

# On Power Factor Improvement by Series Compensation: a Cyclodissipativity Framework

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Abstract: This paper presents a reformulation of the power factor compensation problem by series compensation for alternating current power systems with ideal distorted sources. In particular, it establishes that power factor improvement for nonlinear loads with nonsinusoidal (but periodic) source voltage is equivalent to imposing a certain cyclodissipativity condition to the compensated system. Applications of the framework are illustrated through a simple linear example.

*Keywords:* Reactive power, Cyclodissipativity, Power-Factor Improvement, Series Compensation

# 1. INTRODUCTION

Generally, nonlinear loads have been represented as current sources because their current waveforms are distorted from pure sinusoidal at fundamental frequency. However, there is another common type of harmonic source which can be characterized as harmonic voltage sources, as shown in Fig. 1. A typical example of this kind of harmonic source is diode rectifier with smoothing dc capacitors which is used in electronic equipment, household appliances, and AC drivers, see Peng (2001). Moreover, in Peng et al. (1999) it has been shown that shunt-compensation is not effective for compensating such voltage-source types of nonlinear loads and a seriescompensation has been used to compensate such nonlinear loads instead.  $i<sub>s</sub>$ 



Fig. 1. (a) Power delivery system with (possibly) nonlinear and time varying load and n-phase AC ideal generator. (b) Per-phase equivalent circuit of voltagesource nonlinear load.

Series compensation modifies the transmission of distribution systems parameters, namely, the voltage and impedance of the source, while shunt compensation changes the equivalent impedance load. For instance, in power systems, series capacitance is used mainly to reduce the series inductance between two points and thereby to improve the voltage regulation with inductive load Shepherd and Zand (1979).

Regarding shunt compensation, in Garcia-Canseco et al. (2007) a novel framework for analysis and design of (possibly nonlinear) power factor (PF) compensators for electrical systems operating in non-sinusoidal (but periodic) regimes with nonlinear time-varying loads was presented. This framework proceeds from the definition of PF and does not rely on any axiomatic definition of reactive power. It has been shown that the PF is improved if and only if the compensated system satisfies a certain cyclodissipativity property, Hill and Moylan (1990). Using this framework the classical capacitor or inductor compensators are interpreted in terms of energy equalization. In addition, this energy-equalization approach, which underlies the phase-shifting action of power factor compensation, begins to permeate through electrical engineering textbook, Mahdavi Tabatabaei et al. (2017).

This approach has been applied in Ortega et al. (2008) to analyze passive compensation of a classical half-bridge controlled rectifier with non-sinusoidal source voltage. Afterwards, in del Puerto-Flores et al. (2011) was presented the extension of the cyclodissipative framework for PF compensation to consider arbitrary LTI lossless filters, and demonstrated that for general lossless LTI filters the PF is improved if and only if a certain equalization condition between the weighted powers of inductors and capacitors of the load is ensured. Next, in del Puerto-Flores et al. (2012) we have presented the proof that PF improvement can also be characterized in terms of a new cyclodissipativity property where the supply rate is independent of the load and is solely determined by the compensator—paving the road for compensator design applications. Finally, in del Puerto-Flores et al. (2010) we have also formulated the PF compensation problem in a way that explicitly accounts for the effects of a non-negligible source impedance on the load voltage and current. We have demonstrated that cyclodissipativity provides a rigorous mathematical framework useful to analyze and design power factor compensators for general nonlinear loads operating in nonsinusoidal regimes with significant source impedance<sup>1</sup>.

In this work, our task is to formulate the power factor compensation problem by means of series compensation. Based on the results presented in del Puerto-Flores et al. (2010), we prove that cyclodissipativity provides a mathematical framework useful to analyze and design power factor compensators for general nonlinear loads operating in nonsinusoidal regimes with ideal sources.

First, we briefly review the main result on shunt compensation from Garcia-Canseco et al. (2007) in Section 2. Then, in Section 3 we give a cyclodissipativity characterization of Series Compensation problem under nonsinusoidal regimes, where we provide a geometric interpretation and we end with an example in Section 4.

