

A Comparative Study of State Estimation Methodologies for Electric Power Systems

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Abstract: This paper establishes a design and performance comparison between two different estimation techniques in a Single Machine Infinite Bus (SMIB) system, these techniques are the Extended Kalman Filter which is a classic estimator that has been used in the power systems for almost forty years and a nonlinear observer whose design is based on nonlinear mathematical model.

Keywords: Extended Kalman Filter, Non linear observer, State estimation, SMIB.

1. INTRODUCTION

The complexity of the electric power systems (EPS), its fast grown as well as the incursion of new technologies aimed at the generation of electrical energy, have caused the emergence of new problems that conventional solutions can not solve, for that reason the study of EPS takes major relevance for power system community (Ziping et al., 2018).

The state estimation problem is a topic that has gained relevance in recent years, the emergence of new generation technologies in EPS have motivated that the traditional state estimators lose in some cases either some their features advantages or performance properties, in this way research have been focused on studying other observation techniques based on a modified version of the Kalman Filter, mainly due to their features, *i.e.*, robustness against the noise, versatility and recursive approach (Bishop et al., 2001), most of them used in the chemical industry (Tebianian and Jeyasurya, 2013; Ghahremani and Kamwa, 2015; Zhou et al., 2014).

The application of nonlinear observer techniques is a novel solution in power system's theory as is exposed in Alhelou et al. (2018), also in Shoukry et al. (2018) addresses a fault sensor problem considering cyber-attacks in the measurement systems, it is clear that the estimation problem needs a more robustness and reliable estimators, thus the use of nonlinear observer techniques represent a novel tool for solving in a better way these current problems.

The main contribution of this paper is to design two different estimation methodologies for a SMIB system, in order to make a comparison between these observer techniques so as to analyse the performance of this estimators and their principal features to establish the advantages and disadvantages for both of them. The estimators are based on Extended Kalman Filter (EKF) methodology and a

nonlinear observer theory. The comparison is carried out using a Single Machine Infinite Bus (SMIB) system because this is considered as a classical example in the power system theory and it allows to show the principal features of each estimator including its design.

This paper is organized in five sections, in the first section the SMIB mathematical model and the principal assumptions about it are presented, in the second section is talked about the estimators methodology design, at first an explanation about the EKF is given and the design for a SMIB is considered, in the same way, the nonlinear observer is explained in detail and its application for a SMIB system is introduced, in the following section, two study cases are presented, finally, some concluding remarks are included.

2. SMIB MATHEMATICAL MODEL

The SMIB system is modeled by the three-dimensional Ordinary Differential Equation (ODE) system, this model is represented by Sauer and Pai (1998)

$$\dot{\delta} = \omega \quad (1)$$

$$M\dot{\omega} = P_m - D\omega - \lambda_1 E'_q \sin \delta$$

$$\tau \dot{E}'_q = -\lambda_2 E'_q + \lambda_3 \cos \delta + E_{fd} \quad (2)$$

where λ_1 , λ_2 and λ_3 are power system parameters, δ is the rotor angle, ω is the angular velocity and E'_q is the quadrature voltage, P_m is the mechanical power, D is the factor damping, M is the inertia factor and τ is the transitory time constant. In this case, the dynamic associated with the electric system (2) is considered faster than the mechanical's system dynamic (1) Milano (2010), so that it is possible to analyze (2) through a singular perturbation analysis *i.e.*, the dynamic of E'_q is a nonlinear function of the rotor angle and it can be expressed by

$$E'_q = \frac{\lambda_3}{\lambda_2} \cos \delta + \frac{1}{\lambda_2} E_{fd} \quad (3)$$

The substitution of the quasi-static state E'_q in (1) yields

$$\begin{aligned} \dot{\delta} &= \omega \\ \dot{\omega} &= \frac{1}{M} \left[P_m - D\omega - \lambda_1 \left(\frac{\lambda_3}{\lambda_2} \cos \delta + \frac{1}{\lambda_2} E_{fd} \right) \sin \delta \right] \end{aligned} \quad (4)$$

where the electric power is given by

$$P_e = \lambda_1 \left(\frac{\lambda_3}{\lambda_2} \cos \delta + \frac{1}{\lambda_2} E_{fd} \right) \sin \delta \quad (5)$$

The SMIB model is restricted to the next condition

- The mechanical power must satisfy

$$0 \leq P_m \leq \frac{\lambda_1}{\lambda_2} E_{fd} \quad (6)$$

where the mechanical input power is considered constant.

