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Complex network synchronization and modulation of fractional-order hyperchaotic systems for voice encryption

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Abstract: In this paper, the encryption improvement via modulation of fractional-order hyperchaotic oscillator state variables, is presented. A network of N-coupled fractional-order hyperchaotic oscillators, is synchronized. In order to encrypt a voice message via additive encryption, the state variable with the highest energy value is selected. The state variable is modulated to improve the encryption results.

Keywords: Hyperchaos, synchronization, fractional-order, complex network, voice encryption.

1. INTRODUCTION

There are many methods in the literature to achieve the synchronization between two or more chaotic oscillators, i.e.: Pecora-Carroll method see Pecora and Carroll (1990), via sliding mode control see Xu and Wang (2013), active control see Wu and Yang (2010), backstepping design see Naseri et al. (2009), linear control see Odibat et al. (2010), etc. Since Pecora and Carroll suggested and proved the synchronization of chaotic systems, numerous results on synchronization and its applications in communications and encryption have been reported in the literature, see López-Gutiérrez et al. (2009.), Serrano-Guerrero et al. (2010) and Cruz-Hernández et al. (2010).

The fractional calculus, allows to describe and model a real object more accurately than the classical "integer" methods see Petráš (2011). The main reason for using integer-order models was the absence of solution methods for fractional differential equations see Podlubny et al. (2002). At present, there are many methods for approximation of the fractional derivative and integral see Petráš (2011).

The most important contributions of the fractional calculus were published between 1695 – 1974. In 1695 the fractional derivative concept was introduced with a letter from Leibiniz to L'Hospital about the possibility of generalizing the operation of differentiation to non-integer orders. In 1819, S. F. Lacroix mentions for the first time the arbitrary order derivative. Later, L. Euler and J. B. J. Fourier addressed the issue without applications. In 1823, N. H. Abel applied it to the integral equation related to the problem of isochronous. This motivated J. Liouville to the first major attempt at a formal and consistent definition of the fractional derivative in 1832. Other researchers that addressed the fractional calculus are mentioned such as J. L. Lagrange, P. S. Laplace, O. Heaveside, M. Riesz, H. Weyl, K. B. Oldham, J. Spanier, A. K. Grünwald, M. Caputo, G. F. B. Riemann, I. Podlubny and many others see Petráš (2011).

A complex network is defined as an interconnected set of nodes (two or more). Where each node is a fundamental unit, with its dynamic depending of the nature of the network see Angulo-Guzmán et al. (2012).

In this case, the complex network is composed by fractional-order hyperchaotic oscillators. Their behavior, is totally different for a distinct value in the order of their derivatives. For an unwanted person, the parameters for which the oscillator has a hyperchaotic behavior, the initial conditions, and, the order of the derivatives of the system, are unknown.

In this paper, the encryption improvement by modulating the state variables of the hyperchaotic oscillator will be shown.

In order to encrypt the message using a complex network, it is needed to achieve synchronization. A non-modulated state variable of the selected oscillator is used to encrypt the message, based on the highest energy value. Then, the selected state variable is modulated and used to encrypt the same message, the results are compared.

2. PRELIMINARIES

In this section some mathematical methods to solve fractional derivatives and important definitions are mentioned.

2.1 Fractional calculus

The fractional calculus is a generalization of integration and differentiation to non-integer-order fundamental operator ${}_aD_t^\alpha$, where a and t are the bounds of the operation and $\alpha \in \Re$. The continuous integro-differential operator from Petráš (2011) and Podlubny (1999) is defined as:

$$ {}_aD_t^\alpha = \begin{cases} \frac{d^\alpha}{dt^\alpha}, & \alpha > 0, \\ 1, & \alpha = 0, \\ \int_a^t (d\tau)^{-\alpha}, & \alpha < 0. \end{cases} \tag{1} $$

There exist three common definitions of a fractional-order derivative: the Caputo definition see Petráš (2011), the Grünwald-Letnikov definition see Podlubny (1999), and the

Riemann-Liouville definition see Oldham and Spainer (2006). These definitions are equivalent under some conditions Podlubny (1999). The Grünwald-Letnikov definition is defined in non-integer differentiation given by the following equation:

$$ {}_aD_t^{\alpha} f(t) = \lim_{h\to 0} h^{-\alpha} \sum_{j=0}^{\frac{t-a}{h}} (-1)^j \binom{\alpha}{j} f(t-jh). \tag{2} $$

