

Challenges in the design of a sensorless control for the SR motor^{*}

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Abstract: This article presents the major challenges to design a state observer for the Switched Reluctance motor when the measured signals are only the electrical ones, that is, phase currents and voltages. The results presented in this paper are based on the observability analysis of the small signal model. These are: 1) the model is non-uniformly observable *i.e.*, the observability depends on the input, 2) there are singular points on the observability map which are related to the motor commutation. To overcome the first problem it may be possible to design an immersion-based observer, an example of this method for a nonuniform observable system is presented to show its potential. On the other hand, the observability analysis results are used to propose a very simple example of a sensorless method based on sinusoidal inputs for low speed. The objective is to design a stator phase commutator based in current measurements only.

Keywords: switched reluctance motor, estimator, observers, observability

1. INTRODUCTION

This paper is motivated by the appealing qualities of the Switched Reluctance Motor (SR motor), such as its simple and rugged construction, no need of permanent magnets, and its torque-speed characteristics. In particular, the SR motor has the unique property that it can still operate during fault condition (Saha and Choudhury, 2016), which means that this motor is a reliable source of motion. These properties have made it a serious candidate for traction in Electric Vehicles (EV) and Hybrid Electric Vehicles (HEV), as documented in Rahman et al. (2000), Zeraoulia et al. (2006) and Ehsani et al. (2018). However, there are some disadvantages for using this motor. For example, the control of this machine is more complicated than for other motors. In particular, this motor cannot be open loop controlled. Another drawback is that most industrial controllers present speed ripple and noise. Additionally, it is not as commercially available as other motors.

Another control challenge is the *sensorless control*, *i.e.*, the implementation of a speed control by using only electrical input (voltages) and output (currents) measurements, without any mechanical sensors. The main motivation behind the *sensorless control* is of economic

nature, since it can diminish the cost of the entire drive, while, from a technical viewpoint the elimination of mechanical sensors decreases the complexity and maintenance of the entire installation. On the other hand, as claimed by Ehsani and Fahimi (2002), the self-tuning sensorless techniques for the SR motor may have a better performance than the sensor-based ones when there are parameter variations involved.

The sensorless problem for the switched reluctance motor has been extensively studied and partially solved using different methods, which can be classified as: 1) active phase detection, 2) current gradient, 3) flux linkage reconstruction and 4) state observers. In general, the first two methods are signal-based and the two last ones are model-based methods.

In this article an observability analysis is developed, which shows the two main challenges to design an state observer for this system, namely, the model is non-uniform observable, *i.e.* the observability depends on the input, and the observability map presents singularities, which are linked to the way the motor commutes.

The observability analysis is based on the observability rank condition from Hermann and Krener (1977) and the local weak observability property defined therein. To overcome the first challenge, it may be possible to define an state extension, in the sense of the one presented in Besancon and Ticlea (2007), to get the appropriate

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form for observer design (Torres et al., 2012). The singularities are related to the motor commutation, therefore this information, along with the geometry characteristics of the motor, is used to design an estimator of the mechanical variables. Then, this estimator is used to design a low speed controller which validates the observability analysis results. It must be mentioned that this article is an extension of a previous work (De La Guerra et al., 2015), where an observability condition was stated based on a different observability definition and a *natural* input for the SR motor.

This article is organized as follows: Section 2 presents the SR motor model and its commutation. In Section 3, an observability analysis is developed. Later in the same section, an example of an observer design for a non-uniformly observable system is presented. Section 4 presents the proposed estimator and the corresponding low-speed controller. Section 5 includes simulation results to validate the proposed scheme. Finally, in Section 6 some conclusions and directions for future work are provided.