- It there must be an invariant and compact set D so that

$$D = \{[\delta, \omega] \in \mathbb{R}^2 : 0 \leq \delta \leq \frac{\pi}{2} - \varepsilon\} \quad (7)$$

where $\varepsilon > 0$ and it is arbitrarily small.

In addition, in the power system theory it is well known that the condition (6) and (7) must be accomplished in order to guarantee that EPS operation point has asymptotic stability proprieties Machowski et al. (2011).

The power system may be affected by different kind of faults, one of them is when the generated electric power by the SMIB is zero in a short lapse of time. This fault is equivalent to have a three-phase short circuit on a transmission line, it may induce a change in the power system structure. For this case, the fault is treated as an unknown input ι in (4) whose dynamic is represented by

$$\frac{d}{dt} \iota = \frac{\lambda_1}{M\lambda_2} (\lambda_3 \cos 2\delta + E_{fd} \cos \delta) \omega \quad (8)$$

The augmented state comprises $[x_1, x_2, x_3] = [\delta, \omega, \iota]$, these are given by (4) and (8). The three state model of SMIB in compact form is written as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= a_1 P_m - a_2 x_2 - a_3 (\lambda_3 \cos x_1 + E_{fd}) \sin x_1 + x_3 f_a(t) \\ \dot{x}_3 &= a_3 (\lambda_3 \cos 2x_1 + E_{fd} \cos x_1) x_2 \end{aligned} \quad (9)$$

where $a_1 = \frac{P_m}{M}$, $a_2 = \frac{D}{M}$, $a_3 = \frac{\lambda_1}{M\lambda_2}$ and f_a is an auxiliary function that represents the fault's time (t_f)

$$f_a(t) = \begin{cases} 1 & \text{if } t_f \neq 0 \\ 0 & \text{if } t_f = 0 \end{cases} \quad (10)$$

In the next section the estimation methodologies are explained in detail, the former corresponds a classic Extended Kalman Filter and the latter is a nonlinear

observer methodology that works for a special class of nonlinear systems.

3. ESTIMATORS METHODOLOGY DESIGN

3.1 Extended Kalman Filter design

The state estimation problem is focused on an Ordinary Differential Equation (ODE) model and it is studied within a global (non local) framework, including:

- The attainment of a comparatively (with respect to previous approaches) of a **better compromise** between state reconstruction speed, accuracy and robustness as well as on-line computational burden (determined by the number of equations and their ill conditioning).
- The identification (with physical meaning) of the underlying **robust solvability** condition in terms of robust observability/detectability.
- A priori (before numerical implementation) guarantee of **robust functioning** in terms of control gains and meaningful (parametric and input) error bounds.
- **Simple** (conventional-like) and **systematic construction tuning** procedure.
- Fair benchmark **comparison** through numerical simulation of the proposed **versus existing** (mostly EKF) **approach**.

Consider the mathematical model given by

$$\dot{x} = f(x, t) + Bw \quad x(0) = x_0; \quad \forall t \geq 0 \quad (11)$$

where $x \in \mathbb{R}^n$ is the state variable of the system, $B \in \mathbb{R}^{n \times m}$ is the matrix that indicates which states are affected by the incident noise and $w \in \mathbb{R}^m$ is a zero-mean Gaussian white noise with constant intensity Q . The measured output signal is defined by

$$y = g(x) + v \quad (12)$$

where $g(x) \in \mathbb{R}^p$ is the output signal and $v \in \mathbb{R}^p$ is a zero-mean Gaussian white noise with constant intensity matrix r .