For binomial coefficients calculation, the relation between Euler's *Gamma* function represented by Γ_e and factorial is used and it is defined as:

$$ \binom{\alpha}{j} = \frac{\alpha!}{j!(\alpha-j)!} = \frac{\Gamma_e(\alpha+1)}{\Gamma_e(j+1)\Gamma_e(\alpha-j+1)}, \tag{3} $$

For numerical solution of fractional-order derivatives, the relation derived from Grünwald-Letnikov definition (2) given by the following expression from Petráš (2011) and Podlubny (1999) is used:

$$ {}_{k-L_m/h}D_{t_k}^{q} f(t) \approx h^{-q} \sum_{j=0}^{k} (-1)^j \binom{\alpha}{j} f(t_k - j), \tag{4} $$

where L_m is the "memory length", $t_k = kh$, h is the time step of calculation and $(-1)^j \binom{q}{j}$ are binomial coefficients $c_j^{(q)} (j = 0,1,\ldots)$. The calculation of binomial coefficients is given by:

$$ \begin{aligned} c_0^{(q)} &= 1, \\ c_j^{(q)} &= \left(1 - \frac{1+q}{j}\right) c_{j-1}^{(q)}. \end{aligned} \tag{5} $$

The general numerical solution of the fractional differential equation

$$ {}_aD_t^{q} y(t) = f(y(t),t), \tag{6} $$

can be expressed as follows:

$$ y(t_k) = f(y(t_k),t_k)h^q - \sum_{j=v}^{k} c_j^{(q)} y(t_k - j). \tag{7} $$

2.2 *Complex networks and synchronization*

Consider a network of N identical oscillators, with each oscillator being an n-dimensional dynamic subsystem.

Each oscillator is defined as follows:

$$ {}_aD_t^{\alpha} x_{ni}(t) = f_n(x_i,t) + u_i, \quad i = 1,2,\ldots,N, \tag{8} $$

where $x_i = (x_{1i}, x_{2i}, \ldots, x_{ni})^T \in \Re^n$ are the state variables of the oscillator i and u_i establishes the synchronization between two or more oscillators and is defined as follows see Wang and Chen (2002), Wang et al. (2006) and Posadas-Castillo et al. (2014):

$$ u_i = c \sum_{j=1}^{N} a_{ij} \Gamma x_j, \ i = 1,2,\ldots,N. \tag{9} $$

The constant $c > 0$ represents the coupling strenght, and Γ is a constant matrix linking coupled state variables. Assume that $\Gamma = \mathrm{diag}(r_1, r_2, \ldots, r_n) \in \Re^n$ is a diagonal matrix with $r_n = 1$ if it is the linking coupling state of the network, and $r_n = 0$ otherwise.

The matrix $A = (a_{ij}) \in \Re^{N\times N}$ is the coupling matrix which shows a connection between the oscillator i and j, then $a_{ij} = 1$, otherwise $a_{ij} = 0$ for $i \neq j$. The diagonal elements of A are defined as:

$$ a_{ii} = -\sum_{j=1, j\neq i}^{N} a_{ij} = -\sum_{j=1, j\neq i}^{N} a_{ji}, \ i = 1,2,\ldots,N. \tag{10} $$

The dynamical network achieves identical synchronization if

$$ x_1(t) = x_2(t) = \ldots = x_N(t), \ t \to \infty, \tag{11} $$

which means that the following equation Bocaletti et al. (2002):

$$ \lim_{t\to\infty} \parallel x_i(t) - x_j(t) \parallel = 0, \tag{12} $$

where $i = 1,\ldots,N-1$, $j = j+1,\ldots,N$ holds for every pair of fractional-order hyperchaotic oscillators. This, implies that the complex network has reached synchrony.

3. SYNCHRONIZATION OF N-COUPLED FRACTIONAL-ORDER HYPERCHAOTIC OSCILLATORS VIA COUPLING MATRIX

In this section, the synchronization of a non-directed complex network of N identical fractional-order hyperchaotic oscillators coupled in irregular topology is achieved.

