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An energy-based approach for modeling water distribution networks with faults

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Abstract: This article introduces an energy-based approach for modeling water distribution networks with faults. The flow in each network pipeline is described by the rigid water column model (RWC), which can be obtained by assuming that the walls of the pipelines are rigid and the flow is incompressible. The key feature of the proposed approach is the modeling of the interactions of the network components (pipelines, faults, and sources), which is done through the use of the graph theory and from an energy point of view. Three examples are given: a pipeline with a partial blockage, a pipeline with a leak, and a pipeline with both faults. The models resulting from this approach can be used for the implementation of real-time applications, for example, for fault diagnosis or for control of valves and pumps in case of faults.

Keywords: Modeling, rigid water column model, graph theory, water distribution networks.

1. INTRODUCTION

Most of the models used for managing water distribution networks (WDN) are in steady state (Rossman et al., 2000), which can limit both the implementation of real-time applications and the exploitation of new technologies for the good management of water resources, such as the Internet of Things (IoT). For this reason, WDN models must involve unsteady state conditions in their formulation. Moreover, for implementing real-time diagnosis tools, models should include the effects of possible faults.

For modeling faulty networks some approaches have been proposed such as the impedance, the matrix and the admittance methods. The frequency-domain models that result from the application of these approaches have been employed for the conception of fault diagnosis algorithms based on optimization methods (Capponi et al., 2017; Duan, 2017). Their limitation is due to the linearization of both the quasi-steady friction and the faults (leaks in particular). Furthermore, such models cannot be used for real-time algorithms because of their nature. Regarding this constraint, an alternative was proposed by Torres and Besançon (2019): the modeling of faulty WDN by assuming that the walls of the pipelines are rigid and the flow is incompressible (Wood et al., 1990; Ivanov and Bournaski, 1996; Axworthy, 1998; De Persis and Kallesoe, 2011; Nault and Karney, 2016; Kaltenbacher et al., 2017; Scholten et al., 2017). Concretely, Torres and Besançon (2019) presented models for single pipelines with two classes of faults: leaks and blockages. The novelty of the proposed models is their formulation via the port-Hamiltonian formalism, which underlines the physics of systems by highlighting the relation between energy storage, dissipation, and interconnection structure (Van der Schaft, 2004; Van Der Schaft and Maschke, 2013; Van der Schaft et al., 2014).

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This contribution complements the work of Torres and Besançon by involving the graph theory for the systemic modeling of large water distribution networks. For showing the applicability of the proposed approach, three examples are presented at the end of this paper, which indeed is organized as follows: Section 2 presents the considered elements of a WDN and their constitutive relations. Based on graph theory, Section 3 presents the main result of the paper which is the energy-based modeling, while in Section 4 some examples are presented. Finally, in Section 5 some concluding remarks are included.

2. PRELIMINARIES

In order to describe the flow in a pressurized pipeline in terms of lumped elements, it is necessary to make the next assumptions:

- **(A1)** The flow is one-dimensional.
- **(A2)** The cross-sectional area is constant along the pipeline.
- **(A3)** The conduit walls are rigid and the flow is incompressible.
- **(A4)** Convective changes in velocity are negligible.

Throughout the paper, the terms fluid and water are used interchangeably to refer to the content of the pipelines.

2.1 Storing and Resistive Elements

In a pipeline, there is a set of parameters that together process the energy. This process is related to the dissipation, storage, and generation of energy. Regardless of the elements' nature, each element has associated the pressure drop ΔP *across* its terminals and the flow rate Q that flows *through* it.

Energy Dissipation The hydraulic resistance can be characterized by the steady loss along the pipeline, which can be expressed as a function of the flow rate and the hydraulic resistance (Adamkowski and Lewandowski, 2006). This later gathers two main class of losses: major losses (or friction resistance), which are caused by the effect of the fluid's viscosity near the wall of the pipeline, and minor losses (or local resistance), which are caused by the dissipation of mechanical energy when the configuration or the direction of flow is sharply changed because of the presence of bends, fittings, and valves. This steady loss can be then expressed as follows (White, 1999):

$$\Delta P_R = \Delta P_f + \Delta P_m = \underbrace{L_x \bar{\Omega} Q_R^{\gamma}}_{R(Q_R)} |Q_R|, \tag{1}$$