# 2. PROBLEM FORMULATION AND BACKGROUND

We consider the energy transfer from an  $n$ -phase AC generator to a load, as in Fig. 1a, where we assume  $v_s$ is ideal. In particular, we make the following assumptions throughout this work.

*Assumption 1.* The source is ideal, in the sense that  $v_s$ remains unchanged for all loads.

*Assumption 2.* All signals are assumed to be periodic, with fundamental period  $T$ , and have finite power, that is, they belong to

$$
\mathcal{L}_2^n = \left\{ x : [0, T) \to \mathbb{R}^n : ||x||^2 := \frac{1}{T} \int_0^T |x(\tau)|^2 d\tau < \infty \right\}
$$

where  $|\cdot|$  is the Euclidean norm. We also define the inner product in  $\mathcal{L}_2^n$  as

$$
\langle x, y \rangle := \frac{1}{T} \int_0^T x^\top(t) y(t) dt.
$$

Under Assumption 2, the waveforms can be represented by a complex exponential Fourier series, viz.,

$$
x(t) = \sum_{k=-\infty}^{\infty} \hat{X}_k \exp(jk\omega_0 t)
$$

where  $\omega_0 := 2\pi/T$  is the fundamental frequency and, for integers k, the vector Fourier coefficients  $\hat{X}_k$  are given by

$$
\hat{X}_k = \frac{1}{T} \int_T x(t) \exp(-jk\omega_0 t) dt.
$$

If  $x(t)$  is real, then its Fourier coefficients satisfy  $\hat{X}_k =$  $\hat{X}^*_k,$  where  $\hat{X}^*_k$  denotes the conjugate of  $\hat{X}_k.$ 

The process of power factor correction is an attempt to reduce the apparent power of a load to the value of the active power consumed. The accepted definition of PF is given as IEEE Standard-1459 (2010):

*Definition 3.* (Power factor) Consider the power delivery systems of the Fig. 1a. The PF of an AC electric power system is defined by

$$
PF := \frac{P}{S},\tag{1}
$$

where  $P := \langle v, i \rangle$  is the active (real) power, also called average power, and  $S := ||v|| ||i||$  is the apparent power.

From (1), it follows that  $P \leq S$ . Hence  $PF \in [-1,1]$  is a dimensionless measure of the energy-transmission efficiency <sup>2</sup> . Cauchy–Schwartz also tells us that a necessary and sufficient condition for the apparent power to equal the active power is that  $v$  and  $i$  are collinear—see Lemma 3.1 in Luenberger (1969). If this is not the case,  $P < S$ and compensation schemes are introduced to maximize the PF. The condition for unity power factor is that the input current to a systems is proportional at all times to the instantaneous supplied voltage.

*2.1 On power factor improvement by shunt lossless compensation*



Fig. 2. (a) Parallel load compensation and (b) Series compensation schemes in a power delivery system with ideal source, where the load and compensator are represented by their admittances  $Y_{\ell}$  and  $Y_c$ , and/or impedance  $Z_c$ , resp.

The PF compensation configuration considered in this subsection is depicted in Fig. 2a, where  $Y_c, Y_{\ell}: \mathcal{L}_2^n \to \mathcal{L}_2^n$ are the admittance operators of the compensator and the load  $\mathfrak{N}$ , respectively. That is,

$$
i_c = Y_c(v_s), \quad i_\ell = Y_\ell(v_s), \tag{2}
$$

<sup>&</sup>lt;sup>1</sup> An AC power system with significant impedance is also often referred to as "weak grid". This kind of grids are usually found in more remote places where the feeders are long and operated at a medium voltage level, e.g. Bindner (1999).