Following the proposed methodology of Álvarez and Fernández (2009), the EKF is given by (13).

$$\dot{\hat{x}} = f(\hat{x}, t) + K[y - g(\hat{x})] \quad \hat{x}(0) = x_0, K = \vartheta H^\top(\hat{x}, t) r^{-1} \quad (13)$$

where K is a Kalman filter gain and ϑ is the solution of the Riccati equation

$$\dot{\vartheta} = F(\hat{x}, t)\vartheta + \vartheta F^\top(\hat{x}, t) + Q - \vartheta H^\top(\hat{x}, t) r^{-1} H(\hat{x}, t)\vartheta, \quad \vartheta(0) = \vartheta_0 \quad (14)$$

where

$$F(x, t) = \frac{\partial f(x, t)}{\partial x}, \quad H(x, t) = \frac{\partial g(x)}{\partial x} \quad (15)$$

The Kalman filter gain may be adjusted through standard linear stochastic optimal filtering techniques Kwakernaak and Sivan (1972). To simplify the filter design, (14) is multiplied by r^{-1} , it allows to have just one design parameter

$$r^{-1} \left(\dot{\vartheta} = F(\hat{x}, t)\vartheta + \vartheta F^\top(\hat{x}, t) + Q - \vartheta H^\top(\hat{x}, t)r^{-1}H(\hat{x}, t)\vartheta \right) \quad (16)$$

Defining $\vartheta r^{-1} = S$ and Q is a constant diagonal matrix which may be rewritten as qI

$$\begin{aligned} \dot{\hat{x}} &= f(\hat{x}, t) + c(\hat{x}, S)(y - g(\hat{x})) \\ \dot{S} &= F(\hat{x}, t)S + SF^\top(\hat{x}, t) + \left(\frac{q}{r}\right)I - SH^\top(\hat{x}, t)H(\hat{x}, t)S \end{aligned}$$

where $c(\hat{x}, S) = SH^\top(\hat{x}, t)$ is the gain vector of the EKF and (q/r) is the only tuning parameter (motivated of geometric control) $w = (q/r)$ where w is defined as ten times the frequency of the system.

EKF for a SMIB In the particular case of the SMIB presented in this work, the design of the EKF is based on the augmented state model (9) and considering the rotor angle as the measured output:

$$\dot{\hat{x}}_1 = \hat{x}_2 + c_1(\hat{x}, S)(y - \hat{x}_1) \quad (17)$$

$$\begin{aligned} \dot{\hat{x}}_2 &= a_1 P_m - a_2 \hat{x}_2 - a_3 (\lambda_3 \cos \hat{x}_1 + E_{fd}) \sin \hat{x}_1 + \hat{x}_3 f_a(t) \\ &= +c_2(\hat{x}, S)(y - \hat{x}_1) \end{aligned} \quad (18)$$

$$\dot{\hat{x}}_3 = a_3 (\lambda_3 \cos 2\hat{x}_1 + E_{fd} \cos \hat{x}_1) \hat{x}_2 + c_3(\hat{x}, S)(y - \hat{x}_1) \quad (19)$$

where $c_1(\hat{x}, S)$, $c_2(\hat{x}, S)$ and $c_3(\hat{x}, S)$, are the i -th component of the Kalman filter gain vector, considering $\hat{x} = [\hat{x}_1 \ \hat{x}_2 \ \hat{x}_3]^\top$.