3.1 *Fractional-order hyperchaotic oscillator*

The fractional-order hyperchaotic oscillator used in this paper, considering $L_m = 85$, is described as follows Xiang et al. (2008):

$$ \begin{cases} {}_0D_t^{q_1} x(t) &= a(y(t) - x(t)) + \mu y(t)z(t), \\ {}_0D_t^{q_2} y(t) &= cx(t) - dx(t)z(t) + y(t) + w(t), \\ {}_0D_t^{q_3} z(t) &= x(t)y(t) - bz(t), \\ {}_0D_t^{q_4} w(t) &= -\nu y(t). \end{cases} \tag{13} $$

The set of equations (12) exhibits hyperchaotic behavior for parameters $(a,b,c,d,\mu,\nu) = (35,4,25,5,35,100)$ and orders $q_1 = q_2 = q_3 = q_4 = 0.95$ Petráš (2011). Fig. 1 shows the attractor of the fractional-order hyperchaotic oscillator.

Fig. 2 shows the topology and configuration of the complex network. The corresponding coupling matrix for the network in Fig. 2 is given by:

$$ A = \begin{pmatrix} -2 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & -2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & -3 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & -5 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -2 \end{pmatrix}. \tag{14} $$

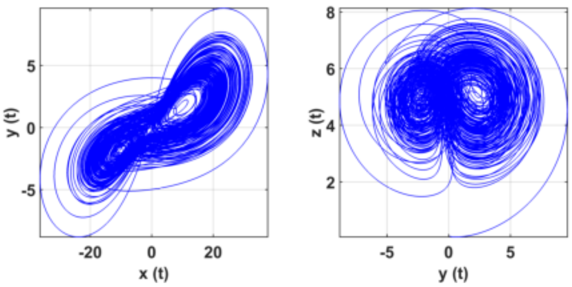

Fig. 1. Phase space of the hyperchaotic strange attractors equ.(12) for parameters: $(a,b,c,d,\mu,\nu) = (35,4,25,5,35,100)$, and initial conditions $(x(0),y(0),z(0),w(0)) = (0.1,0.1,0.1,0.1)$ proyected onto $(x(t),y(t),z(t))$ space.

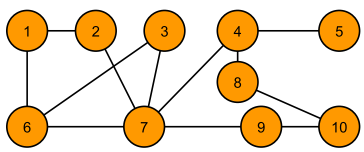

Fig. 2. Topology considered to arrange the fractional-order hyperchaotic oscillators (irregular coupled network).

For this case, the Γ matrix is defined as $\Gamma = \mathrm{diag}(0,1,0,0)$ because the synchronization is achieved by the $y(t)$ state. By means of the equations (8), (9) and (10) the mathematical model of the complex network is described as follows:

$$\begin{cases} {}_0D_t^{q_1} x_i(t) &= a(y_i(t)-x_i(t)) + \mu y_i(t) z_i(t), \\ {}_0D_t^{q_2} y_i(t) &= c x_i(t) - d x_i(t) z(t) + y_i(t) + w_i(t) + u_{i,2}, \\ {}_0D_t^{q_3} z_i(t) &= x_i(t) y_i(t) - b z_i(t), \\ {}_0D_t^{q_4} w_i(t) &= -\nu y_i(t). \end{cases} \tag{15}$$

where $i = 1,2,3,\ldots,10$.

The control laws are applied to the state $y_i(t)$ of the complex network. By using the equation (9) the control laws $u_{i,2}$ for $i = 1,\ldots,10$ are given by:

$$\begin{aligned} u_{1,2} &= c(-2y_1(t)+y_2(t)+y_6(t)), \\ u_{2,2} &= c(-2y_2(t)+y_1(t)+y_5(t)), \\ u_{3,2} &= c(-2y_3(t)+y_6(t)+y_7(t)), \\ u_{4,2} &= c(-3y_4(t)+y_5(t)+y_7(t)+y_8(t)), \\ u_{5,2} &= c(-y_5(t)+y_4(t)), \\ u_{6,2} &= c(-3y_6(t)+y_1(t)+y_2(t)+y_7(t)), \\ u_{7,2} &= c(-5y_7(t)+y_2(t)+y_3(t)+y_4(t)+y_6(t)+y_9(t)), \\ u_{8,2} &= c(-2y_8(t)+y_4(t)+y_{10}(t)), \\ u_{9,2} &= c(-2y_9(t)+y_7(t)+y_{10}(t)), \\ u_{10,2} &= c(-2y_{10}(t)+y_8(t)+y_9(t)), \end{aligned} \tag{16}$$

The initial conditions for each oscillator of the complex network randomly obtained in a range $[-5,5]$, are shown in Table 1.