where L_x is the length of the pipeline, ΔP_f denotes the pressure drop due to the friction resistance, ΔP_m represents the pressure drop due to the local resistance, $R(Q_R)$ denotes the hydraulic resistance, whereas $\bar{\Omega}$ and $\gamma \leq 1$ are parameters that can be associated to physical parameters of both the pipeline and water by means of the formula employed to describe the steady head loss. If the Darcy-Weisbach (DW) equation is used for a pipeline with length L_x, then $\gamma = 1$ and

$$\bar{\Omega} = \rho f(\mathrm{Re})/2\phi A_r^2, \tag{2}$$

where A_r is the cross-sectional area of the pipeline, ϕ its diameter, ρ is the mass density of the water, and $f(\mathrm{Re})$ is the friction factor that depends on the *Reynolds number* Re defined as

$$\mathrm{Re} = \frac{Q_R \phi}{A_r \nu}, \tag{3}$$

where ν is the kinematic viscosity that depends on temperature. Note then that if the DW equation is used, $\bar{\Omega}$ is a function of both the flow rate, through Re, and the temperature, via ν.

In this paper, it is considered that $f(\mathrm{Re})$ can be computed by using the Swamee-Jain equation given as follows

$$f(\mathrm{Re}) = \frac{0.25}{\left(\log\left(\frac{\frac{\kappa}{\phi}}{3.7} + \frac{5.74}{Re^{0.9}}\right)\right)^2}, \tag{4}$$

with κ as the material roughness.

To simplify the notation, the following variables are defined:

$$\alpha := \frac{0.25 L_x \rho}{2\phi A_r^2}, \quad a := \frac{\frac{\kappa}{\phi}}{3.7}, \quad b := 5.74\left(\frac{\phi}{A_r \nu}\right)^{-0.9}, \tag{5}$$

such that the hydraulic resistance $R(Q_R)$ takes the form

$$R(Q_R) = \alpha\left(\log\left(a + bQ_R^{-0.9}\right)\right)^{-2} Q_R^{\gamma}, \tag{6}$$

and the pressure drop across the resistance ΔP_R can be defined by

$$\Delta P_R = \alpha\left(\log\left(a + bQ_R^{-0.9}\right)\right)^{-2} Q_R^{\gamma} |Q_R|, \tag{7}$$

Flow Store In a fluid flow store, as the hydraulic capacitor, the form to store energy is the potential energy $\mathcal{V}(V\!\!\!/)$, with $V\!\!\!/$ denoting water volume. According to Wellstead (1979), the port variables of the hydraulic capacitor can be obtained as

$$\Delta P_C = \frac{\partial \mathcal{V}(V\!\!\!/)}{\partial V\!\!\!/}, \qquad Q_C = \frac{dV\!\!\!/}{dt}. \tag{8}$$

In the linear case, the energy function can be written as

$$\mathcal{V}(V\!\!\!/) = \frac{\rho g}{2A_r} V\!\!\!/^2, \tag{9}$$

with g as the acceleration due to gravity, such that (8) takes the following particular form

$$\Delta P_C = C^{-1} V\!\!\!/. \tag{10}$$

with the constant $C := A_r/\rho g$. The substitution of the time derivative of (10) into (8) gives the constitutive relation for the fluid flow store:

$$Q_C = C \Delta \dot{P}_C. \tag{11}$$

Effort Store A fluid effort store or hydraulic inductor, stores kinetic energy $\mathcal{T}(\varphi_p)$, with momentum φ_p. The port variables of the inductor can be obtained as

$$Q_L = \frac{\partial \mathcal{T}(\varphi_p)}{\partial \varphi_p}, \qquad \Delta P_L = \frac{d\varphi_p}{dt}. \tag{12}$$

For a linear hydraulic inductance, $\mathcal{T}(\varphi_p)$ takes the form

$$\mathcal{T}(\varphi_p) = \frac{A_r}{2\rho L_x} \varphi_p^2, \tag{13}$$

which means that [1]

$$Q_L = L^{-1} \varphi_p, \tag{14}$$

with $L := \rho L_x / A_r$. Finally, substitution of the time derivative of (14) into (12) gives the constitutive relation for the fluid effort store:

$$\Delta P_L = L \dot{Q}_L. \tag{15}$$

The latter equation characterizes the pressure drop that is required to accelerate the water between the pipeline ends when the flow varies at the rate $\dot{Q}_L$; see Maré (2016).