<sup>&</sup>lt;sup>2</sup> For a passive load the measure is defined as  $PF \in [0, 1]$ , for the current lagging or leading the voltage, since  $P > 0$ . However, for an active load, or under nonsinusoidal or unbalanced conditions the active power may take negative values, more details in IEEE Standard-1459 (2010), therefore  $PF \in [-1, 1]$  is a general measure.

where  $i_c, i_{\ell} \in \mathcal{L}_2^n$  are the compensator and load currents, respectively. In the simplest LTI case the operators  $Y_c, Y_\ell$ can be described by their admittance transfer matrices, which we denote respectively by  $\hat{Y}_c(s)$ ,  $\hat{Y}_\ell(s) \in \mathbb{C}^{n \times n}$ , where s represents the complex frequency variable  $s = j\omega$ .

Following standard practice, we consider only *shunt lossless compensators*, that is,

$$
\langle Y_c(v_s), v_s \rangle = 0, \quad \forall v_s \in \mathcal{L}_2^n. \tag{3}
$$

## *2.2 On PF improvement by series lossless compensation*

The PF compensation configuration considered in this subsection is depicted in Fig. 2b, where  $Z_c, Y_{\ell}: \mathcal{L}_2^n \to$  $\mathcal{L}_2^n$  are the impedance and admitance operators of the compensator and the load  $\mathfrak{N}$ , respectively. That is,

$$
v_c = Z_c(i_s), \quad i_s = Y_\ell(v_{\ell_c}), \tag{4}
$$

where  $v_c, v_{\ell_c} \in \mathcal{L}_2^n$ , are the compensator and load voltages, respectively. In the simplest LTI case the operators  $Z_c, Y_{\ell}$  can be described by their impedance and admittance transfer matrices, which we denote by  $\hat{Z}_c(s)$ ,  $\hat{Y}_{\ell}(s) \in \mathbb{C}^{n \times n}$ , respectively.

Following standard practice, we consider only *series lossless compensators*, that is,

$$
\langle Z_c(i_s), i_s \rangle = 0, \quad \forall i_s \in \mathcal{L}_2^n. \tag{5}
$$

For the shunt and series compensation schemes, the uncompensated  $PF$ , i.e., the value of  $PF$  is given by

$$
PF_u := \frac{\langle v_s, i_{\ell} \rangle}{\|v_s\| \|i_{\ell}\|},\tag{6}
$$

that is, the value of PF with  $Y_c = 0$  or  $Z_c = 0$  and where, by Kirchhoff's Current and Voltage Laws (KCL and KVL), the uncompensated current and voltage are  $i_s = i_\ell$  and  $v_s = v_\ell$ , respectively.

*Definition 4.* (Power factor improvement). Given a nphase source voltage  $v_s(t)$  with a fixed load  $Y_\ell : \mathcal{L}_2^n \to \mathcal{L}_2^n$ , as in Fig. 1a, power-factor improvement is achieved with the lossless compensator  $Y_c$ ,  $Z_c$ :  $\mathcal{L}_2^n \to \mathcal{L}_2^n$  if and only if  $PF > PF_u$  (7)

where  $PF_u$  denotes the uncompensated power factor.

*2.3 A cyclodissipativity characterization of power factor compensation*

In this work, the framework for analysis of PF compensation for non-sinusoidal nonlinear networks is based on a cyclodissipativity condition introduced in Garcia-Canseco et al. (2007), which is recalled to contextualize the results. Then, we first present the definition of cyclodissipativity in the context of electrical networks.

*Definition 5.* Given a mapping  $w : \mathcal{L}_2^n \times \mathcal{L}_2^n \to \mathbb{R}$ . The n-port system of Fig. 1a is cyclodissipative with respect to the supply rate  $w(v, i)$  if and only if

$$
\int_0^T w(v(t), i(t))dt > 0.
$$
\nfor all

\n
$$
(v, i) \in \mathcal{L}_2^n \times \mathcal{L}_2^n.
$$
\n(8)

Notice that for cyclopassivity the supply rate function  $w(v, i)$  is of a specific form, namely  $w(v, i) := v^{\top}(t)i(t)$ . By this concept, the next results from Garcia-Canseco et al. (2007) were introduced for the shunt compensation.

