Energy management system for autonomous vehicle

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Abstract: The present work focuses on forecasting energy generation for an autonomous vehicle. The objective is to find the optimal management of the storage system using a genetic algorithm as an optimization method to be able to use any type of objective function, whether continuous or non-continuous. The energy comes from renewable sources and storage devices. The aim of this paper consists of optimal energy management of the storage system within the hybrid renewable energy system considering known predictions of solar radiation and wind speed for a period, T. The optimization model is solved using a hybrid genetic algorithm with interior point (GA-IP) provided by the optimization toolbox MATLAB®. The case study considers the hybrid renewable energy system with the photovoltaic panels, the wind turbines and the batteries for energy storage. In this test, the power supplied by renewable sources is enough to feed the motors and charge the storage system.

Keywords: Energy storage, genetic algorithm, autonomous vehicle.

1. INTRODUCTION

This work is motivated by the incursion renewable energy in autonomous vehicles. Nowadays, renewable energy technology has almost daily advances, and this technology had been implemented to be used in marine transportation which has caused a decline in prices for power generated based on non-renewable resources. In this case we studied the hybrid renewable energy system or microgrid in autonomous mode (Olivares et al. 2014). Inthis operation mode, the microgrid is controlled through a three-level hierarchical control system which can be split in a primary, secondary and tertiary controllers (Bidram et al. 2012). This paper is part of the tertiary controller of the microgrid operating in autonomous mode. In this scenario, the objectives that mainly are considered for the tertiary controller are the optimal handling of the storage system and the optimization of the voltages profile within the microgrid, through of an optimization method used by Katiraei et al. 2008. As a result, optimization model analysis is the cornerstone of the tertiary controller. Therefore, this work is particularly focused on the implementation of a model for optimal power flows of the microgrid for autonomous operating mode and one optimization method for the resolution of optimization model of microgrid. On the basis of profiles of electric demand curves for the marine vehicle, solar radiation and predicted wind speed for a period of time T, the analysis of optimal power flows can be used to determine the optimum supply from storage energy devices, with the aim of maximize the benefit of the energy generated by the renewable energy devices installed in the microgrid (Mahyiddin et al. 2016).

This paper we focus on optimization method for a unified model of hybrid renewable energy system, which considers basic models of the elements of the photovoltaic panels, wind turbines, as well as batteries for energy storage. The model is solved by one method of optimization: hybrid genetic algorithm with interior point (GA-IP) provided by the optimization toolbox included in MatLab®. The solutions obtained allows us to evaluate the optimal supply of energy storage devices.

2. MODEL FOR THE HYBRID RENEWABLE ENERGY SYSTEM COMPONENTS

The models for the hybrid renewable energy system components presented in this section are considered in order to explicitly formulate the unified model of optimization for microgrid presented in Section 4.

2.1 Batteries

The batteries are elements that can operate in charge or discharge mode to provide or consume a net amount of active power in their connection node. Then, the *j-th* battery is represented by two sources of active power, as shown in Fig. 1 (Gill et al. 2014). One of them represents the charge power $P_{Bcj}^{t_z} \le 0$ and the other the discharge power $P_{Bdj}^{t_z} > 0$, the sum of both powers represents the net power $P_{Bnj}^{t_z}$ provided or consumed by the battery in their connection node.

$$P_{Bnj}^{t_z} = P_{Bcj}^{t_z} + P_{Bdj}^{t_z}; 0 \le P_{Bdj}^{t_z}; P_{Bcj}^{t_z} \le 0; \forall t_z \in t \qquad (1)$$

In addition, the voltage in the connection node is represented by its magnitude $V_j^{t_z}$. Thus, the decision variables of the *j-th* battery are $[P_{Bcj}^{t_z}, P_{Bdj}^{t_z}] \in y_B; (\forall t_z \in t)$ being y_B the batteries variables, while $[V_j^{t_z}] \in \boldsymbol{y_{RD}}$ Moreover, the State Of Charge *(SOC)* of the *j-th* battery in the time t_z, $(SOC_{Bj}^{t_z})$ can be approximated by means of Eq. (2)