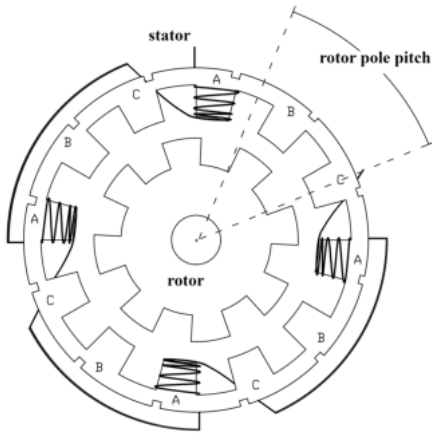


Fig. 1. Three-phases SR motor and rotor pole pitch

2. SWITCHED RELUCTANCE MOTOR MODEL

A mathematical model for a m -phases SR motor is

$$\frac{d\mathbf{i}}{dt} = \mathbf{L}^{-1}(\theta)(-\omega\mathbf{C}(\theta)\mathbf{i} - \mathbf{R}\mathbf{i} + \mathbf{u}) \quad (1a)$$

$$\dot{\theta} = \omega \quad (1b)$$

$$\dot{\omega} = \frac{1}{2J}\mathbf{i}^T\mathbf{C}(\theta)\mathbf{i} - \frac{d}{J}\omega - \frac{1}{J}\tau_L, \quad (1c)$$

where $\mathbf{i} \in \mathbb{R}^m$ is the vector of stator currents, $\mathbf{u} \in \mathbb{R}^m$ is the vector of voltage inputs, $\omega \in \mathbb{R}$ is the angular velocity, $\mathbf{R} \in \mathbb{R}^{m \times m}$ is a diagonal matrix accounting for the winding resistances, $\tau_L \in \mathbb{R}$ is the load torque, $J \in \mathbb{R}$ is the rotor inertia and $d \in \mathbb{R}$ the viscous friction coefficient. The winding inductance is defined as

$$L_j(\theta) = l_0 - l_1 \cos\left(N_r \theta - (j-1)\frac{2\pi}{m}\right), \quad (2)$$

where $j = 1, 2, 3, \dots, m$, $\theta \in \mathbb{R}$ is the angular rotor position and $l_0 > l_1 > 0$ are the static winding coefficients.

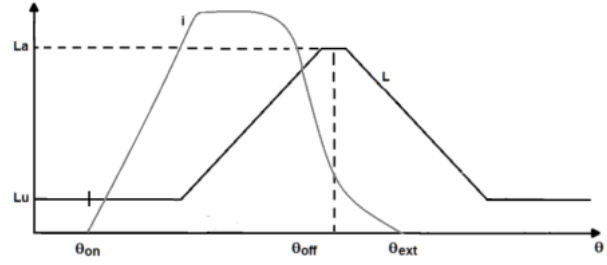


Fig. 2. Stator phase inductance, L_j , and current, i_j , in a power cycle. The magnitudes of the signals have been modified for the sake of clarity.

Thus, $\mathbf{L}(\theta) \in \mathbb{R}^{m \times m}$ is a diagonal matrix of winding inductances and $\mathbf{C}(\theta) \in \mathbb{R}^{m \times m}$ is given by

$$\mathbf{C}(\theta) = \frac{\partial \mathbf{L}(\theta)}{\partial \theta}.$$

Fact 1. The winding inductances matrix is symmetric and positive definite $\mathbf{L}(\theta) = \mathbf{L}(\theta)^T > 0$, which means that its elements are different from zero for every value of θ .

2.1 Commutation

To move the rotor shaft, a current must be circulating in the j -th phase when the poles of the rotor shaft are between the position corresponding to the minimum inductance value for this phase, θ_{on} , and the position corresponding to the maximum inductance value, θ_{off} (see Figure 2). In this figure, θ_{ext} is the angle where the phase current is extinguished. During each fundamental switching period, named rotor pole-pitch, which is show in Figure 1, all the phases are excited once and the interval between the excitation of two consecutive phases is called the *stroke angle* (Miller, 2001). One stroke is equal to $S = mN_r$, so the stroke angle is

$$\theta_S = \frac{2\pi}{mN_r},$$

which defines the fundamental switching frequency

$$f = \frac{N}{60}N_r, \quad (3)$$

with N the angular speed in [RPM]. The total produced torque is the sum of the torque produced for each phase. There must be one stroke per rotor pole-pitch for each phase, and the current in each phase flows in a fraction of the cycle.

