginning, the HPR converges to the maximum value in only two days. On the other hand, Figure 5 shows in green the HRT computed by the super-twisting controller (4). As can be verified, the HRT respect the constraint imposed all along the simulation.

Let us now compare the super-twisting controller (4) with the model-based real-time optimization strategy proposed by Torres et al. (2018), which has considerably shorter convergence time with respect to the optimization algorithm (3). The influent glucose considered is the same as in the previous case (see Figure 3), while the restriction

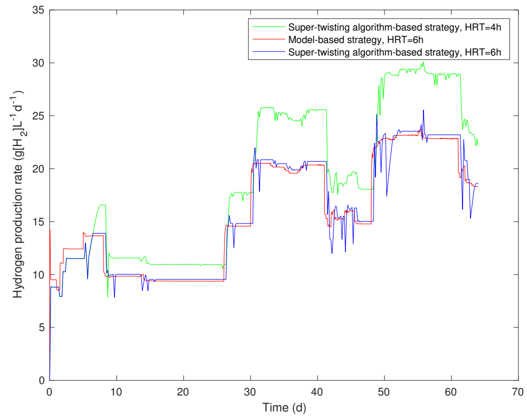

Fig. 4. HPR computed by the super-twisting algorithm (4).

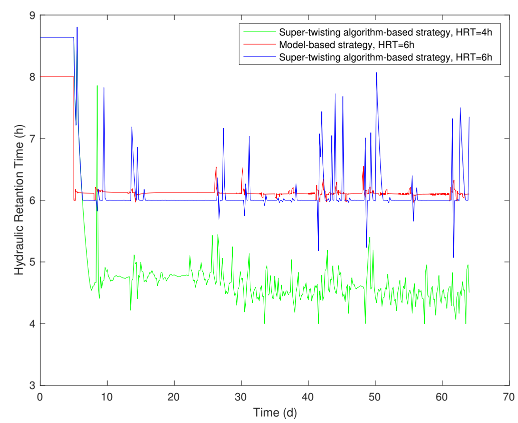

Fig. 5. HRT computed by the super-twisting algorithm (4).

on the input is updated to $6h \leq HRT \leq 12h$. Figure 4 shows in red the HPR by the model-based real-time optimization strategy while in blue the HPR by the super-twisting-based real-time optimization strategy. As can be observed, the HPR computed by the super-twisting controller is slightly higher than the HPR by the model-based optimizer. However, the convergence time of the model-based optimizer is shorter than the convergence time of the super-twisting controller. This is due to the fact that the super-twisting controller uses only output measurements but does not consider information about the model of the biohydrogen production process. As can be observed in Figure 5, both strategies respect the constraint imposed practically all along the simulation. Nevertheless, when the glucose concentration at the bioreactor input chenges, the control input computed by the super-twisting controller presents larger oscillations than those produced by the control input computed by the model-based optimization strategy (see Figure 5).

5. CONCLUSIONS AND PERSPECTIVES

In this article a real-time optimization strategy based on the super-twisting algorithm was presented. Compared with the gradient-based line search algorithm used by classical extremum seeking control, the convergence time of the super-twisting-based optimizer is shorter (both strategies use the same gradient approximation). Nevertheless, compared with a model-based optimizer, the convergence time is larger. This is expected because the super-twisting-based optimizer only uses measurements of the output and the controlled input generated to approximate the gradient. It is well known that the more information about the process model, the shorter is the convergence time of the optimization strategy.

At this point some questions arise: How the proportional term of the super-twisting controller is helping to improve the convergence time? The convergence of the super-twisting-based optimizer can be assured for any dynamic system (1) respecting Assumptions (1)-(3)? How the stability of the closed-loop system can be assured? Can the gradient $\nabla_u l(u, w)$ be better estimated? These questions trace a path to follow by our research group.

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