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An Adaptive Gravity Compensation Controller For the Leaderless Consensus of Uncertain Euler-Lagrange Agents

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Abstract: Consensus is the most basic synchronization behavior in multiagent systems. For networks of Euler-Lagrange (EL) agents different controllers have been proposed to achieve consensus, requiring in all cases, either the cancellation or the estimation of the gravity forces. In the latter case, it is necessary to estimate, not just the gravity forces, but the parameters of the whole dynamics. This requires the computation of a complicated regressor matrix, that grows in complexity as the degrees-of-freedom of the EL-agents increase. In this paper, we propose an adaptive controllers to solve the leaderless consensus problem by only estimating the gravitational term of the agents and hence without requiring the complete regressor matrix. To the best of our knowledge, this is the first work that achieves such an objective. The controller is a simple Proportional plus damping (P+d) scheme that does not require to exchange velocity information between the agents. Simulation results demonstrate the performance of the proposed controllers.

Keywords: Euler-Lagrange Agents; Multiagent Systems; Leaderless Consensus; Adaptive Control

1. INTRODUCTION

The objective in cooperative control is to design a distributed controller so that the aggregate system achieves specified behaviors, such as flocking (Lee and Spong, 2007; Gu and Wang, 2009), synchronization (Rodriguez-Angeles and Nijmeijer, 2004; Abdessameud et al., 2012), coordination (Wang et al., 2012; Qin et al., 2012), rendezvous and formation control. The fundamental cooperative behavior is *consensus*, where all agents in a network reach an agreement in some coordinates of interest. Consensus control can be split into two classes, namely, the leader-follower, where a network of agents agree at a given leader reference; and the leaderless, where in the absence of a leader, the agents converge to a certain common value (Arcak, 2007; Ren and Cao, 2011; Liu et al., 2014; Aldana et al., 2015; Klotz et al., 2015).

Consensus control of dynamical systems presents significant theoretical and practical challenges, specially when dealing with networks of nonlinear systems, as Euler-Lagrange (EL) agents (Ren and Cao, 2011; Hatanaka et al., 2015). The control of EL-agents is of practical interest because these systems describe the behavior of several physical systems—including mechanical, electrical and electromechanical systems (Ortega et al., 1998). The first results on consensus (synchronization) of a particular class of EL-agents has been reported in (Chopra and Spong, 2005) and the case of general, nonidentical, EL-systems has been reported in (Nuño et al., 2011). Since then, a plethora of different controllers have been proposed to solve both consensus problems. See (Wang, 2014) for a unified treatment of different consensus controllers for EL-systems and (Hatanaka et al., 2015) for a survey of recent developments on this topic.

As is well-known (Ortega et al., 1998) the Coriolis and centrifugal forces that appear in EL-systems are workless, therefore they don't play any role on the regulation of the position of the system—a feature that is encrypted in the well-known "Skew symmetry property" of EL-systems. On the other hand, the presence of the gravity forces has to be taken into account when solving this task, because they affect the equilibrium point. For fully actuated EL-systems it is possible to propose the exact cancellation of these forces and then solve the regulation task with a simple Proportional plus damping (P+d) controller (Takegaki and Arimoto, 1981). Since these forces are usually uncertain it is necessary to appeal to an adaptive version of the controller. Within the context of robotics, this problem was first solved in (Tomei, 1991) adding an ingenious gravity estimation feature.

In the context of multiagent systems P+d schemes with gravity cancellation have been proposed in (Ren, 2009; Nuño et al., 2013b; Nuño and Ortega, 2018; Ye et al., 2017). Besides having the advantage of a simple implementation they do not require the agents to exchange their velocities with their neighbours. However, their disadvantage is that they are not robust to parameter uncertainty—refer to (Nuño et al., 2013a) where the uncertainty problem has been analyzed. To deal with parametric uncertainty adaptive schemes have been proposed in (Chung and Slotine, 2009; Nuño et al., 2011; Meng et al., 2014; Chen et al., 2015a; Abdessameud et al., 2017; Chen et al., 2015b; Wang, 2017). The fundamental paper (Hernández-Guzmán and Orrante-Sakanassi, 2019) reports a model independent PID scheme that solves the consensus problem provided that sufficiently large gains are chosen.