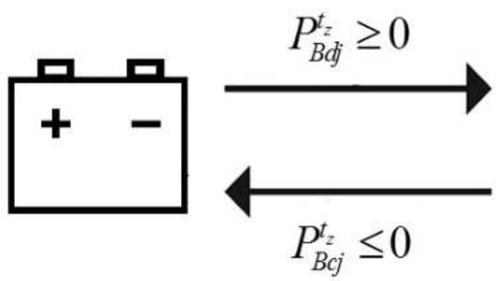

Fig. 1. Battery model

$$SOC_{Bj}^{t_z} = SOC_{Bj}^{t_0} - \frac{\varepsilon_{cj}\Delta t}{E_{Bnom\,j}} \sum_{t=1}^{t_z} P_{Bc\,j}^{t} - \frac{\Delta t}{E_{Bnom\,j}\,\varepsilon_{dj}} \sum_{t=1}^{t_z} P_{Bd\,j}^{t} \qquad (2)$$

It is assumed that the energy provided by the batteries has no cost since it is absorbed and provided in the same connection node. However, since the optimization algorithm will manage the power, batteries will be charged in periods of renewable sources generate more energy and it will be discharged in periods of low energy generation. This fact involves an economic and environmental benefit in the operation of the microgrid

2.2 Vertical Wind turbine

For the aim of the stationary analysis, the vertical wind turbine can be considered a source of controlled active power $P_{Aj}^{t_z}$, depending on its density δ, wind speed S^{t_z}, as well as the area A covered by the blades of the wind turbine (Wang et al. 2009) .

$$P_{Aj}^{t_z} = \delta A\left(S^{t_z}\right)^3 / 2;\ \forall t_z \in T \qquad (3)$$

Since it is considered that the power delivered by the vertical wind turbine is controllable, this element does not introduce decision variables. However, Eq. (10) is required to assess the contribution of the power *j-th* of the wind turbine for a wind speed curve, $P^{t_z} \forall t_z \in t$, density, δ, and area, A, given. Clearly, the magnitude $V_j^{t_z}$ of the voltage of the connection node is considered decision variable, such that $[V_j^{t_z}] \in \boldsymbol{y_{RD}}$. Finally, it is assumed that the energy provided by the vertical wind turbine has no cost.

2.3 Photovoltaic modules

Fig. 2 shows the schematic model of a photovoltaic module connected to the node *k-th* through a converter DC/DC (Bellini et al. 2009). The implicit expression showed in Eq. (4) models the behavior of the current DC in $I_{CD_k}^{t_z}$ panel terminals. Where I_{ph} is the current of the photovoltaic panel, I_0 is the current of saturation, $V_{CD_k}^{t_z}$ is the DC voltage in the module terminals, R_S represent the resistance in series, n_S is the number of cells in series and n_p is the number of cells in parallel. The term R_S is evaluated from Eq. (5), where V_{oc} represent the open circuit voltage, V_{mp} is the voltage of the point of maximum power, I_{sc} is short circuit electricity and I_{mp} is maximum power point electricity. Terms I_{sc} and V_{oc} are evaluated through Eq. (6) and Eq. (7), respectively. Where $I_{sc,stc}$, G, G_{stc}, k_i, T, T_{stc}, $V_{oc,stc}$ and k_v represent the short circuit current electricity standard under conditions of test, irradiance, irradiance under test, electricity temperature coefficient, temperature of the panel, standard temperature under test, standard low open circuit voltage conditions test and voltage and temperature coefficient, respectively.