*Proposition 6.* Consider the system of Fig. 2a with fixed  $Y_{\ell}$ . The compensator  $Y_c$  improves the PF if and only if the system is cyclodissipative with respect to the supply rate

$$
w(v_s, i_s) := (Y_{\ell}(v_s) + i_s)^{\top} (Y_{\ell}(v_s) - i_s).
$$
 (9)

The proof follows from (7) and the fact the compensator is lossless.

From Proposition 6, the next corollary characterizes the set of all compensators  $Y_c$  that improve the power-factor for a given  $Y_{\ell}$ .

*Corollary 7.* Consider the system of Fig. 2. Then  $Y_c$ improves the PF for a given  $Y_{\ell}$  if and only if  $Y_c$  satisfies

 $2\langle Y_\ell(v_s), Y_c(v_s) \rangle + ||Y_c(v_s)||^2 < 0, \quad \forall v_s \in \mathcal{L}_2^n$  $(10)$ Dually, given  $Y_c$ , the PF is improved for all  $Y_\ell$  that satisfy  $(10).$ 

#### 3. MAIN RESULT

In this section we present a cyclodissipativity characterization of the series compensation, where we also provide a feedback configuration and a geometric interpretation.

Firstly, although the problem at hand is posed as a problem in networks, it can be equally well interpreted as a feedback problem; the circuit of Fig. 2b is represented by the system of Fig. 3, which consists of two systems in a feedback loop. Specifically, the input is  $v_s$ , since  $i'_{\ell} = 0$ , the outputs are  $i_{\ell_c}$  and  $v_c$ , and the (error) outputs are  $i_c$ and  $v_{\ell_c}$ . The products are related with the instantaneous delivered power by the source  $i_s^{\top} v_s$  and the instantaneous input power into the load  $i_{\ell_c}^{\top} v_{\ell_c}$ , where  $i_s = i_{\ell_c}$ .



Fig. 3. Feedback configuration of the series compensation scheme in a power delivery system with ideal source.

We are in position to formulate the series PF improvement problem in terms of cyclodissipativity.

*Proposition 8.* Consider the system of Fig. 2b. Given a *n*-phase source voltage  $v_s(t)$  with a fixed load  $Y_{\ell}$ . The compensator  $Z_c$  improves the PF if and only if the system has finite gain and is cyclodissipative with respect to the supply rate

$$
w(v_s, i_s) := \delta^2 (Y_\ell v_s)^\top Y_\ell v_s - i_s^\top i_s. \tag{11}
$$

for all  $(v_s, i_s) \in \mathcal{L}_2^n \times \mathcal{L}_2^n$ , where  $\delta$  is the *upper gain bound* <sup>3</sup> and is given by

<sup>3</sup> See Table 1 of Hill and Moylan (1977), Definition 2.1 of Dower and James (1998), and Definition 2 of Polushin and Marquez (2004).

$$
\delta = \frac{\langle v_{\ell_c}, i_s \rangle}{\|v_s\|} \frac{\|(I + Z_c Y_{\ell}) v_{\ell_c}\|}{\langle (I + Z_c Y_{\ell}) v_{\ell_c}, i_{\ell} \rangle},\tag{12}
$$

with  $1 < \delta < \infty$ .

**Proof.** From KCL and KVL, we have for the uncompensated case,

$$
i_s = i_\ell \tag{13}
$$

$$
v_s = v_\ell, \tag{14}
$$

and, with series compensation, i.e.,  $Z_c \neq 0$ ,

$$
i_s = i_c = i_{\ell_c} \tag{15}
$$

$$
v_s = v_{\ell_c} + Z_c i_c. \tag{16}
$$

Substituting  $(14)$  and  $(15)$  into  $(16)$ , we obtain

$$
v_{\ell} = (I + Z_c Y_{\ell}) v_{\ell_c}, \qquad (17)
$$

where, we use  $i_{\ell_c} = Y_{\ell} v_{\ell_c}$ .