Unlike the single EL-system case of Tomei (1991)—see also (Kelly, 1993)—the adaptive multiagent schemes estimate, not just the gravity forces, but the parameters of the whole dynamics. There are two important drawbacks of estimating all dynamical parameters. First, to ensure a successful parameter search in a bigger dimensional space requires higher excitation levels and longer convergence times—both factors stymying the achievement of good transient performances. Second, it requires the computation of a complicated regressor matrix, that grows in complexity as the degrees-of-freedom of the EL-agents increase, rendering the controllers of limited practical interest.

In this paper, we propose an adaptive controller that solves the leaderless consensus problem by only estimating the gravitational term of the agents, and hence without requiring the complete regressor matrix. To the best of our knowledge, along adaptive controllers this is the first controller for multiagent systems with only adaptive gravity compensation. The controller is a simple adaptive P+d scheme that does not require velocity information exchange between the agents. Since this scheme does not estimate the whole EL-dynamics its complexity is significantly smaller than all the previously reported adaptive controllers that require the computation of the regressor matrix for the full dynamics.

2. BACKGROUND

Notation. $\mathbb{R} := (-\infty, \infty)$, $\mathbb{R}_{>0} := (0, \infty)$, $\mathbb{R}_{\geq 0} := [0, \infty)$ and $\mathbb{N} := \{1, 2, 3, \ldots\}$. $\mathbf{I}_n \in \mathbb{R}^{n\times n}$ is the $n \times n$ identity matrix, $\mathbf{1}_n \in \mathbb{R}^n$ is the vector of n elements equal to one and $\mathbf{0}_n \in \mathbb{R}^n$ is the all-zeros vector. For any $\mathbf{x} \in \mathbb{R}^n$, $|\mathbf{x}|$ is its Euclidean norm and $\mathbf{th}(\mathbf{x}) := [\tanh(x_1), \ldots, \tanh(x_n)]^\top$, where $\tanh(x)$ is the standard hyperbolic tangent. $\lambda_m\{\mathbf{A}\}$ and $\lambda_M\{\mathbf{A}\}$ are the minimum and the maximum eigenvalues of the symmetric matrix $\mathbf{A} \in \mathbb{R}^{n\times n}$. $\bar{N} := \{1, 2, ..., N\}$ for $N \in \mathbb{N}$.

2.1 Euler-Lagrange Agents

We consider a network composed of N *fully-actuated and conservative* EL-agents, with n-Degrees-of-Freedom (DoF). The dynamics of the ith-agent is given by

$$\mathbf{M}_i(\mathbf{q}_i)\ddot{\mathbf{q}}_i + \mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i + \mathbf{g}_i(\mathbf{q}_i) = \boldsymbol{\tau}_i \tag{1}$$

where $\mathbf{q}_i, \dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i \in \mathbb{R}^n$ are the generalized position, velocity and acceleration, respectively; $\mathbf{M}_i(\mathbf{q}_i) \in \mathbb{R}^{n\times n}$ is the generalized inertia matrix, which is symmetric positive definite and lower bounded for all $\mathbf{q}_i \in \mathbb{R}^n$; $\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) \in \mathbb{R}^{n\times n}$ is the Coriolis and centrifugal forces matrix, defined via the Christoffel symbols of the first kind; and $\boldsymbol{\tau}_i \in \mathbb{R}^n$ is the control input.

We restrict to EL-agents (1) that satisfy the following assumption:

Assumption A1: There exists a known $m_{2i} \in \mathbb{R}_{>0}$ such that, for all $\mathbf{q}_i \in \mathbb{R}^n$, $\mathbf{M}_i(\mathbf{q}_i) \leq m_{2i}\mathbf{I}_n$. ◇

Model (1) has the following properties (Kelly et al., 2005; Spong et al., 2005):

Property P1: Matrix $\dot{\mathbf{M}}_i(\mathbf{q}_i) - 2\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)$ is skew-symmetric. Further, $\dot{\mathbf{M}}_i(\mathbf{q}_i) = \mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) + \mathbf{C}_i^\top(\mathbf{q}_i, \dot{\mathbf{q}}_i)$. ◁

Property P2: There exists $k_{ci} \in \mathbb{R}_{>0}$ such that, for all $\mathbf{q}_i \in \mathbb{R}^n$, $\|\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i\| \leq k_{ci}\|\dot{\mathbf{q}}_i\|^2$. ◁

Property P3: The gravity vector $\mathbf{g}_i(\mathbf{q}_i)$ is linearly parameterizable. Thus $\mathbf{g}_i(\mathbf{q}_i) = \mathbf{Y}_i(\mathbf{q}_i)\boldsymbol{\theta}_i$, where $\mathbf{Y}_i(\mathbf{q}_i) \in \mathbb{R}^{n\times m}$ is a matrix of known functions and $\boldsymbol{\theta}_i \in \mathbb{R}^m$ is a constant vector of the manipulator physical parameters. ◁