The Jacobian matrices are computed at each time step to determine the local linearized model of the system.

$$F(x, t) = \begin{bmatrix} 0 & 1 & 0 \\ \varphi_1 & -a_2 & f_a(t) \\ \Delta_1 & a_3(\lambda_3 \cos 2x_1 + E_{fd} \cos x_1) & 0 \end{bmatrix} \quad (20)$$

$$H(x, t) = [1 \ 0 \ 0] \quad (21)$$

where

$$\varphi_1 = -a_3(\lambda_3 \cos 2x_1 + E_{fd} \cos x_1) \quad (22)$$

$$\Delta_1 = a_3(2\lambda_3 \sin 2x_1 + E_{fd} \sin x_1)x_2 \quad (23)$$

3.2 Nonlinear observer design

For explaining the methodology observer design, we will consider systems whose mathematical model can be represented by the next equations

$$\dot{\eta} = A(y, u)\eta + B(y, u) \quad (24)$$

$$\dot{y} = \psi_0(y, u) + \psi_1(y, u)\eta$$

where $[\eta, y] \in \mathbb{R}^n \times \mathbb{R}^p$ is the system state, $\eta \in \mathbb{R}^{n-p}$ is the unmeasured state, $y \in \mathbb{R}^p$ is the measured output and $u \in \mathbb{R}^m$ is the control input. The system (24) can be represented as

$$\dot{\eta} = A(y, u)\eta + B(y, u) \quad (25)$$

$$z = \psi_1(y, u)\eta \quad (26)$$

where $z = \dot{y} - \psi_0(y, u)$.

Notice that one possible observer is given by

$$\dot{\hat{\eta}} = A(y, u)\hat{\eta} + B(y, u) + K_0(z - \hat{z}) \quad (27)$$

where

$$\hat{z} = \psi_1(y, u)\hat{\eta} \quad (28)$$

Substitution of (27) into (28) leads to the equivalent representation

$$\begin{aligned} \dot{s} &= (A(y, u) - K_0\psi_1(y, u))\hat{\eta} + B(y, u) - K_0\psi_0(y, u) \\ \hat{\eta} &= s + \beta(y) \end{aligned} \quad (29)$$

If it is defined the observation error by

$$\varepsilon = s + \beta(y) - \eta, \quad (30)$$

then the error derivative follows that

$$\dot{\varepsilon} = \left(A(y, u) - \frac{\partial \beta(y)}{\partial y} \varphi_1(y, u) \right) \varepsilon \quad (31)$$

if $A_e = \left(A(y, u) - \frac{\partial \beta(y)}{\partial y} \varphi_1(y, u) \right)$ then the error dynamic is given by the next expression

$$\dot{\varepsilon} = A_e \varepsilon \quad (32)$$

Proposition 1. There is a mapping $\beta(y) : \mathbb{R}^q \rightarrow \mathbb{R}^{n-q}$ so that the matrix A_e is continuous, differentiable and non singular \square .

If A_e is continuous, bounded and non singular matrix, there is an only equilibrium point that is $\varepsilon = 0$. In order to prove Lyapunov stability it is proposed the next candidate Lyapunov function

$$V(\varepsilon, t) = \varepsilon^\top P(t)\varepsilon \quad (33)$$

where $V(\varepsilon, t) : \mathbb{R}^{n-p} \times \mathbb{R} \rightarrow \mathbb{R}$ also there is a matrix $P(t) \in \mathbb{R}^{n-p}$ that is a positive symmetric defined continuously differentiable and bounded matrix. The derivative of $V(\varepsilon, t)$ along to the system trajectories is given by

$$\dot{V}(\varepsilon, t) = \varepsilon^\top (A_e^\top P(t) + \dot{P}(t) + P(t)A_e)\varepsilon \quad (34)$$

as A_e is continuous and bounded, the matrix $P(t)$ meets the following

$$\dot{P}(t) + A_e^\top P + PA_e = -Q(t)$$

where $Q(t)$ is a positive symmetric continuous matrix, therefore the Lyapunov derivative function along of the system trajectories is presented by the next equation

$$\dot{V}(\varepsilon, t) = -\varepsilon^\top Q(t)\varepsilon \quad (35)$$

Theorem 2. Let $\varepsilon = 0$ an equilibrium point of the system (32) and $D \subset \mathbb{R}^{n-p}$ a domain which contains $\varepsilon = 0$. Let a function $V : [0, \infty) \times D \rightarrow \mathbb{R}$ that is a continuously differentiable function so that

$$k_1 \|\varepsilon\|^a \leq V(t, \varepsilon) \leq k_2 \|\varepsilon\|^a \quad (36)$$