From the definition of power factor (1) and the lossless condition of the series compensator (5), we have

$$
PF = \frac{\langle v_{\ell_c}, i_s \rangle}{\|v_s\| \|i_s\|},\tag{18}
$$

and, we define

$$
\alpha = \frac{\langle v_{\ell_c}, i_s \rangle}{\|v_s\|}.
$$
\n(19)

The uncompensated power factor (6) is given by

$$
PF_u := \frac{\langle v_\ell, i_\ell \rangle}{\|v_\ell\| \|i_\ell\|},
$$

and by using (17), then

$$
PF_u = \frac{\langle (I + Z_c Y_\ell) v_{\ell_c}, i_\ell \rangle}{\|(I + Z_c Y_\ell) v_{\ell_c}\| \|i_\ell\|},\tag{20}
$$

and we define

$$
\alpha_u := \frac{\langle (I + Z_c Y_\ell) v_{\ell_c}, i_\ell \rangle}{\|(I + Z_c Y_\ell) v_{\ell_c}\|}.
$$
\n(21)

From Definition (7), we conclude that  $PF > PF_u$  if and only if

$$
\frac{\langle v_{\ell_c}, i_s \rangle}{\|v_s\| \|i_s\|} > \frac{\langle (I + Z_c Y_{\ell}) v_{\ell_c}, i_{\ell} \rangle}{\|(I + Z_c Y_{\ell}) v_{\ell_c}\| \|i_{\ell}\|},
$$

or, by (19) and (21), the inequality becomes

$$
\|i_s\|^2 < \delta^2 \|Y_\ell v_s\|^2,\tag{22}
$$

where we use  $i_{\ell} = Y_{\ell} v_s$  and  $\delta := \frac{\alpha}{\alpha_u}$ . Consequently, note that  $(8)$  with  $(11)$  is equivalent to  $(22)$ , which yields the desired result. Moreover, if  $Z_c = 0$ , i.e., the uncompensated case, from (12) and (17) we have that  $\delta = 1$ , and if  $Z_c \rightarrow \infty$ , then  $\delta \rightarrow 0$ , since  $v_{\ell_c} = (I + \mathbf{Z} \times \mathbf{C})^{-1}$  $Z_cY_{\ell}$ )<sup>-1</sup>v<sub>s</sub>. Finally, because the fact that  $\delta$  depends only on bounded signals,  $i_{\ell}, v_{\ell_c}, v_s$  and the operators  $|Z_c|$ ,  $|Y_{\ell}| \in \mathcal{L}_2^n$ , we can conclude that  $0 \leq \delta < \infty$  for  $Z_c \neq 0$ and  $1 < \delta < \infty$  for  $Z_c \neq 0$  that improves the power factor.

The next corollary of this result is the characterization of all compensators that improve the power factor.

*Corollary 9.* Consider the system of Fig. 2b. Then  $Z_c$ improves the PF for a given  $Y_{\ell}$  if and only if  $Z_c$  satisfies  $||Y_{\ell}Z_{c}i_{s}||^{2} - 2\langle i_{\ell}, Y_{\ell}Z_{c}i_{s}\rangle < (\delta^{2}-1)||i_{\ell}||^{2}, \forall i_{s}, i_{\ell} \in \mathcal{L}_{2}^{n}.$  (23) **Proof.** From (22), using the facts  $i_s = Y_\ell v_{\ell_c}, v_c = Z_c i_s$ , and  $v_s = v_c + v_{\ell_c}$ ,

$$
||Y_{\ell}v_{\ell_{c}}||^{2} < \delta^{2}||i_{\ell}||^{2},
$$
  
\n
$$
||Y_{\ell}(v_{s} - v_{c})||^{2} < \delta^{2}||i_{\ell}||^{2},
$$
  
\n
$$
||Y_{\ell}v_{s}||^{2} - 2\langle Y_{\ell}v_{s}, Y_{\ell}Z_{c}i_{s}\rangle + ||Y_{\ell}Z_{c}i_{s}||^{2} < \delta^{2}||i_{\ell}||^{2}.
$$
  
\nand, from  $i_{\ell} = Y_{\ell}v_{s}$ , then we have  
\n
$$
||Y_{\ell}Z_{c}i_{s}||^{2} - 2\langle i_{\ell}, Y_{\ell}Z_{c}i_{s}\rangle < (\delta^{2} - 1)||i_{\ell}||^{2}.
$$
 (24)