2.2 Interconnection Topology

As it is customary, we use graphs to represent the interconnection topology among the N EL-agents. In particular, we employ the graph Laplacian matrix $\mathbf{L} := \{L_{ij}\} \in \mathbb{R}^{N\times N}$ that is defined as $L_{ii} = \sum_{j\in\mathcal{N}_i} a_{ij}$ and $L_{ij} = -a_{ij}$, where $a_{ij} > 0$ if $j \in \mathcal{N}_i$ and $a_{ij} = 0$ otherwise. The set $\mathcal{N}_i$ contains all the neighbours of the ith-EL-agent. Note that, by construction, $\mathbf{L}$ has a zero row sum. Therefore $\mathbf{L}\mathbf{1}_N = \mathbf{0}_n$.

We assume that the agents exchange information according to the following assumption.

Assumption A2. The EL-agents interconnection graph is *undirected, static* and *connected.* ◇

From **A2**, the Laplacian $\mathbf{L}$ is symmetric; positive semi-definite; it has a single zero-eigenvalue, with the associated eigenvector $\mathbf{1}_N$, and all of the other eigenvalues are strictly positive; and $\mathrm{rank}(\mathbf{L}) = N - 1$. Further, $\ker(\mathbf{L}) = \alpha\mathbf{1}_N$, $\forall\alpha \in \mathbb{R}$.

2.3 Problem Definition

We say that leaderless consensus problem is solved if there exists $\mathbf{q}_c \in \mathbb{R}^n$ such that

$$\lim_{t\to\infty} |\dot{\mathbf{q}}_i(t)| = 0, \qquad \lim_{t\to\infty} \mathbf{q}_i(t) = \mathbf{q}_c \tag{2}$$

for all $i \in \bar{N}$.

In order to achieve (2), let us define the position error of the ith-agent as

$$\mathbf{e}_i := \sum_{j\in\mathcal{N}_i} a_{ij}(\mathbf{q}_i - \mathbf{q}_j). \tag{3}$$

Defining $\mathbf{q} := [\mathbf{q}_1^\top, \ldots, \mathbf{q}_N^\top]^\top$ and using the Laplacian matrix, we can write $\mathbf{e} = (\mathbf{L} \otimes \mathbf{I}_n)\mathbf{q}$. Thus

$$\lim_{t\to\infty} |\mathbf{e}_i(t)| = 0 \Leftrightarrow \lim_{t\to\infty} |\mathbf{e}(t)| = \lim_{t\to\infty} |(\mathbf{L} \otimes \mathbf{I}_n)\mathbf{q}(t)| = 0.$$

Moreover, $(\mathbf{L} \otimes \mathbf{I}_n)\mathbf{q} = \mathbf{0} \Leftrightarrow \mathbf{q} = \mathbf{1}_N \otimes \mathbf{q}_c$. Therefore, (2) holds if we can prove that

$$\lim_{t\to\infty} |\dot{\mathbf{q}}_i(t)| = 0, \quad \lim_{t\to\infty} |\mathbf{e}_i(t)| = 0.$$

In what follows we propose an adaptive controller capable of ensuring that velocities and position errors globally and asymptotically converge to zero.

3. ADAPTIVE P+D CONTROLLER

Inspired in (Tomei, 1991), the proposed adaptive P+d scheme is given by

$$\boldsymbol{\tau}_i = -d_i\dot{\mathbf{q}}_i - p_i\mathbf{e}_i + \mathbf{Y}_i(\mathbf{q}_i)\hat{\boldsymbol{\theta}}_i, \tag{4}$$

where $p_i, d_i \in \mathbb{R}_{>0}$ are the proportional and damping injection gains. The parameter estimation law is

$$\dot{\hat{\boldsymbol{\theta}}}_i := -\boldsymbol{\Gamma}_i\mathbf{Y}_i(\mathbf{q}_i)^\top\left(\frac{1}{p_i}\dot{\mathbf{q}}_i + \epsilon\mathbf{th}(\mathbf{e}_i)\right), \tag{5}$$

where $\epsilon > 0$, $\boldsymbol{\Gamma}_i$ is a positive definite diagonal matrix and $\mathbf{th}(\mathbf{e}_i)$ is a vector defined with the standard hyperbolic tangent as $\mathbf{th}(\mathbf{e}_i) := [\tanh(e_{i1}), \ldots, \tanh(e_{in})]^\top$.