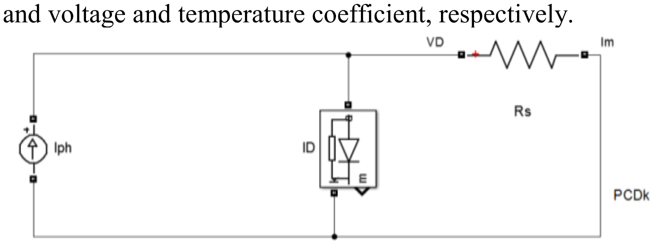

Fig. 2. Schematic model of a photovoltaic module

$$I_{CDk}^{t_z} = \left[I_{ph} - I_0 \left(\exp\left(\frac{\frac{V_{CD\,k}^{t_z}}{n_s} + \frac{I_{CDk}^{t_z}}{n_p} R_s}{V_{CD}^{t_z}} \right) - 1 \right) \right] n_p; \forall t_z \in T \qquad (4)$$

$$R_s = \frac{\frac{V_{oc} - V_{mp}}{n_s} + V_{CD}^{t_z} \ln\left(\frac{I_{sc} - I_{mp}}{I_{sc}}\right)}{\frac{I_{mp}}{n_p}} \qquad (5)$$

$$I_{sc}(T,G) = I_{sc,stc} - \frac{G}{G_{stc}}\left[1 + \frac{k_i}{100}(T - T_{stc})\right] \qquad (6)$$

$$V_{oc}(T) = V_{oc,stc}\left[1 + \frac{k_v}{100}(T - T_{stc})\right] \qquad (7)$$

In this work, the photovoltaic module parameters involved in Eq. (4)-(7) were taken from the features of the Monocrystalline module brand Silfab model SLAM 300 (Silfab, 2015). Moreover, a good forecast of solar irradiance curves is considered available.

According to the model in Fig. 2, the injected power in DC terminals (generated by module) can be expressed directly as (Zúñiga, 2009).

$$P_{CDk}^{t_z}\left(V_{CDk}^{t_z},\, I_{CDk}^{t_z}\right) = I_{CDk}^{t_z} V_{CDk}^{t_z}\ ;\ \forall t_z \in t \qquad (8)$$

2.4 Electrical load

Energy consumption in load nodes of the microgrid is represented by a model of constant power for any interval of time t_z. The power *i-th* $P_{li}^{t_z}$ consumed in the node is then represented by Eq. (9) where $P_{li}^{t_z}$ represent the power consumption active on that node at the instant , respectively.

$$P_{li}^{t_z}\ \ \forall t_z \in t \qquad (9)$$

Model of energy demand does not introduce decision variables to the optimization problem. In addition, it is considered that a good forecast of the energy demand curves is available $P_{li}^{t_z} \forall l, t_z \in t$.

3 GENERIC MODEL OF OPTIMIZATION FOR MICROGRID

For demand curves of motor, solar radiation and predicted wind speed for an interval of time t, the general model of optimization is given by Eq (10)-(13). It should be noted that, in this model, it is considered that the period of time t is composed of a set of time stages t_z ($\forall = 1......end$), such that $t = [t_1...........t_{end}]$ (Polanco et al. 2018).

$$\text{Minimize} \quad F_T = \sum_{t_z=1}^{t_{end}} f^{\,t_z}(\boldsymbol{y}^{t_z}) \qquad (10)$$

$$\text{Subject to} \quad \boldsymbol{h}^{t_z}(\boldsymbol{y}^{t_z}) = \boldsymbol{0}\ ;\ \forall t_z \in t \qquad (11)$$

$$\boldsymbol{g}^{t_z}(\boldsymbol{y}^{t_z}) \le \boldsymbol{0}\ ;\ \forall t_z \in t \qquad (12)$$

$$\underline{\boldsymbol{y}} \le \boldsymbol{y}^{t_z} \le \overline{\boldsymbol{y}}\ ;\ \forall t_z \in t \qquad (13)$$