3. OBSERVABILITY ANALYSIS

In this section, it is presented an observability analysis of the SR motor model using the so-called *observability rank condition* (Hermann and Krener, 1977). System (1) can be rewritten in the form (7) as

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}_0(\mathbf{x}) + \mathbf{f}_1(\mathbf{x})\mathbf{u} \\ \mathbf{y} &= \mathbf{h}_0(\mathbf{x}), \end{aligned} \quad (4)$$

where $\mathbf{x} = [i \ \theta \ \omega]$. The fields $\mathbf{f}_0, \mathbf{f}_1 \in \mathbb{R}^{m+2}$ and the output function $h_0 \in \mathbb{R}^m$ are defined as

$$\mathbf{f}_0 = \begin{bmatrix} -\frac{x_{11}}{L_1(x_2)}(C_1(x_2)x_3 + r) - \frac{x_{12}}{L_2(x_2)}(C_2(x_2)x_3 + r) \\ \dots \\ -\frac{x_{1m}}{L_m(x_2)}(C_m(x_2)x_3 + r), x_3, T_e - \frac{d}{J}x_3 \end{bmatrix}^T$$

$$\mathbf{f}_1 = \left[\frac{1}{L_1(x_2)} \ \frac{1}{L_2(x_2)} \ \dots \ \frac{1}{L_m(x_2)}, 0, -\tau_L(t)/J \right]^T$$

$$h_0 = [x_{11} \ x_{12} \ \dots \ x_{1m}]^T,$$

with

$$T_e = \frac{1}{2J} \mathbf{x}_1^T \mathbf{C}(x_2) \mathbf{x}_1.$$

For system (4), the observation space, $\mathcal{O}(h)$, which is formed with h_j and their Lie derivatives, is given by

$$\mathcal{O}(h) = [h_1 \ \dots \ h_m \ L_{f_1}h_1 \ L_{f_0}h_1 \ \dots \ L_{f_1}h_m \ L_{f_0}h_m]^T.$$

For a distribution based on these fields to be non singular at some point x_0 , the observability matrix, \mathbf{O} , constructed with the differentials of the elements of the observation space, has to be non singular at x_0 . There are multiple ways to construct \mathbf{O} , depending on the choice of outputs and Lie derivatives, in this case using the three outputs and the Lie derivatives of $h_1 = x_{11}$,

$$L_{f_0}h_1 = -\frac{x_{11}}{L_1(x_2)}(C_1(x_2)x_3 + r)$$

$$L_{f_1}h_1 = \frac{1}{L_1(x_2)}.$$

it is obtained the matrix,

$$\mathbf{O} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial L_{f_1}h_1}{\partial x_2} & 0 \\ \frac{\partial L_{f_0}h_1}{\partial x_{11}} & 0 & 0 & \frac{\partial L_{f_0}h_1}{\partial x_2} & \frac{\partial L_{f_0}h_1}{\partial x_3} \end{bmatrix}.$$

After some calculations, it can be shown that the rank condition is fulfilled if the determinant of matrix \mathbf{O} satisfies

$$\det(\mathbf{O}) = -\frac{C_1^2(x_2)x_{11}}{L_1^3(x_2)} \neq 0. \quad (5)$$

In other words, given Fact 1, $C_1(x_2)$ and x_{11} must be different from zero simultaneously.

Proposition 2. The SR motor model (1) is locally and weak observable, for the j -th phase, with $u_j \neq 0$, if the following conditions are satisfied

$$i_j \neq 0, -\frac{\theta_S}{2} < \theta < \frac{\theta_S}{2} \quad (6)$$

with θ_S the stroke angle. ∇

Remark 1. Condition (6) is directly related with the way in which the phases of the SR motor are switched to generate continuous movement of the rotor shaft, because $C_j(x_2) = 0$ determines the switching instants of the j -th phase. This means that the mechanical variables can be

reconstructed with the sum of the contributions of each phase in a power generation cycle (rotor pole-pitch) given the current and voltage for the corresponding phase are different from zero.

Remark 2. When the input of a given phase is identically zero, that is, $u_j \equiv 0$, it can be seen from (1a) that the corresponding phase current tends exponentially to zero. Thus, eventually \mathbf{O} will be singular. This implies that (6) will be fulfilled only if $u_j \neq 0$ which means that the observability depends on the input: model (1) is non-uniformly observable. This result is in accordance with the one obtained in De La Guerra et al. (2015).