Energy dissipation due to a leak A leak behaves like a fixed orifice with free discharge to the atmosphere, and it can be modeled as a variable resistance R_ℓ that depends on the leak flow rate. The pressure drop across the resistance (ΔP_ℓ) represents the loss of pressure in the position of the leak (z_ℓ) due to the water coming out. The hydraulic resistance associated with a leak can be deduced from the Torricelli's equation given as follows

$$Q_\ell = C_d A_\ell (2P_\ell/\rho)^{-1/2}, \tag{16}$$

[1] In a pipeline without extractions, there is a unique flow rate flowing along the pipeline.

where ΔP_ℓ is the pressure drop at the leak, A_ℓ is the leak area and C_d is the dimensionless discharge coefficient. From Eq. (16), the following expression for the pressure at the leak position (z_ℓ) is obtained

$$\Delta P_\ell = R_\ell(Q_\ell)Q_\ell, \tag{17}$$

where $R_\ell(Q_\ell) = \left(\rho Q_\ell / 2C_d^2 A_\ell^2\right)$ is the leak resistance.

3. WATER DISTRIBUTION NETWORK MODEL

In this paper, it is assumed that a water distribution network is composed of p pipelines that under assumptions **A1-A4** can be represented by the interconnection of lumped elements. In addition, it is assumed the following.

(A5) A pipeline is analogous to a series $R-L$ circuit with a hydraulic capacitor parallel-connected as shown Fig. 1. $R(Q_R)$ (or $R(Q_L)$ since $Q_R = Q_L$) is an energy dissipator and L is a hydraulic inductor.

(A6) A leak is modeled by a variable resistance $R_\ell(Q_\ell)$ that depends on the leak flow rate Q_ℓ expressed by Eq.(16).

(A7) A blockage is modeled as three pipelines connected in series, where the in-between pipeline has a reduced cross-sectional area and different roughness.

(A8) At the inlet of a pipeline there is a water source (e.g. a reservoir), with pressure ΔP_{in} and flow rate Q_{in}, and at the outlet there is a water extraction, with pressure ΔP_{out} and flow rate Q_{out}, which can represent a water demand by users [2].

Remark 1. Fig. 1 shows a circuit that under assumptions **A1-A8** is analogous to the behavior of the flow in a pressurized pipeline. Since the pressure drop along the pipe is equal to the pressure drop associated with the hydraulic capacitance, the capacitor is in parallel to the potential. The ground represents the atmospheric pressure.

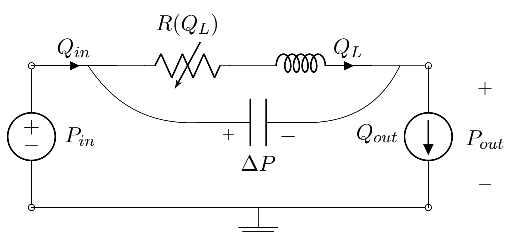

Fig. 1. Nonlinear RLC circuit analogous to a pipeline.

Remark 2. Fig. 2 shows a nonlinear RLC circuit that under assumptions **A1-A8** is analogous to the flow in a pressurized pipeline with a leak. The first loop represents the flow dynamics in the pipeline section before the leak and the second loop, the flow dynamics in the pipeline section after the leak, while Q_{in} and Q_{out} denote, respectively, the flow rate across the first and second sections of the pipeline. Q_ℓ is the leak discharge.

[2] For a pipeline with an open output the pressure P_{out} is the atmospheric, such that in Fig. 1 the output P_{out} is short-circuited with the ground.

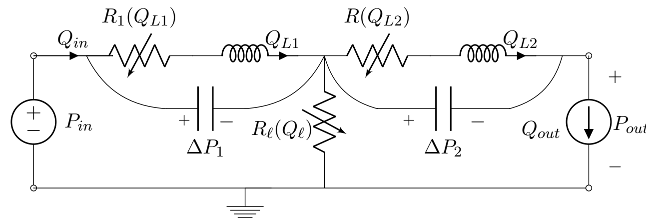

Fig. 2. Nonlinear RLC circuit analogous to a pipeline with a leak. The pressure drop across the resistance ΔP_ℓ represents the loss of pressure in z_ℓ.