Taking into account that, tz represents the *z-th* time stage, the description of the terms of the model, Eq. (10)-(13), is as follows: F_T is the objective function to optimize along the interval t, ***h(y)*** is a set of equality constraints, which represents the balance equations of active power on all the nodes in the microgrid, as well as other operating conditions that must be fulfilled unconditionally, ***g(y)*** is a set of inequality constraints of that represents the physical limits and operating of the elements that make up the microgrid, y is the set of decision variables (to be optimized) composed of subsets $\boldsymbol{y}_{RD}$, $\boldsymbol{y}_{MF}$ and $\boldsymbol{y}_B$ such y=[$\boldsymbol{y}_{MF}$, $\boldsymbol{y}_B$]. As it has been pointed out along Section 2, $\boldsymbol{y}_B$, $\mathbf{y}_{\mathbf{RD}}$ and $\boldsymbol{y}_{MF}$ represent batteries variables, distribution grid and photovoltaic modules, respectively. The upper and lower limits, $\overline{\boldsymbol{y}}$ and $\underline{\boldsymbol{y}}$, of these variables are formulated through the inequality constraints, see in Eq. (13).

4. EXPLICIT MODEL OF OPTIMIZATION

Models of the microgrid components described in Section 2 are considered in this section to formulate explicit model of optimization for autonomous microgrid. For this purpose, is considered one generic microgrid composed of a number of, N_{bCD} nodes in DC, N_B battery, N_A wind turbines, N_{MF} photovoltaic modules and N_{CE} electrical loads. In addition, the nodes in DC represent DC terminals of the photovoltaic modules, converter for wind turbines and battery such N_{bCD}=N_{MF} (Polanco et al. 2018).

4.1 Objective function

The intention is to minimize Eq. (10), which is formulated explicitly taking into account the models of renewable sources, as follows:

$$F_T = \sum_{t_z=1}^{t_{end}} \sum_{j=1}^{N_{pac}} a_j + b_j\ \ (P_{RP\,j}^{t_z}) + c_j\ \ (P_{RP\,j}^{t_z})^2 \qquad (14)$$

Note that Eq. (14) denotes that the objective is to minimize the cost of the total energy of renewable sources during the period of time t to supply the demand curves predicted $P_{li}^{t_z} \forall l, t_z \in t$.

4.2 Equality constraints

The set of equality constraints ***h(y)*** in Eq. (11) is expressed explicitly as follows. The constraints of power active, $\Delta P_{RDi}^{t_z}$, balance corresponding to the nodes of the microgrid in DC are writen in the first block of Eq. (15). In this case, the block does not include the reactive power equations because it is a DC grid.

$$\boldsymbol{h}^{t_z}(\boldsymbol{y}^{t_z}) = \left\{ \begin{array}{c} \Delta P_{RD\,i}^{t_z} = P_{RP\,i}^{t_z} + \sum\limits_{\forall j \in i} P_{Bc\,j}^{t_z} + \sum\limits_{\forall j \in i} P_{Bd\,j}^{t_z} + \sum\limits_{\forall j \in i} P_{A\,j}^{t_z} + \\ \sum\limits_{\forall j \in i} P_{CD\,j}^{t_z} - \sum\limits_{\forall j \in i} P_{lj}^{t_z} - \sum\limits_{\forall j \in i\,|\,j \in N_{Al}, N_T} P_{inj\,j}^{t_z}(\boldsymbol{V},\boldsymbol{I}) = 0, \\ i = 1,2,\ldots,N_{bAC}\ ;\ k = 1,2,\ldots,N_{bAC}\,|\,\forall k \notin N_{GEN} \\ \hline \Delta I_{CD\,k}^{t_z} = I_{CD\,k}^{t_z} - f_{CD}(V_{CD\,k}^{t_z}, I_{CD\,k}^{t_z}) = 0, \\ \Delta P_{CD\,k}^{t_z} = P_{CDk}^{t_z}(\boldsymbol{V},\boldsymbol{I}) - P_{CD\,k}^{t_z}(V_{CD\,k}^{t_z}, I_{CD\,k}^{t_z}) = 0, \\ \Delta V_{CD\,k}^{t_z} = V_{CD\,k}^{t_z} - (\pi/16) V_k^{t_z} = 0 \\ k = N_{bDC}+1,\ldots,N_{bDC} \end{array} \right\}_{\forall t_z \in t} \qquad (15)$$