3.1 A nonuniform observable system

As shown in the last section, model (1) is non-uniformly observable. To surmount this challenge, it may be possible to use a transformation to put model (1) in a suitable form for observer design. Accordingly, in this section it is presented the immersion procedure from Besancon and Ticlea (2007) to *locally immerse* a control-affine system of the form

$$\dot{\mathbf{x}} = f_0(\mathbf{x}) + \sum_{i=1}^n f_i(\mathbf{x})u_i$$

$$\mathbf{y} = h_0(\mathbf{x}) + \sum_{i=1}^n h_i(\mathbf{x})u_i, \quad (7)$$

satisfying the observability rank condition at some point x_0 , into the form,

$$\dot{\mathbf{z}} = \mathbf{A}(u, \mathbf{y}) + \mathbf{B}(u, z)$$

$$\mathbf{y} = \mathbf{C}(u)z + \mathbf{D}(u). \quad (8)$$

Immersion procedure:

- Build a vector $z_1(x)$ of all state-dependent functions $h_i(x)$ of (7), $i = 0 \dots M$.
- At step $k + 1$, assume the vectors z_1 to z_k have been constructed in the previous steps, and choose among the differentials of the elements in z_1, \dots, z_k a basis $\{d\phi_1, \dots, d\phi_k\}$ regular around x^o for the codistribution spanned by these differentials.
 - If $\nu_k = n$; end the procedure.
 - If not, construct a vector $z_{k+1}(x)$ by taking all functions $L_{f_i}z_k^j$, $i = 0 \dots m$, $j = 1 \dots N_k$ that do not satisfy $d\phi_1 \wedge \dots \wedge d\phi_{\nu_k} \wedge dL_{f_i}z_k^j = 0$ around x^o , with

$$z_k = \begin{pmatrix} z_k^1 \\ \vdots \\ z_k^{N_k} \end{pmatrix}.$$

- Notice that by construction, the elements in the vectors z_1, z_2, \dots , belong to the observation space of the system, \mathcal{O} , which means that their differentials are elements of $\Omega_{\mathcal{O}}$. The stopping condition of the procedure requires in other words that a basis of $\Omega_{\mathcal{O}}$ be found.

Next, it is presented an example of a non-uniform observable system, which has similar observability problems of model (1) :

$$\begin{aligned}\dot{x}_1 &= x_2 + (1 + x_3)u - x_1 \\ \dot{x}_2 &= -x_2 + x_1 \\ \dot{x}_3 &= \alpha x_3,\end{aligned}\quad (9)$$

where $\alpha \in \mathbb{R}$. Here, attention must be paid to the term multiplying the input in the first equation, because it is similar to the case of the inductance matrix inverse that multiplies the input in model (1).

The output defines the first variable as

$$h_0 = z_1^1 = x_1.$$

The Lie derivatives along the vector fields f_0 and f_1 are therefore,

$$\begin{aligned}L_{f_0}h_0 &= x_2 - x_1 \\ L_{f_1}h_0 &= 1 + x_3.\end{aligned}$$

In this case the Lie derivatives cannot be expressed only in terms of variable z_1^1 . Thus, both become new variables, z_2^1 and z_2^2 . Given that these three variables already define a basis for the observation space, the procedure stops here. The new state vector is define as $z = [z_1^1 \ z_2^1 \ z_2^2]$ and the system has the form (8) with

$$\dot{z} = \begin{bmatrix} 0 & 1 & u \\ 0 & 0 & -u \\ 0 & 0 & 0 \end{bmatrix} z + \begin{bmatrix} 0 \\ -2z_2^1 \\ \alpha(z_2^2 + 1) \end{bmatrix} u \quad y = [1 \ 0 \ 0] z \quad (10)$$

In this example the transformed system has the same dimension that the original system which, however, is not always the case.