Remark 3. Since the area of the pipeline is constant, the pipeline's capacitance is not modified by the leak, i.e.

$$C_1 = C_2 = A_r/\rho g. \tag{18}$$

Remark 4. Because of the leak, hydraulic resistance in the first and second sections change and can be expressed as follows, respectively:

$$R_1(Q_{L1}) = z_\ell \bar{\Omega} Q_{L1}^\gamma, \quad R_2(Q_{L2}) = (L_x - z_\ell)\bar{\Omega} Q_{L2}^\gamma. \tag{19}$$

Remark 5. Because of the leak, the inertance for both sections of the pipeline also change and can be defined as follows, respectively:

$$L_1 = \frac{\rho z_\ell}{A_r}, \quad L_2 = \frac{\rho(L_x - z_\ell)}{A_r}.$$

According to Avila-Becerril et al. (2016), the effort compatibility and flow continuity constraints, given by the pressure drop balance and flow-rate balance, respectively, are systematically obtained applying graph theory concepts (Bollobás, 2013).

3.1 Graph Theory

A water distribution network can be defined as an oriented graph G consisting of a finite set of *nodes* $\mathcal{V}(G) = \{\mathrm{v}_1, \mathrm{v}_2, \ldots, \mathrm{v}_n\}$ and a finite set of *edges* $\mathcal{E}(G) = \{\mathrm{e}_1, \mathrm{e}_2, \ldots, \mathrm{e}_b\}$ such that $\mathcal{E}$ is a subset of pairs of $\mathcal{V}$ where no self-loops are allowed. The set of nodes are the interconnection points of the elements [3] whereas the set of edges are associated directly to elements.

The interconnection of the elements of the WDN must satisfy the $n-1$ flow balance constraints and the $b-(n-1)$ pressure drop balance, which in this paper are given in terms of *basic cutsets* and *loopsets* for a given *tree* and *co-tree* (Wellstead, 1979; Bollobás, 2013). A *basic cutset* is a set of edges whose elements are one branch and some or all the chords. A *basic loopset* is a set conformed by one chord and some or all the branches such that a closed loop is formed.

Let $Q_t \in \mathbb{R}^{n-1}$ be the flows associated to the tree, $Q_\varsigma \in \mathbb{R}^{b-(n-1)}$ the associated to the co-tree, and $\Delta P_t \in \mathbb{R}^{n-1}$, $\Delta P_\varsigma \in \mathbb{R}^{b-(n-1)}$ the branch and chord drop

[3] The storage, dissipator or sources elements are considered lumped one-port (two-terminal) elements.

pressures, respectively; then the flow and pressure balance constraints can be expressed as

$$\begin{bmatrix}\mathbf{I} \; H\end{bmatrix}\begin{bmatrix}Q_t\\ Q_\varsigma\end{bmatrix}=0;\quad \begin{bmatrix}-H^T \; \mathbf{I}\end{bmatrix}\begin{bmatrix}\Delta P_t\\ \Delta P_\varsigma\end{bmatrix}=0, \tag{20}$$

with $\mathbf{I}$ a generic identity matrix of proper dimensions. The matrix $H \in \mathbb{R}^{(n-1)\times b-(n-1)}$, known as the *Fundamental loop matrix*, completely characterizes the topology of the network. Their entries are equal to 1 if a co–tree current points out a given basic ambit, -1 if points into the basic ambit and 0 if does not belong to the basic cutset.

In the graph, it is now selected the pressure source, the p capacitors, and the p resistances in series with the inductors as tree elements, while the p inductors, the ℓ leaks and the flow source in the co-tree of the graph, i.e.

$$Q_t=\begin{bmatrix}Q_{in}\\ Q_C\\ Q_R\end{bmatrix}, Q_\varsigma=\begin{bmatrix}Q_\ell\\ Q_L\\ Q_{out}\end{bmatrix},$$

$$\Delta P_t=\begin{bmatrix}-\Delta P_{in}\\ \Delta P_C\\ \Delta P_R\end{bmatrix}, \Delta P_\varsigma=\begin{bmatrix}\Delta P_\ell\\ \Delta P_L\\ \Delta P_{out}\end{bmatrix},$$

where $Q_{in}, -\Delta P_{in} \in \mathbb{R}$, $Q_C, \Delta P_C \in \mathbb{R}^p$, $Q_R, \Delta P_R \in \mathbb{R}^p$ are the pressure source's, capacitors' and resistances' flows and pressures respectively, while $Q_\ell, \Delta P_\ell \in \mathbb{R}^\ell$, $Q_L, \Delta P_L \in \mathbb{R}^p$, $Q_{out}, \Delta P_{out} \in \mathbb{R}$ are the leaks', inductors' and flow source's flow rates and pressures, respectively.