According to the Lemma 1 from Wang and Chen (2002), synchronization is achieved with a coupling strength $c = 25$. Fig. 3 shows the time evolution of state variables, the synchronization is achieved. Fig. 4 shows the phase portraits of some state variables of the complex network. Fig. 5 shows the synchronization errors between some of the state variables of the complex network.

Table 1. Initial conditions

Node	$x(0)$	$y(0)$	$z(0)$	$w(0)$
1	3.56	-3.66	-4.09	-4.29
2	2.66	1.92	0.76	0.65
3	-4.11	3.8	0.29	-2.06
4	3.96	-2.59	-4.56	3.45
5	0.87	-0.44	2.68	4.81
6	-3.99	2.37	4.85	-0.56
7	1.8	0.63	-3.64	4.28
8	-0.69	4.61	-2.87	0.81
9	4.95	-4.48	1.04	3.09
10	4.36	3.36	2.53	-0.33

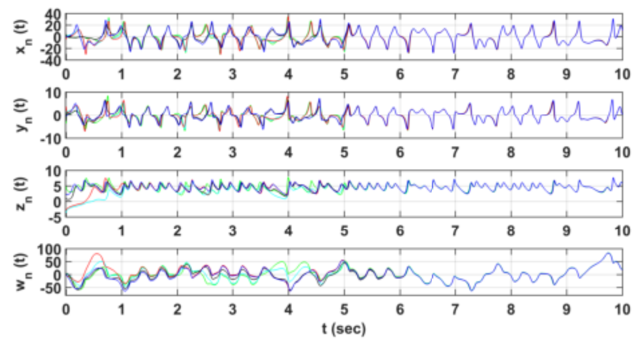

Fig. 3. Evolution of the synchronization of some states in the complex network: $x_i(t)$, $y_i(t)$, (where $i = 1,2,...,5$), and $z_i(t)$, $w_i(t)$, (where $i = 6,7,...,10$).

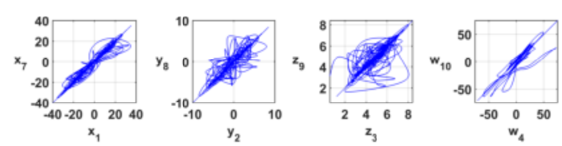

Fig. 4. Phase portrait of some states of the complex network: x_7 vs x_1, y_8 vs y_2, z_9 vs z_3, w_{10} vs w_4.

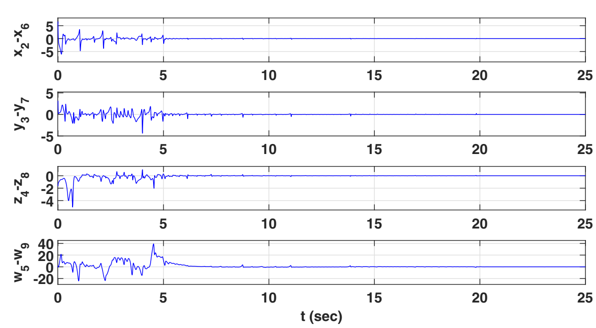

Fig. 5. Time evolution of the synchronization error between some state variables of the complex network: $x_2 - x_6$, $y_3 - y_7$, $z_4 - z_8$, $w_5 - w_9$.

4. HYPERCHAOTIC ENCRYPTION

Once the complex network is synchronized, we have selected the first oscillator which is identical to the rest of the oscillators in the network. The state variable to encrypt the message is selected according to the following critera proposed in Soriano-Sánchez et al. (2015).

Criterion J_1: selection based on the hyperchaotic signal energy:

$$\sum_{n=0}^{N-1} |x_c(n)|^2 \gg \sum_{n=0}^{N-1} |x_m(n)|^2, \tag{17}$$

where $x_c(n)$ is the hyperchaotic sampled signal, and $x_m(n)$ is the message sampled signal. The criterion J_1 shows how many times the energy of $x_c(n)$ exceeds the energy of $x_m(n)$. Therefore, $J_1 \gg 1$ leads to a good encryption.