4.3 Inequality constraints functions

According to the elements models introduced in Section 2, the only inequality constraints function ***g(y)*** in Eq. (12) corresponds to the batteries, as follows. The batteries have capacity of charge and discharge finite. With this point in mind, the SOC is modulated over the period T through Eq. (16) (Gill et al. 2014).

$$\boldsymbol{h}^{t_z}(\boldsymbol{y}^{t_z}) = \left\{ SOC_{Bj}^{\min} \le SOC_{Bj}^{t_z} \le SOC_{Bj}^{\max} \right\}_{\forall j \in N_B; \forall t_z \in t} \qquad (16)$$

4.4 Equality constraints variables

Should be taken into account that, the decision variables must acquire admissible values, otherwise, the solution provided by the model of optimization could not make sense from a practical point of view. For this reason, the decision variables y are limited during the interval of time t by means of the constraints set in Eq. (17).

$$\left\{ \begin{array}{c} \underline{y}_{MF} \le y_{MF}^{t_z} \le \overline{y}_{MF} \\ \underline{y}_B \le y_B^{t_z} \le \overline{y}_B \end{array} \right\}_{\forall t_z \in t} \qquad (17)$$

4.5 Hybrid algorithm composed by genetic algorithm and interior point methods

This algorithm is applied through of the ga function which belongs to the MATLAB® 'Direct Search and Genetic Algorithm Toolbox' and is especially useful for the optimization of global non-linear problems with or without constraints. Also, it provides a great flexibility since it contains multiple parameters of configuration (The MathWorks 2016a, b).

The ga function suggested, specify the values of the arguments of the set of inputs $\mathbf{I}_A$ and outputs $\mathbf{O}_A$ that are established and regulated for its execution. The general form of this function is,

$$[\boldsymbol{O}_A] = ga(\boldsymbol{I}_A) \qquad (18)$$

The tables 1 and 2 describe the main elements of these sets of arguments according to the order in which they must be provided to the ga function for its execution.

Name	Description
@fitnessfcn	The handle of the M-function file contains the objective function.
nvar	The vector contains the numerical value of the initial condition of system variables of OPF.
A	All these parameters are not of interest in this work, they refer to linear equality and inequality constraints. They simply are set as empty arguments (for example, A=[ ];).
B	
Aeq	
Beq	
Lb	Vector of lower bounds value of variables, in this work applied to the sets V and P.
Ub	Vector of upper bounds value of variables, in this work applied to the sets V and P.
@nonlcon	The handle of the M-function file contains the nonlinear equality and inequality functional constraints, here; @(X)constraints OPF(X)
options	This structure provides optional parameters for the optimization process, which are set by means the gaoptimset function of Matlab

Table 1. Input Argument

Name	Description
x	This vector contain the numerical values of the system environment variables in the solution of the optimization model.
fval	Value of the objective function in the solution x
exitflag	Describe the exit conditions of ga; if exitflag> 0, convergence was achieved. If exitflag = 0, the maximum number of iterations was exceeded. If exitflag <0, there was no convergence.
output	The structure that contains information about the results of the optimization process.
population	Matrix whose ranks contain the members of the final population
scores	Column vector of fitness values

Table 2. Output Argument

The description of the implementation of the algorithms is summarized by means of the represented algorithm in Fig. 3.

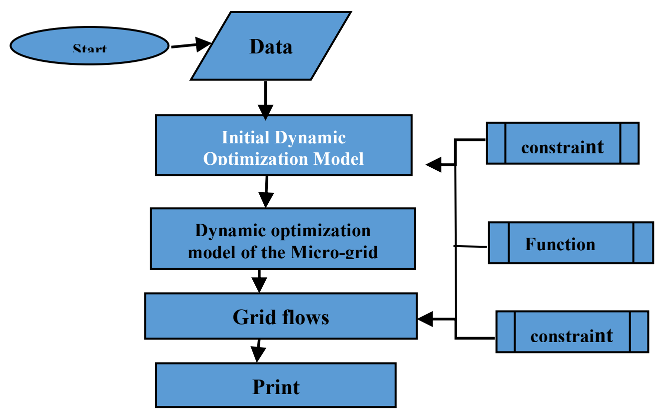

Fig. 3. Optimization computational algorithm

5 RESULT

The numerical results obtained from the simulations of the algorithm in MATLAB® are shown below.