3.2 Water Distribution Network Dynamic

The main contribution of the paper is presented in the next proposition.

Proposition 1. Consider a network composed of p pipelines and ℓ leaks, such that assumptions **A1-A8** are fulfilled. Define the state vector

$$x := \begin{bmatrix}\Delta P_C\\ Q_L\end{bmatrix} \in \mathbb{R}^{2p\times 1}, \tag{21}$$

then the model of the network is given by the Hamiltonian system

$$\mathcal{P}\dot{x} = \left[J - \bar{R}(x)\right]x + G(x)E, \tag{22}$$

subject to the algebraic constraints

$$Q_{in} = \mathbf{1}_\ell^\top Q_\ell + Q_{out}, \tag{23a}$$

$$\Delta P_{out} = \Delta P_{in} + H_{CO}^\top \Delta P_C, \tag{23b}$$

with the parameter matrix $\mathcal{P} = diag\{C, L\} \in \mathbb{R}^{2p\times 2p}$, $C = diag\{C_i\}$, $L = diag\{L_i\}$, $i = 1, \ldots, p$, and the matrices

$$J=\begin{bmatrix}\mathbf{0} & -\mathbf{I}_p\\ \mathbf{I}_p & \mathbf{0}\end{bmatrix}=-J^T,\; G(x)=\begin{bmatrix}H_{C\ell}R_\ell^{-1}\mathbf{1}_\ell & -H_{CO}\\ \mathbf{0}_{p\times 1} & \mathbf{0}_{p\times 1}\end{bmatrix},$$

$$\bar{R}(x)=\begin{bmatrix}H_{C\ell}R_\ell^{-1}H_{C\ell}^\top & \mathbf{0}\\ \mathbf{0} & R_t\end{bmatrix}=\bar{R}^\top(x)\geq 0,\; E=\begin{bmatrix}-\Delta P_{in}\\ Q_{out}\end{bmatrix},$$

where $R_\ell(Q_\ell) = diag\{R_{\ell j}(Q_{\ell j})\}$, for $j = 1, \ldots, \ell$, represents the leaks, while $R_t(Q_L) = diag\{R_i(Q_{Li})\}$ for $i = 1, \ldots, p$, are the hydraulic resistances of the pipelines (tree resistances) which have non-linear constitutive relations given by (1).

Proof. On the one hand, with assumptions **A5-A8** at hand, the fundamental loop matrix can be partitioned as

$$H=\begin{bmatrix}-\mathbf{1}_\ell^\top & \mathbf{0}_{1\times p} & -1\\ H_{C\ell} & \mathbf{I}_p & H_{CO}\\ \mathbf{0}_{p\times\ell} & -\mathbf{I}_p & \mathbf{0}_{p\times 1}\end{bmatrix}, \tag{24}$$

where $\mathbf{I}$, $\mathbf{0}$, $\mathbf{1}_\ell$ represent an identity matrix, a matrix filled with zeros, and a vector filled of ones, respectively. The structure of $H_{C\ell} \in \mathbb{R}^{p\times\ell}$ and $H_{CO} \in \mathbb{R}^{p\times 1}$ depends on the topology of a particular network, showing the relation between the leaks and the flow source with each of the capacitors; see Avila-Becerril et al. (2016). Now, a direct substitution of (11) and (15) in (20) leads to the dynamical model of the system (22). □

4. EXAMPLES

In this section some examples are presented. The examples illustrate the simplicity in the modeling when blockages and faults are included along the pipeline.

4.1 Example 1: a pipeline with a blockage

A pipeline with a blockage can be treated as three pipelines connected in series ($p = 3$), where the in-between pipeline has a reduced cross-sectional area and different roughness. Fig. 3 shows an analogous circuit that under assumptions **A1-A8** represents a partially blocked pipeline.