Under the assumption of a sufficient persistent input an observer for system (10) can be designed as

$$\begin{aligned}\dot{\hat{z}} &= \mathbf{A}(u, y) + \mathbf{B}(u, \hat{z}) - \mathbf{\Gamma}(\lambda) \mathbf{S}^{-1} \mathbf{C}^T(u)(\hat{y} - y) \\ \dot{\mathbf{S}} &= -\lambda(-\gamma \mathbf{S} - \mathbf{A}(u, y)^T \mathbf{S} - \mathbf{S} \mathbf{A}(u, y) + \mathbf{C}^T \mathbf{C}) \\ \hat{y} &= \mathbf{C}(u) \hat{z} + \mathbf{D}(u)\end{aligned}\quad (11)$$

with

$$\mathbf{\Gamma}(\lambda) = \begin{bmatrix} \lambda I_{N1} & & 0 \\ & \lambda^2 I_{N2} & \\ & & \ddots \\ 0 & 0 & \lambda^q I_{Nq} \end{bmatrix}. \quad (12)$$

For the observer designed for this example, the state and their estimates are shown in Figure 1 and 2, where the input is obtained using a uniform random number generator .

Remark 3. The observer design for the SR motor becomes more involved since the observability map turns to be infinitely dimensional, in contrast with the one of the example, which is finite dimensional. This poses a major challenge to develop a sensorless observer for the SR motor. Moreover, the term that multiplies the input becomes a state for the transformed system. In the case of the SR motor model, another objective is to find the minimum number of variables to obtain a transformed system with the structure defined in Besancon and Ticlea (2007).

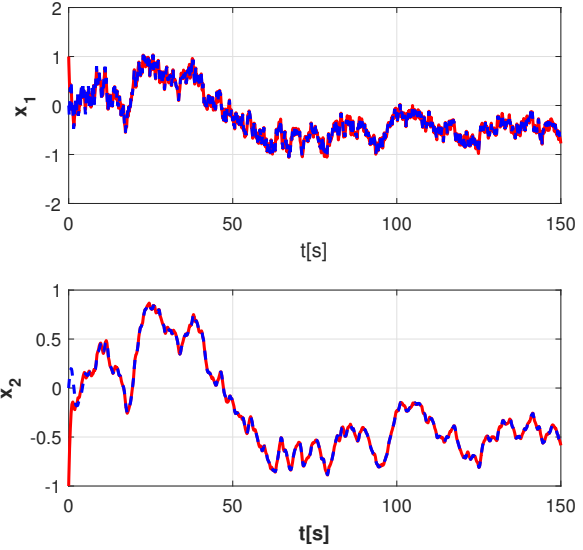


Fig. 3. Top : x_1 —, \hat{z}_1 - - -, Bottom : x_2 —, \hat{z}_2 - - -.

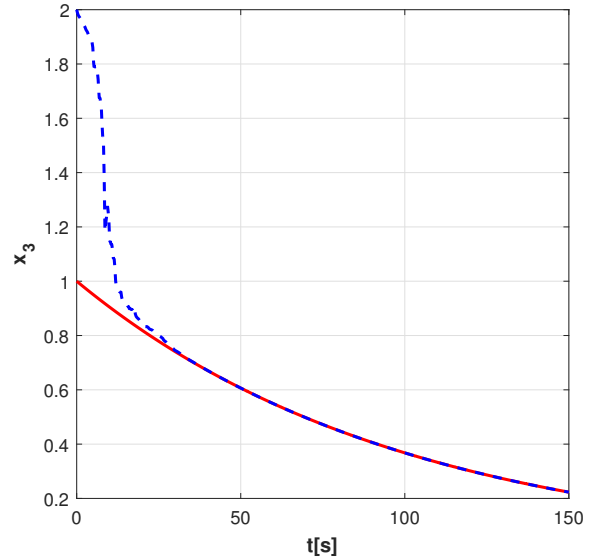


Fig. 4. Estimation x_3 —, \hat{z}_3 - - -, parameter $\alpha < 0$.

4. LOW SPEED SENSORLESS CONTROL

From the relations between inductance, current and position defined by the operation cycle described above, always fulfilling condition (6), it can be designed a method to reconstruct the rotor shaft angular position in a rotor pole pitch by detecting changes of the phase current and its derivative. This is also the basic idea behind the current gradient estimators as the one reported in Gallegos-Lopez et al. (1998), where the current is regulated close to a reference value, i_{ref} using PWM with a current gradient estimator used to appropriately commute the stator phases.