Criterion J_2: selection based on the hyperchaotic signal energy in the frequency domain:

$$\sum_{k=0}^{N-1} \eta(k)|X_c(k)|^2 \gg \sum_{k=0}^{N-1} \eta(k)|X_m(k)|^2, \tag{18}$$

where $X_c(k)$ are the spectrum samples of the hyperchaotic signal, $X_m(k)$ are the spectrum samples of the message, and $\eta(k)$ is the frequency weighting function that selects the frequency band in which the message is located. The criterion J_2 shows how many times the weighted energy of $X_c(k)$ exceeds the weighted energy of $X_m(k)$ in a selected frequency band if $\alpha(k) = 1$ in $K \in [k_1, k_2]$. Thereby, $J_2 \gg 1$ leads to a good encryption in the frequency band where the message is located.

Discrete time criteria are used because, in the numerical simulation, values in discrete time are obtained, due to integration step. The signals were transformed from the time domain to the frequency domain by using the Fast Fourier Transform.

The resulting values of criteria J_1 and J_2 are shown in Table 2. E_c represents the total energy of the hyperchaotic signal, J_1 obtained from (17) represents how many times the hyperchaotic signal energy is higher than the message energy. E_w represents the weighted energy of the hyperchaotic signal, J_2 obtained from (18) shows how many times the hyperchaotic signal energy is higher than the message one in the frequency range in which our message is located.

In this case our data is a voice recording: "data encrypted with fractional-order chaotic oscillators". It is located in a frequency band of 0.3 kHz - 3 kHz Tomasi (2001) with a sampling frecuency $F_s = 11.025$ kHz.

Table 2. Criteria values for the hyperchaotic signals of the complex network after synchronization.

State	$E_c(10^7)$	$J1(10^4)$	$E_w(10^3)$	$J2$
$x(t)$	0.9629	0.7871	4.0814	4.2398
$y(t)$	0.0448	0.0366	0.4508	0.4683
$z(t)$	0.1788	0.1462	0.1143	0.1188
$w(t)$	4.2211	3.4506	0.2680	0.2784

According to J_1 from Table 2, $w(t)$ provides the highest value. However, by comparing $w(t)$ with $x(t)$ according to J_2 criterion, $x(t)$ provides the highest value in the frequency band in which the message is located. Thereby, by using $w(t)$ leads to a poor encryption. We have selected the $w(t)$ signal to encrypt the message in order to compare the results after the states modulation.

Once the state variable is been selected to encrypt the message, an hyperchaotic version of the additive encryption is used. This method consist on the application of autonomous hyperchaotic oscillators whose output signal is added to the information signal. This sum is sent over a communication channel. A second hyperchaotic signal of the transmitter is sent and used by the receptor to synchronize an equivalent hyperchaotic oscillator with the transmitter system. The reconstructed hyperchaotic signal is then subtracted from the sum transmitted which finally restores the information see Dachselt and Schwarz (2001), this is illustrated in Fig. 6.

The communication diagram for the transmission process in two channels and the retrieval in multi-user modality is shown in Fig. 7.

Fig. 8 shows the encryption results, where (a) $m(t)$, is the message to encrypt previously mentioned, (b) $s_1(t) = w(t) + m(t)$, represents the encrypted message, and (c) $m'(t) = s_1(t) - w'(t)$, is the retrieved message. Fig. 9 shows the error between the message and retrieved message $e(t) = m(t) - m'(t)$.

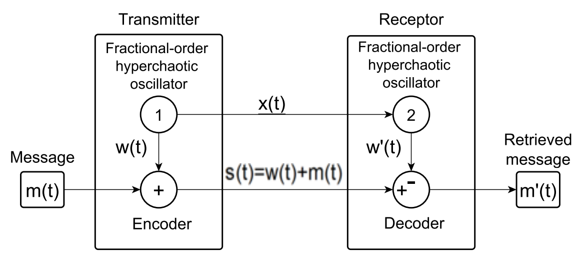

Fig. 6. Basic diagram of additive encryption. Message to encrypt $m(t)$, encrypted message $s(t)$ and, retrieved message $m'(t)$.

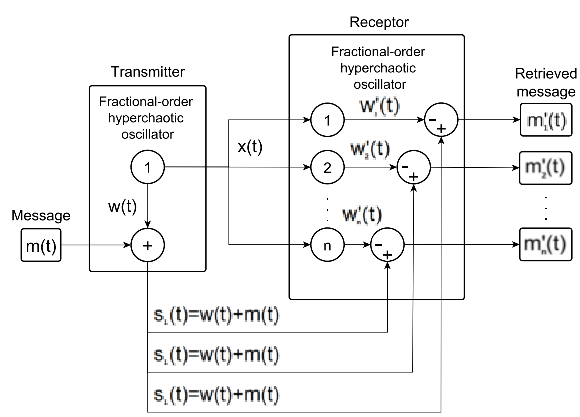

Fig. 7. Multi-user communication diagram with two transmission channels.