### *3.1 Geometrical interpretation of the PF compensation*

Referring to Fig. 4 we have a geometric interpretation of power factor compensation. Fig. 4 depicts the vector  $v_s$ ,  $v_{\ell_c}, v_c, i_s$ , and  $i_{\ell}$ . The angles  $\beta$  and  $\beta_u$  are defined as

$$
\beta := \cos^{-1} PF, \ \ \beta_u := \cos^{-1} PF_u,
$$

or,  $\beta = \angle(v_{\ell_c}, i_s)$  and  $\beta_u = \angle(v_{\ell_c}, i_\ell)$ . Then, it is clear from Fig. 4 that  $PF > PF_u$  if only if  $\beta < \beta_u$ . From (12), by assuming that  $\langle v_{\ell_c}, i_s \rangle > 0$  and  $\langle (I +$ 

 $Z_c Y_{\ell} | v_{\ell_c}, i_{\ell} \rangle > 0$ , then we have that  $1 < \delta < \infty$ .



Fig. 4. Geometric interpretation of the PF compensation.

*Remark 10.* The projection of  $i_s = i_{\ell_c}$  onto  $v_s$  is the vector denoted and defined by

$$
\text{proj}(i_s, v_s) := \frac{\langle v_s, i_s \rangle}{\|v_s\|^2} v_s,
$$

with magnitude

$$
\alpha := \frac{\langle v_s, i_s \rangle}{\|v_s\|} = \frac{\langle v_{\ell_c}, i_s \rangle}{\|v_s\|}
$$

*Remark 11.* Consider the projection of  $i_\ell$  onto  $v_s = v_\ell$  is the vector denoted and defined by

$$
\text{proj}(i_{\ell}, v_s) := \frac{\langle v_s, i_{\ell} \rangle}{\|v_s\|^2} v_s,
$$

with magnitude

$$
\alpha_u := \frac{\langle v_s, i_\ell \rangle}{\|v_s\|} = \frac{\langle (I + Z_c Y_\ell) v_{\ell_c}, i_\ell \rangle}{\|(I + Z_c Y_\ell) v_{\ell_c}\|}.
$$

where we use  $v_{\ell} = (I + Z_c Y_{\ell}) v_{\ell_c}$ .

## 4. EXAMPLE: LINEAR CIRCUIT WITH NONSINUSOIDAL SUPPLY VOLTAGE

In this section we present an example that illustrate some of the points discussed in the paper.



Fig. 5. Capacitive series compensation of a RL-series circuit.

Consider the circuits of Fig 5, namely, a linear R-L loads, with admittance  $Y_{\ell}$ , and with a capacitive compensator  $\hat{Y}_c(s) = sC$ , where  $v_s(t)$  is the supply instantaneous voltages with the following harmonics:



Let  $V_{s_k}$  denote the k-th harmonic component of the source voltage,

$$
Y_{\ell_k} = G_{\ell_k} - j B_{\ell_k}
$$

the load admittance for the k-harmonic. The network equations yield to the following relations:

$$
\begin{array}{l} V_{\ell_{c,k}}^2=\frac{V_{s_k}^2}{|I+Z_{c_k}Y_{\ell_k}|^2},\ I_{\ell_{c,k}}^2=|Y_{\ell_k}|^2V_{\ell_{c,k}}^2=\frac{|Y_{\ell_k}|^2V_{s_k}^2}{|I+Z_{c_k}Y_{\ell_k}|^2}\\ Z_{c_k}=1/jk2\pi fC. \end{array}
$$

and, the rms values of the source current and (compensated) load voltage and the power delivered to the load are given by, respectively,

$$
||i_s|| = \sqrt{\sum_k I_{s_k}^2}, ||v_{\ell_c}|| = \sqrt{\sum_k V_{\ell_{c,k}}^2}, \ P_{\ell_c} = \sum_k G_{\ell_k} V_{\ell_{c,k}}^2.
$$

The R-L load is assumed to be lumped resistance  $R = \frac{0.65}{10}$ <br> $\Omega$  in series with lumped, pure inductance  $L = \frac{0.76}{10}$  H, with 0.65 PF lagging at 50 Hz.