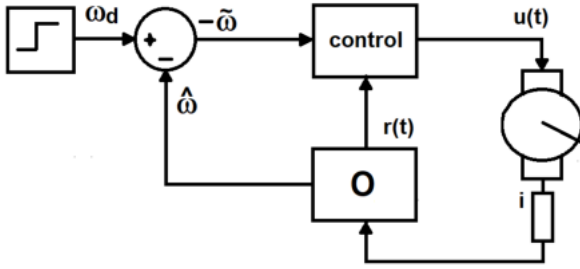


Fig. 5. Control system, block O represents the estimator.

4.1 Angular position/speed estimation

The estimation method is based on the comparison of the phase current derivative residuals for each phase with a constant threshold. This residuals are obtained by employing equation (1a) and the measured current, and are defined by

$$r_j(t) = u_j(t) - (l_0 - l_1) \frac{di_{jm}}{dt} - ri_{jm},$$

where i_{jm} is the measured j -th phase current, r is the winding resistance and $u_j(t)$ is the j -th phase input. The first measured phase residual that is lower than the threshold will define the 0 [rad] value of the estimated angular position, $\hat{\theta}$. The next phase that is below the threshold will add up one stroke angle to the estimated angular position, and so on. Thus, the estimator resolution is equal to θ_S , which implies that this method will work better for SR motors with a large number of stator phases and/or rotor poles. Also, from the estimated angular position, $\hat{\theta}$, it is obtained the estimated angular velocity, $\hat{\omega}$.

4.2 Controller

The estimator is simulated along with a close loop controller based on a modified version of an *universal input* defined in De La Guerra et al. (2015),

$$u_j(t) = u_0 + u_1 \cos\left(\sigma t - (j-1) \frac{2\pi}{m}\right), \quad (13)$$

where the frequency is tuned using an integral term defined as $\sigma = \omega_d N_r - K_i \frac{d\hat{\omega}}{dt}$ with ω_d the set point value, K_i an integral gain, $\hat{\omega} = \hat{\omega} - \omega$ is the comparison between the set point and the estimated angular speed, $u_0 = 5/\omega_d$, $u_1 = 0.1\omega_d^2$, $\theta_0 = -\pi/mN_r$. It must be noted that the term $\omega_d N_r$ is based on the fundamental frequency of the currents (3).

The commutation for each phase is defined using the residuals. The phase voltage $u_j(t)$ will be turned on when the residual $r_j(t)$ is different from zero. Under these conditions the SR motor can reach a constant velocity in steady state in the interval $\omega_d = [0.1, 7]$ [rad/s].

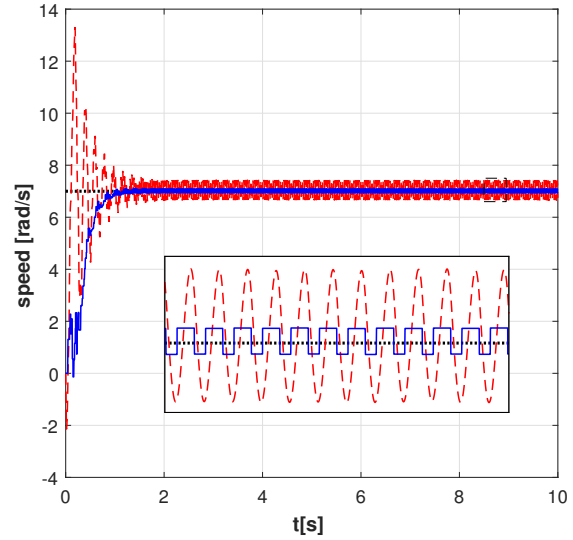


Fig. 6. Speed regulation $\hat{\omega}$ —, ω - - -, Set point ...

5. SIMULATIONS

The simulations were carried out in *MATLAB/Simulink*, with a fixed step $T = 0.0001$ [s], Runge-Kutta solver and model (1) with $N_r = 8$, $J = 0.001$ [Kgm²], $d = 0.001$ [Kgm²/s], $r = 1.7$ [Ω], $l_0 = 0.0121$, $l_1 = 0.0115$. The parameters of the input are $u_0 = 0.7143$, $u_1 = 4.9$ and $\sigma = 56$ [Hz].