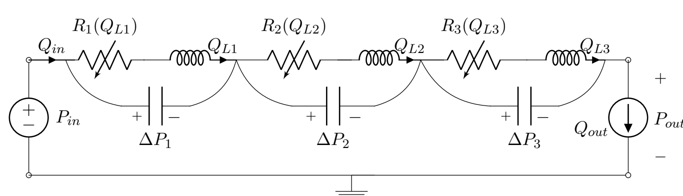

Fig. 3. Nonlinear RLC circuit representing the case of a pipeline with a partial blockage.

The related graph is shown in Fig. 4. Here, the graph has associated a fundamental loop matrix given by

$$H=\left[\begin{array}{ccc|c}0 & 0 & 0 & -1\\ \hline 1 & 0 & 0 & -1\\ 0 & 1 & 0 & -1\\ 0 & 0 & 1 & -1\\ \hline -1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & -1 & 0\end{array}\right],\ \text{s.t. } H_{CO}=\begin{bmatrix}-1\\ -1\\ -1\end{bmatrix}. \tag{25}$$

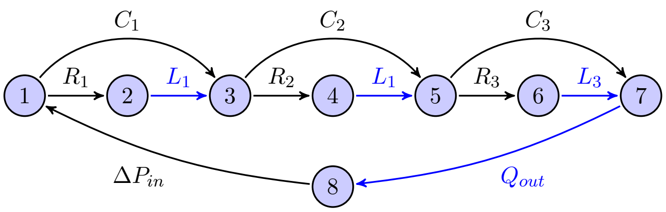

Fig. 4. Graph associated with the pipeline with a partial blockage illustrated in Fig. 3

Notice that, since there is no leak $\ell = 0$, the first column of matrix H in (24) does not appear. To obtain the model, define the state vector

$$x = [\Delta P_{C1}\ \Delta P_{C2}\ \Delta P_{C3}\ Q_{L1}\ Q_{L2}\ Q_{L3}]^\top \in \mathbb{R}^6,$$

the matrix $\mathcal{P} = diag\{C_1, C_2, C_3, L_1, L_2, L_3\}$ and $E = [\Delta P_{in}\ Q_{out}]^\top \in \mathbb{R}^2$. The resulting dynamical model has the form given by (22) with matrix $\bar{R}(x)$ and the product $G(x)E$ given by

$$\bar{R}(x) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & R_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & R_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & R_3 \end{bmatrix}, \ G(x)E = \begin{bmatrix} Q_{out} \\ Q_{out} \\ Q_{out} \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

where $R_i(Q_{Li})$, for $i = 1, 2, 3$, have a non-linear constitutive relations given by (1), and is subject to

$$Q_{in} = Q_{out},$$
$$\Delta P_{out} = -\mathbf{1}_3^\top \Delta P_C = -(\Delta P_{C1} + \Delta P_{C2} + \Delta P_{C3}).$$

4.2 Example 2: a pipeline with a leak

In this example, a pipeline with a leak is modeled. The analogous circuit to this case is shown in Fig. 2, where $p = 2$. This case can also be represented by the graph in Fig. 5, which has a fundamental loop matrix given by

$$H = \begin{bmatrix} -1 & 0 & 0 & -1 \\ -1 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}, \qquad (26)$$

with $H_{CO} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$ and $H_{C\ell} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$.

In this case, the state vector is defined as follows

$$x = [\Delta P_{C1}\ \Delta P_{C2}\ Q_{L1}\ Q_{L2}]^\top \in \mathbb{R}^4,$$

the parameters matrix is defined as

$$\mathcal{P} = diag\{C_1, C_2, L_1, L_2\}$$

and $E = [\Delta P_{in}\ Q_{out}]^\top \in \mathbb{R}^2$. Thus, the dynamical model of a pipeline with a leak has the form expressed by (22) with a matrix $\bar{R}(x)$ and the product $G(x)E$ given by

$$\bar{R}(x) = \begin{bmatrix} R_\ell^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & R_1 & 0 \\ 0 & 0 & 0 & R_2 \end{bmatrix}, \ G(x)E = \begin{bmatrix} R_\ell^{-1}\Delta P_{in} + Q_{out} \\ Q_{out} \\ 0 \\ 0 \end{bmatrix},$$