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial \varepsilon} f(t, \varepsilon) \leq -k_3 \|\varepsilon\|^a \quad (37)$$

$\forall t \geq 0$ and $\forall \varepsilon \in D$ where k_1, k_2, k_3 and a are positive constants. Then $\varepsilon = 0$ is exponentially stable; if the assumptions hold globally, then $x = 0$ is globally exponentially stable. \diamond

The purpose is to design the gain $A(y, u) - \frac{\partial \beta(y)}{\partial y} \psi_1(y, u)$ in such a way that the time derivative of the function V fulfills (36) and (37).

Observer design for a SMIB. The observer design for the system which is represented by (9), it will be shown in this section. At first if it considers that the measured state is δ i.e. $y = x_1$ and unmeasured state is ω and ι i.e. $\eta = [\eta_1, \eta_2] = [x_2, x_3]$ so the system representation at the structure (24) is given by

$$\begin{aligned} \begin{bmatrix} \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} &= \underbrace{\begin{bmatrix} -a_3 & f_a(t) \\ a_3(\lambda_3 \cos 2x_1 + E_{fd} \cos x_1) & 0 \end{bmatrix}}_{A(y, u)} \begin{bmatrix} x_2 \\ x_3 \end{bmatrix} \\ &+ \underbrace{\begin{bmatrix} a_1 - a_3(\lambda_3 \cos x_1 + E_{fd} \sin x_1) \\ 0 \end{bmatrix}}_{B(y, u)} \\ \dot{x}_1 &= \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{\psi_1(y, u)} \begin{bmatrix} x_2 \\ x_3 \end{bmatrix} \end{aligned} \quad (38)$$

Thus, the observer's structure for the system (9) takes the next form

$$\begin{aligned} \dot{s}_1 &= - \left(a_3 + \frac{\partial \beta_1(x_1)}{\partial x_1} \right) \hat{x}_2 + f_a(t) \hat{x}_3 + a_1 \\ &- a_3(\lambda_3 \cos x_1 + E_{fd}) \sin x_1 \\ \dot{s}_2 &= \left(a_3(\lambda_3 \cos 2x_1 + E_{fd} \cos x_1) - \frac{\partial \beta_2(x_1)}{\partial x_1} \right) \hat{x}_2 \end{aligned} \quad (39)$$

also

$$\begin{aligned} \hat{x}_2 &= s_1 + \beta_1(x_1) \\ \hat{x}_3 &= s_2 + \beta_2(x_1) \end{aligned} \quad (40)$$

For this specific system, the function $\beta(y)$ is

$$\begin{aligned} \beta_1(x_1) &= k_1 a_2 (x_1 - x_1^*) \\ \beta_2(x_1) &= k_2 a_3 \left(\frac{1}{2} \lambda_3 (\sin 2x_1 - \sin 2x_1^*) \right) \\ &+ E_{fd} (\sin x_1 - \sin x_1^*) \end{aligned} \quad (41)$$

where k_1 and $k_2 \in \mathbb{R}$ and they are positive.

4. ADVANTAGES AND DISADVANTAGES OF DESIGNED ESTIMATORS

It should be highlighted some aspects about each estimation technique:

- The Extended Kalman Filter is optimal when the system is affected by Gaussian white-noise, also is a well studied state estimation methodology.
- The Kalman Filter gain is not parameterized, in this sense its implementation is nontrivial.
- One of the main drawbacks, is that the EKF requires an extra computational burden due to the computation of Jacobian expressions as well as by the Riccati equations solution, whose dimension increases quadratically with respect to the state dimension.
- The proposed nonlinear observer has a formal convergence proof, so it is easy to tune.
- The nonlinear observer gain is parameterized, in this sense its sintonization and implementation is easier in comparison with the EKF.
- The nonlinear observer design needs a specialized knowledge to be tuned.