The speed regulation is presented in Figure 6, where the motor speed transitory response can not be improved, but the set point is achieved at 2 [s] approximately. The estimated speed has a better transitory response and, from the zoom window it has less ripple than the motor speed. The motor speed ripple may imply that the conduction angle must be modified, perhaps the current phase must conduct in a smaller fraction of the power generation cycle.

In Figure 7 the input and current of phase-1 are compared to show how the voltage defines the form of the phase current. From Figure 2, the stator currents must be positive between the θ_{on} and θ_{off} angles, and must be zero immediately after the change of sign of the derivative of the inductance, Figure 7 shows that the current must be extinguished a little earlier in order to reduce the ripple and the negative current.

Lastly, Figure 8 shows the relation between the residuals and the phase currents, where oscillations near the extinction angle seems to be a source of negative current and thus the speed ripple.

The results of the simulations show that the observability conditions can be exploited, along with the geometric characteristics of the motor and the motor model, to design a simple estimator of the mechanical variables. However, this results are valid for model (1) with in-

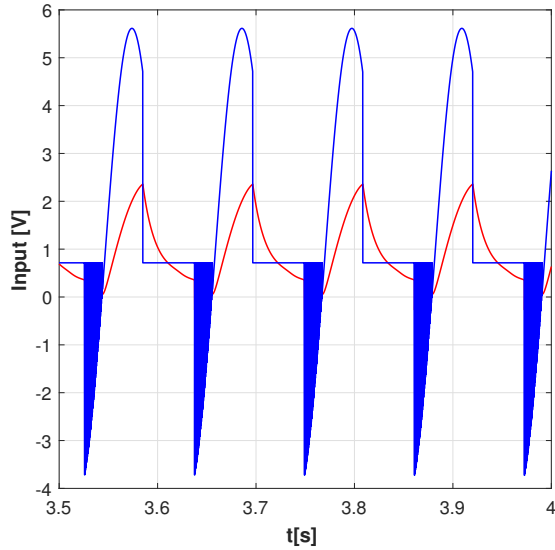


Fig. 7. Comparison $u_1(t)$ —, i_1 —

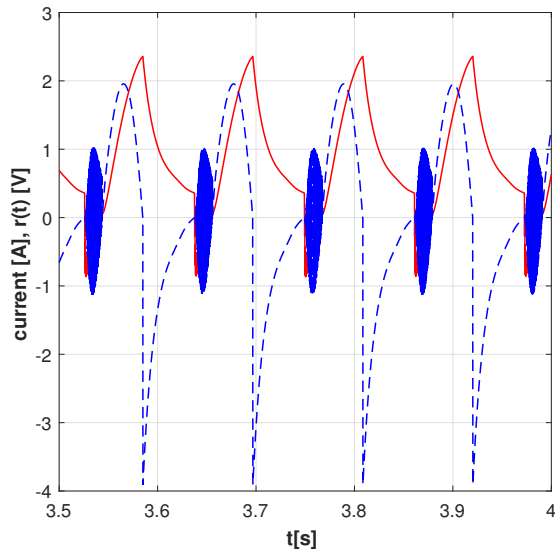


Fig. 8. Comparison $r_1(t)$ - - -, i_1 —

ductances defined as in (2). What this example reveals is that to design a commutator for the electric motor, which has trapezoidal inductances, a more realistic inductance model is necessary, because the singular points of model (1) are sets of singular points for the real inductances as show in Figure 2.

6. FINAL COMMENTS

The present work explains the major challenges to design an observer for the mechanical variables of the SR motor measuring only phase currents and voltages.

The first one, is the nonuniform observability of the small signal motor model, which may be overcome by finding a set of variables to define a transformed model, in the sense of Besancon and Ticlea (2007), with the final goal of designing a state observer.

The second problem is to design a commutator to define the conduction angles of each phase, based only on current and voltage measurements. It has been proven that it is possible to define a residual based on currents measurements to define the conduction region using the geometrical characteristics of the motor. However, the real shape of the inductance must be accounted to obtain a better performance and a larger interval of operation.

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