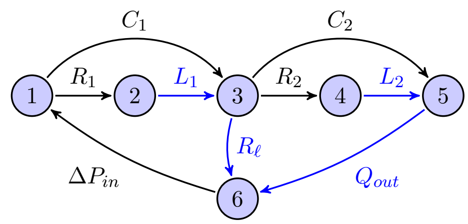

Fig. 5. Graph associated with a pipeline with a leak. See Fig. 2

where $R_i(Q_{Li})$, for $i = 1, 2$, have nonlinear constitutive relations given by (1). The constraints in this case are

$$Q_{in} = Q_\ell + Q_{out},$$
$$\Delta P_{out} = \Delta P_{in} - (\Delta P_{C1} + \Delta P_{C2}).$$

4.3 Example 3: a pipeline with a blockage and a leak

Fig. 6 shows an analogous circuit for illustrating the case of this example. The graph associated to this case is shown in Fig. 7.

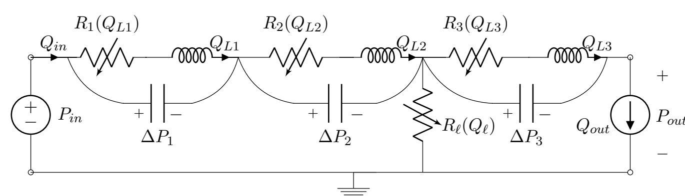

Fig. 6. Non linear RLC circuit representing a pipeline with a reduced cross-sectional area and a leak R_ℓ. Notice that in this case $p = 3$

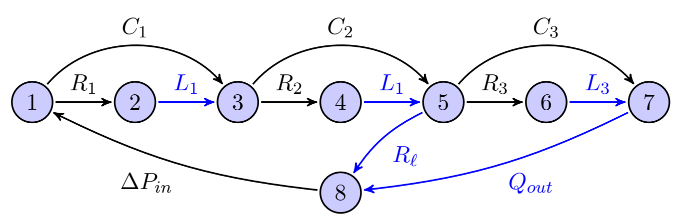

Fig. 7. Graph associated with the case in Example 3. See Fig. 6.

Here, the graph involves a fundamental loop matrix given by

$$H = \begin{bmatrix} -1 & 0 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 & -1 \\ -1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \end{bmatrix}, \qquad (27)$$

and in this case,

$$H_{C\ell} = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}, \quad H_{CO} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}.$$

Since the order of the model is six, it is defined the same state vector of the Example 1:

$$x = [\Delta P_{C1}\ \Delta P_{C2}\ \Delta P_{C3}\ Q_{L1}\ Q_{L2}\ Q_{L3}]^\top \in \mathbb{R}^6.$$

The dynamical model for this case has the form given by (22) with matrices $\bar{R}(x)$ and $G(x)$ given by

$$\bar{R}(x) = \left[\begin{array}{ccc|ccc} R_\ell^{-1} & R_\ell^{-1} & 0 & 0 & 0 & 0 \\ R_\ell^{-1} & R_\ell^{-1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & R_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & R_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & R_3 \end{array}\right],$$

$$G(x) = \begin{bmatrix} -R_\ell^{-1} & 1 \\ -R_\ell^{-1} & 1 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

where the leak has a constitutive relation given by $\Delta P_{R_\ell} = R_\ell(Q_\ell)Q_\ell$. To complete the model, according to (22), the dynamic of this example is subject to the following constraints:

$$\begin{aligned} Q_{in} &= Q_\ell + Q_{out}, \\ \Delta P_{out} &= \Delta P_{in} - \mathbf{1}_3^\top \Delta P_C \\ &= \Delta P_{in} - (\Delta P_{C1} + \Delta P_{C2} + \Delta P_{C3}). \end{aligned}$$

It is important to mention that the mathematical models that were shown in this section fulfilled the rigid pipeline dynamic characteristics.

5. CONCLUSION

This paper presented the modeling of faulty WDN within an energy-based framework. The modeling is based on graph theory concepts, and it is shown that, through the fundamental loop matrix, faulty WDN models can be systematically obtained. The first advantage of the obtained models is that, due to its structure, different faults can be added along the WDN, preserving the structure, which provides flexibility and modularity. Likewise, the form of the models allows employing strategies such as control by interconnection, passivity-based approaches, and the design of state observers for parameter calibration and fault diagnosis.

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