The microgrid is called microgrid test, which is integrated by a photovoltaic panel, a vertical wind turbine and storage system. The microgrid test is selected as a case study to show the results of the energy management optimal obtained from the selected tools. The values of the grid models, photovoltaic module, vertical wind turbine components, as well as the cost coefficients associated with the energy generated are presented in the appendix. For the case study has been considered the unit variables as per unit (pu), for which it took a base power and voltage of 10 KVA and 100 V, respectively. For the analysis, the forecasted curves of active power demand, wind speed and solar irradiation from the city of Cancun (Mexico) were considered. These signals are displayed in Fig. 5a, 5b and 5c, respectively, and were provided by the National Meteorological Service of Mexico. The limits of the magnitudes for the DC voltage for all nodes in the microgrid are 0.95≤V≤1.05 pu. The limits of batteries SOC are 0.2 ≤SOC (tz) ≤0.95 (in %) while the initial state of charge is SOC(t0) = 0.2 (in %). The results, which have been obtained with a PC DELL with 8 GB of RAM and an Intel processor i5 - 3230 M CPU @ 2.40 GHz, from the solution of the model of optimization are shown and discussed below.

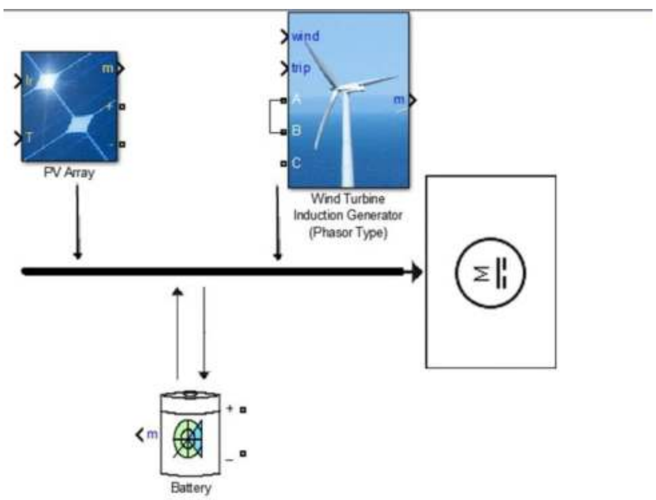

Fig. 4. Microgrid scheme

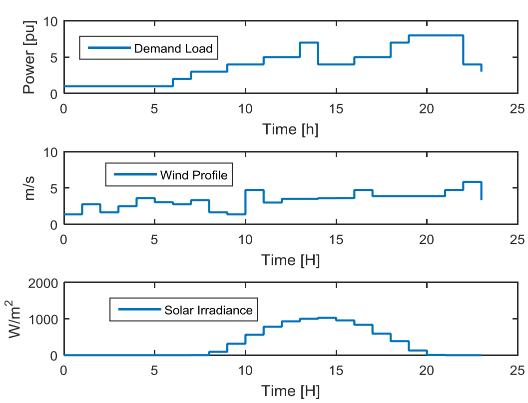

Fig. 5. Forecasted signals: a) active power demand, b) wind speed, c) solar irradiance

Fig. 6a shows the active power provided by the battery, wind turbine and module PV. There is a greater percentage of the demand curve, see Fig. 5a, which is supplied from the wind turbine, is clear that the power provided by the wind turbine has a similar shape than the wind speed shown in Fig. 5b. It is important to highlight that, the photovoltaic panel has a relevant role in the energy contribution between the 8:00 and 19:00 hours, which matches with the period of high irradiation, see Fig. 5c. The battery experiences periods of charge $\left(P_{Bnj}^{t_z} < 0\right)$ and discharge $\left(P_{Bnj}^{t_z} > 0\right)$ In particular, it is possible to see that in periods with high wind power generation (e.g. at hours 10, 18 and 23) the battery is charged $\left(P_{Bnj}^{t_z} < 0\right)$. This power is useful to reduce the energy cost in hours of peak demand (e.g. at hours 10 and 20). This fact fits with the battery SOC shown in Fig. 6b. In Fig. 7. shows the profiles of voltages, it is we see clearly that the voltages are maintained at the admissible levels.