5. SIMULATION RESULTS

The tests evaluate the performance between the observers presented in the previous section. The experiment consists in to disturb a SMIB to prove the observer robustness. In order to get our target, two scenarios are established; in the former the SMIB is disturbed in such a way that the power system recovers its stability. In the second study case, the SMIB is again affected by a three-phase fault, but in this case the electric power system can not recover its stability, so the generator unit loses synchrony. In both cases it has been considered Gaussian white noise affecting the states and measurements.

The considered SMIB system has the next parameters

Parameter	Value[p.u.]
Mechanic Power (P_m)	0.815
Inertia constant M	0.0147
Power system parameter λ_1	2
Power system parameter λ_2	2.7
Power system parameter λ_3	1.7
Field voltage	1.22
Damping factor	0.0588

Table 1: SMIB parameters

5.1 Three-phase fault liberation

Among the principal points that have been considered of the observers performances are:

- The convergence time.
- The robustness observer to measurements with noise.
- The over impulse in the estimation state.
- The observer tuning to get the estimation gains and its ease implementation.

In this study case, it is assumed that the SMIB and the observers start in a steady state condition; the three-phase fault takes place at 5[s], where its fault liberation time is 0.5[s], according to the noise affecting the states and measurements, the variances related to the process and measurement noise are set to 1.5×10^{-5} and 2×10^{-6} . For the EKF the tuning parameter is set to $q/r = 800$. For the nonlinear observer the design parameters are the functions $\beta_1(y)$ and $\beta_2(y)$.

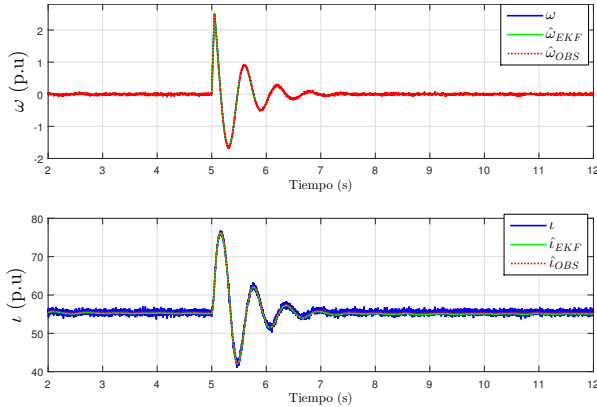


Fig. 1. SMIB affected by a three-phase fault.

It is possible to appreciate in the Figure 1 that the performance of both estimators is very similar, both are capable to estimate the transit state that is induced by the three-phase fault disturbance. The estimation of ω through the nonlinear observer is susceptible to the noise. In this case the EKF shows a better performance than the nonlinear observer because it reduces the incident noise in the state; with respect to the estimation of the disturbance ι both observers carried out a good transient estimation, also both estimators can reduce in a great manner the incident noise in the system.

5.2 Three-phase fault without liberation

In this study case the fault which use to disturb the power system has a duration of more than five system cycles *i.e.*, the SMIB can not recover its stability, in other words, the synchronous machine losses its synchronism. The experiment in this section consists to study the observers performance in the transient stability when the synchronous machine can not return to a steady state, additionally, it has been considered that the states and measurements are affected by Gaussian white-noise whose variances were established with the same values of the previous section.

The Figure 2 shows the estimation of ω and ι , both observers are capable to estimate the persistent transient state present in both states. In the Figure 2 can be appreciated that the noise incidence does not affect the observers performances, they make that the noise in the states would be imperceptible. It must refer to the right tuning of the observer gains avoid apparition of overdrafts in the estimation of the states and of the disturbances.