Condition (22) helps us to obtain the parameters for a given compensator  $Z_c$ , i.e., the capacitance for this example, such that the power factor is improved. Where, the bounded gain is

$$
\delta(C) = \frac{\sum_{k} G_{\ell_{k}} V_{\ell_{c,k}}^{2}}{\sqrt{\sum_{k} V_{s_{k}}^{2}}} \frac{\sqrt{\sum_{k} |I + Z_{c_{k}} Y_{\ell_{k}}|^{2} V_{\ell_{c,k}}^{2}}}{\sum_{k} G_{\ell_{k}} |I + Z_{c_{k}} Y_{\ell_{k}}|^{2} V_{\ell_{c,k}}^{2}},
$$
(25)

the rms value of the input and uncompensated load current are, respectively,

$$
||i_s(C)|| = \sqrt{\sum_k \frac{|Y_{\ell_k}|^2 V_{s_k}^2}{|I + Z_{c_k} Y_{\ell_k}|^2}}, \text{ and } ||i_\ell|| = \sqrt{\sum_k |Y_{\ell_k}|^2 V_{s_k}^2}.
$$
  
Variation of the power factor. load voltage, source current

Variation of the power factor, load voltage, source current and load power against capacitance is shown in Fig. 6,



Fig. 6. Plots of power factor, source current, load voltage and load power. For the upper plot, blue solid line is plotted the power-factor under the distorted voltage source and red dashed line for the fundamental component only

where it also shows, in the upper plot, that the optimal compensation is achieved at the capacitance  $C_{\text{opt}}$  =  $41.8 \,\mathrm{mF}$  and a lower bound on the capacitance  $C_{\text{min}}$  = 20.7 mF is obtained as a consequence of the condition (24) for power-factor improvement (22) for this example. Moreover, the power factor has several corresponding local minima, but for this example the global maximum of the power factor under distorted and fundamental voltage sources does correspond to the same mode. In Fig. 6 the plots of the compensated voltage across the load and the power delivered to the load are also shown. Theses show considerable peaks of load voltage at the capacitor values of approximately 1.667 mF and 4.703 mF. These values are corresponde to the resonant conditions between the compensating capacitance and the load inductance for the 3-th and 5-th harmonic. This yields a current peak for each harmonic. However, because the small load conductance at such frequencies, the average power delivered by these current components is small such that almost non-effect on the delivered power at the resonant conditions.

From Fig. 7, for a fixed LTI capacitor compensator with admittance  $\hat{Z}_c(s) = 1/sC$ , the power factor is improved for all  $C > C_{\text{min}}$ . Through this example we



Fig. 7. The plots of the power factor and the inequality  $\delta(C) > ||i_s|| / ||i_{\ell}||$  against capacitance. Blue  $\delta(C)$ and red  $\|i_s\|/\|i_{\ell}\|.$ 

illustrate that the result reported in this work can be used for the formulation of a problem of optimization of the compensator. However, the major drawback for power factor compensation by means of a series connected capacitor for an equivalent load, as a series RLC circuit, is observed when  $\omega L \gg R$  and this approach may not be feasible since  $\hat{V}_c \gg \hat{V}_s$ .

## 5. CONCLUSIONS AND FUTURE WORK

In this paper, a cyclodissipativity characterization of power factor improvement by means of series compensation for ideal non-sinusoidal networks was presented. Our main goal is to point out that the cyclodissipative framework benefits the design by giving additional physical insights: namely, we show that the series compensation can be interpreted as feedback interconnection between the series compensator and the uncompensated system. Based on this, the obtained results with the dissipativity framework can be used in order to increase system efficiency.

While we just concentrated in this work on series compensation problem, we are aware that there are side effects, namely, some important issues regarding to minimize oscillatory interactions with transformers and motors in practical applications, Miske (2001), and voltage stability in transmission and distribution lines, Manchowski et al. (2008), need to be addressed.

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