As an alternative way to improve the resulting encryption $s_1(t)$, the modulation of the fractional-order hyperchaotic oscillator state variables, is proposed. This shifts the energy of the hyperchaotic signals to the frequency band in which our voice message is located. The encryption improvement is reflected in the values of J_2, since a higher value in J_2 means increased energy in the frequency band of the message.

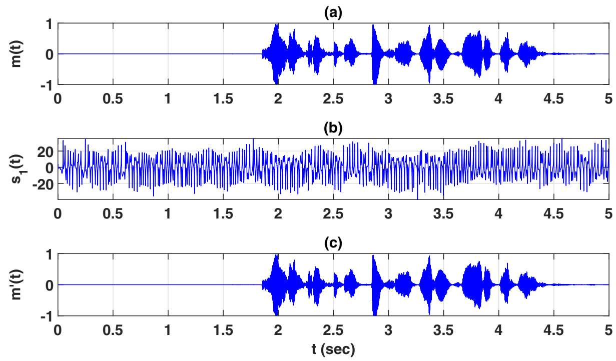

Fig. 8. Resulting signals of the communication process: (a) The message $m(t)$ to be encrypted, (b) The transmitted signal $s_1(t)$ which is the sum of the $w(t)$ hyperchaotic state and the confidential message $m(t)$, (c) Retrieved message $m'(t)$.

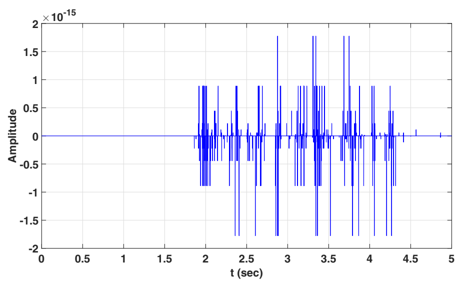

Fig. 9. Error between message and the retrieved message $e(t) = m(t) - m'(t)$.

4.1 Modulation of the fractional-order hyperchaotic oscillator state variables

In this section, the first fractional-order hyperchaotic oscillator state variables from the complex network previously synchronized, are modulated. This, in order to take advantage of the energy provided by the oscillator. Then the selection criteria of the hyperchaotic signal to encrypt the message are used again. The message to encrypt is the same voice recording in order to observe the difference on encryption quality.

By frequency shifting and the modulation theorem see Proakis and Manolakis (1996), if

$$x(n) \overset{F}{\longleftrightarrow} X(\omega), \tag{19}$$

then

$$e^{j\omega_0 n} x(n) \overset{F}{\longleftrightarrow} X(\omega - \omega_0). \tag{20}$$

Therefore, the multiplication of a sequence $x(n)$ by $e^{j\omega_0 n}$ is equivalent to a frequency traslation of the spectrum $X(\omega)$ by ω_0. Since the spectrum $X(\omega)$ is periodic, the shift ω_0 applies to the spectrum signal in every period.

From equation (19):

$$x(n)\cos(\omega_0 n) \overset{F}{\longleftrightarrow} \frac{1}{2}[X(\omega+\omega_0) + X(\omega-\omega_0)]. \tag{21}$$

Once mentioned this, the modulation of the state variables of the first oscillator was performed by using the equation (20) as follows:

$$\begin{aligned} x_{f_0}(n) &= x_1(n)\cos(\omega_0 n), \\ y_{f_0}(n) &= y_1(n)\cos(\omega_0 n), \\ z_{f_0}(n) &= z_1(n)\cos(\omega_0 n), \\ w_{f_0}(n) &= w_1(n)\cos(\omega_0 n), \end{aligned} \tag{22}$$

with $\omega_0 = \frac{2\pi f_0}{F_s}$, where f_0 represents a frequency in the band of the message, i.e. the one to which the hyperchaotic signal is wanted to shift.