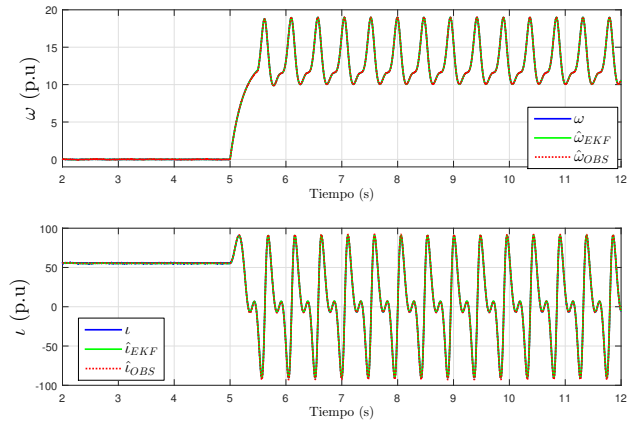


Fig. 2. SMIB against a three-phase fault without liberation

To complement the below explanation, in the Figure 3 shows that the estimation error in the synchronous relative frequency, it can be said that the convergence error of both estimators tends to zero in a short time lapse but the convergence error of the nonlinear observer tends approximately for a second faster than the EKF estimator.

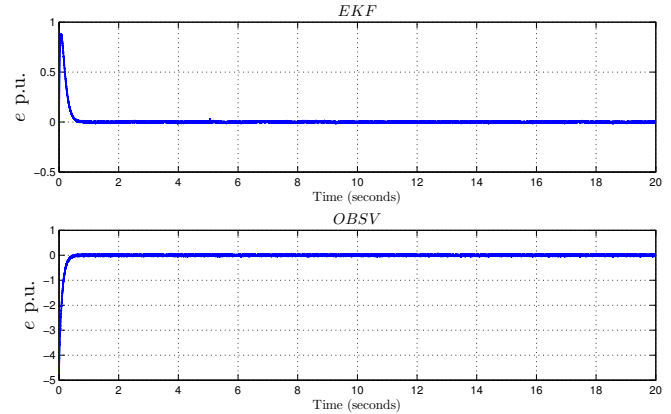


Fig. 3. Error state estimation of ω .

In the same way, the error associated with the estimation of the disturbance which is shown in the Figure 4, it is possible to conclude that the convergence error of the nonlinear observer is faster than the error obtained by the EKF.

6. CONCLUSIONS

In this paper was contrasted two methodologies for solving the state estimation problem in EPS in order to show the principal features of a EKF estimator and a nonlinear observer, the principal results are cited below:

- The mathematical representation of the three-phase fault allowed to reconstruct it with both estimators.
- The performance analysis may be divided in three sections
 - The nonlinear observer convergence time is less than the EKF's convergence time.

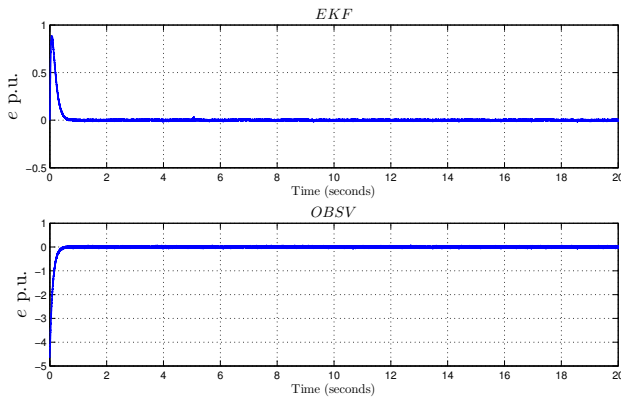


Fig. 4. State estimation error of v .

- The over impulse of the nonlinear observer is bigger than the obtained by the EKF.
- Both estimators are robust against the noise even though the EKF presents a better performance due to in the tests the estimated states by this technique show a lower noise incidence in comparison to those obtained by the nonlinear observer.
- The design methodology is easier in the case of the EKF due to it is a recognized technique and it has been thoroughly used it does not need a especial knowledge contrary to the non linear observer design that needs an especial knowledge to design and tuning.
- The transient response is reconstructed by both observer however the nonlinear observer presents a better transient reconstruction in comparison to the EKF estimator.

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