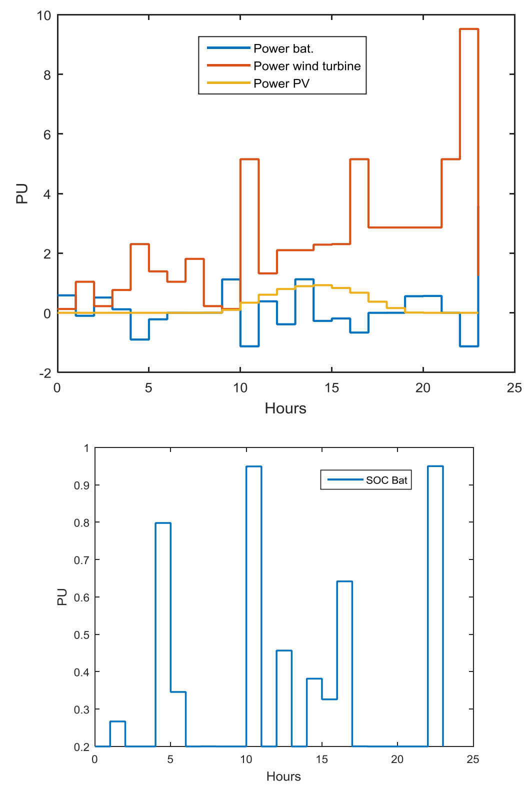

Fig. 6. Case study solved: a) generated power, b) SOC Battery.

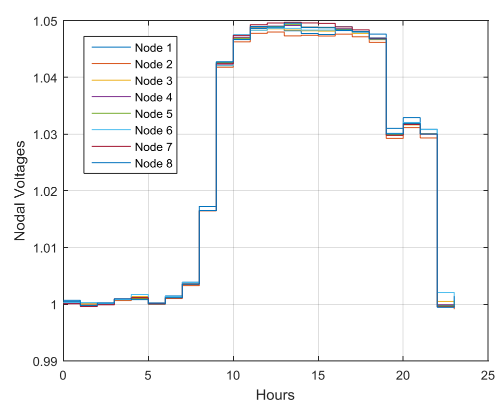

Fig. 7. Case study solved, Voltages Nodes.

6 CONCLUSIONS

In this work a model for the analysis of the hybrid renewable energy system for autonomous vehicle is presented. The model allows us to manage the energy storage elements to improve the economic operation of the microgrid. Likewise, through it is possible to determine in which periods is optimal to discharger or charger the storage system. The model considers basic models of the elements of the photovoltaic panels, wind turbines, as well as batteries for energy storage. The model is solved by one optimization method: a hybrid genetic algorithm with interior point (GA-IP). The presented numerical results illustrate the potential analysis of model of optimization to determine values that could be used as a reference by the secondary controller of a microgrid. In addition, the validation of the optimization methods has been performed throughout the evaluation of a case study which has been efficiently solved.

In this work the objective function is continuous. Therefore, using the method of genetic algorithms for optimization we have the flexibility to use non-continuous functions, this due to the optimization method of genetic algorithms does not need the gradient of the objective function to find the minimum.

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Appendix A.

This appendix shows the parameters of the hybrid renewable energy system components of the case study, which are the following:

Performance data of the microgrid components

$P_{wind\ turbine} = 10000\ watt \quad -100 < Q_{wind\ turbine} > 1000 var$

$P_{bat} = 2000\ watt$

$P_{PV} = 2000\ watt$

Grid cost coefficients

$a = 0.014,\ b = 0.020,$ c=0.0060

Voltage general bounds

$Min = 0.95$ pu, $Max = 1.05$,