Since the message $m(t)$ is located in a frequency band of 0.3 kHz - 3 kHz, we consider $\omega_0 = \pi\frac{300Hz}{5512.5Hz}$ in order to shift the first oscillator state variables to the frequency band of the message. Fig. 10 shows a diagram about how the $w(t)$ state is modulated and used to encrypt the message.

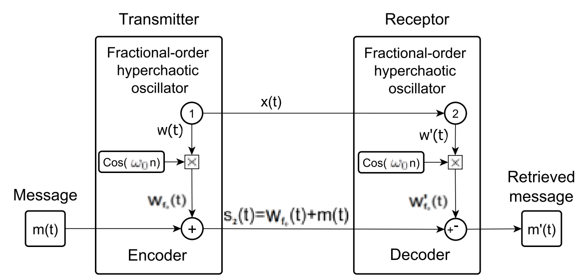

Fig. 10. Diagram of additive encryption and $w(t)$ state modulation.

By using (17), other values were obtained. The second and third column of Table 3 show the resulting values of J_2 before and after the modulation of the first oscillator state variables respectively. In the fourth column, B shows the ratio between the resulting $J_{2,m}$ by modulating the state variables, and J_2 from Table 2. The energy of the hyperchaotic signals is located in the frequency band of the message, improving the encryption quality.

Table 3. Frequency criteria values for the modulated hyperchaotic signals of the complex network after synchronization

State	J_2	$J_{2,m}(10^4)$	$B(10^3)$
$x(t)$	4.2398	0.3457	0.815
$y(t)$	0.4683	0.0154	0.329
$z(t)$	0.1188	0.0869	7.315
$w(t)$	0.2784	1.9079	68.531

Fig. 11 shows the encryption results with the modulated $w(t)$ state of the first oscillator of the network, being (a) $m(t)$ the message, (b) $s_2(t) = w_{f_0}(t) + m(t)$ the encrypted message, and (c) $m'(t) = s_2(t) - w'_{f_0}(t)$ the retrieved message. Fig. 12 shows the error between message and retrieved message $e_{f_0}(t) = m(t) - m'(t)$ when $w_{f_0}(t)$ is used for encryption.

5. CONCLUSIONS

In the encryption using fractional-order hyperchaotic oscillators, the specific parameters for which the system exhibits hyperchaotic behaviour, the initial conditions and the order of the fractional differential equations, are unknown parameters against attackers. The selected frequency band to which the energy of the hyperchaotic signals are shifted to, represents

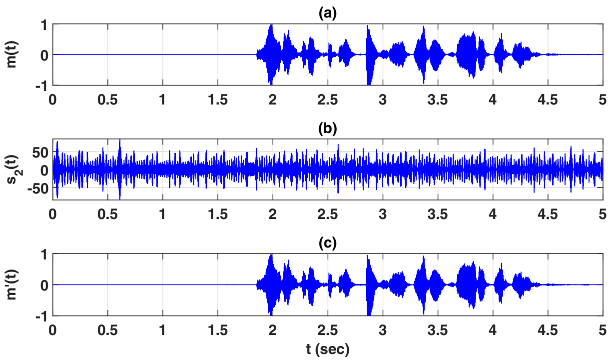

Fig. 11. Resulting signals of the communication process: (a) The message $m(t)$ to be encrypted, (b) The transmitted signal $s_2(t)$ which is the sum of the $w_{f_0}(t)$ hyperchaotic state and the confidential message $m(t)$, (c) Retrieved message $m'(t)$.

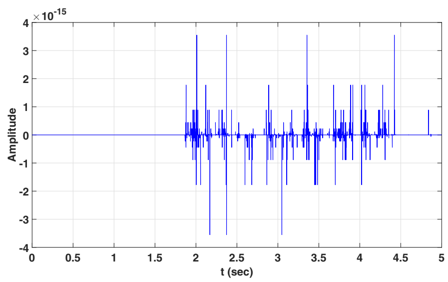

Fig. 12. Error between message and the retrieved message $e_{f_0}(t) = m(t) - m'(t)$ after modulating $w(t)$.

an additional unknown parameter. The fractional-order hyperchaotic oscillator shown in this paper presents its energy at lower frequencies than the voice message. The energy of the hyperchaotic signals was not located in the frequency band of the message, obtaining a poor encryption. By modulating the state variables, we shifted the energy to the frequency band of the message, helping us to improve the encryption quality by an approximate factor $B = 68.5 \times 10^3$.

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