Proposition 1. For each $i \in \bar{N}$, consider the system (1) in closed-loop with (4) and (5). Then, under Assumptions **A1** and **A2**, there exists $\mathbf{q}_c \in \mathbb{R}^n$ such that (2) holds provided that ϵ is set satisfying

$$\epsilon < \min_{i\in\bar{N}}\left\{\sqrt{\frac{1}{2m_{2i}L_{ii}p_i}}; \frac{d_i}{p_i\left(m_2^{\max}\lambda_M\{\mathbf{L}\} + \sqrt{n}k_{ci} + \frac{d_i}{2\mu_i}\right)}\right\}$$
$$\mu_i < \frac{2p_i}{d_i}, \tag{6}$$

where $m_2^{\max} := \max_{i\in\bar{N}}\{m_{2i}\}$. △

Proof. Consider the following Lyapunov candidate function

$$\mathcal{W} = \sum_{i\in\bar{N}}(V_i + \epsilon U_i),$$

where V_i is a positive definite function and U_i is a cross term between position error and velocity. These functions are

$$V_i = \frac{1}{2}\left[\frac{1}{p_i}\dot{\mathbf{q}}_i^\top\mathbf{M}_i\dot{\mathbf{q}}_i + \frac{1}{2}\sum_{j\in\mathcal{N}_i}a_{ij}|\mathbf{q}_i - \mathbf{q}_j|^2 + \tilde{\boldsymbol{\theta}}_i^\top\boldsymbol{\Gamma}_i^{-1}\tilde{\boldsymbol{\theta}}_i\right], \tag{7}$$

and

$$U_i = \mathbf{th}^\top(\mathbf{e}_i)\mathbf{M}_i(\mathbf{q}_i)\dot{\mathbf{q}}_i, \tag{8}$$

respectively. Then, $\mathcal{W}$ can be written as

$$\begin{aligned}\mathcal{W} =& \frac{1}{2}\sum_{i\in\bar{N}}\Bigg[\frac{1}{p_i}(\dot{\mathbf{q}}_i + \epsilon p_i\mathbf{th}(\mathbf{e}_i))^\top\mathbf{M}_i(\dot{\mathbf{q}}_i + \epsilon p_i\mathbf{th}(\mathbf{e}_i))\\ &+ \tilde{\boldsymbol{\theta}}_i^\top\boldsymbol{\Gamma}_i^{-1}\tilde{\boldsymbol{\theta}}_i - \epsilon^2 p_i\mathbf{th}^\top(\mathbf{e}_i)\mathbf{M}_i\mathbf{th}(\mathbf{e}_i)\\ &+ \frac{1}{2}\sum_{j\in\mathcal{N}_i}a_{ij}|\mathbf{q}_i - \mathbf{q}_j|^2\Bigg].\end{aligned}$$

Further, using the fact that $|\mathbf{th}(\mathbf{e}_i)|^2 \le |\mathbf{e}_i|^2$, the term $\mathbf{th}^\top(\mathbf{e}_i)\mathbf{M}_i\mathbf{th}(\mathbf{e}_i)$ can be bounded as

$$\mathbf{th}^\top(\mathbf{e}_i)\mathbf{M}_i\mathbf{th}(\mathbf{e}_i) \le m_{2i}|\mathbf{e}_i|^2 \le m_{2i}L_{ii}\sum_{j\in\mathcal{N}_i}a_{ij}|\mathbf{q}_i - \mathbf{q}_j|^2.$$

Thus, $\mathcal{W}$ admits the following bound

$$\begin{aligned}\mathcal{W} \ge& \frac{1}{2}\sum_{i\in\bar{N}}\Bigg[\frac{1}{p_i}(\dot{\mathbf{q}}_i + \epsilon p_i\mathbf{th}(\mathbf{e}_i))^\top\mathbf{M}_i(\dot{\mathbf{q}}_i + \epsilon p_i\mathbf{th}(\mathbf{e}_i))\\ &+ \left(\frac{1}{2} - \epsilon^2 m_{2i}L_{ii}p_i\right)\sum_{j\in\mathcal{N}_i}a_{ij}|\mathbf{q}_i - \mathbf{q}_j|^2\\ &+ \tilde{\boldsymbol{\theta}}_i^\top\boldsymbol{\Gamma}_i^{-1}\tilde{\boldsymbol{\theta}}_i\Bigg].\end{aligned}$$

Note that setting

$$\epsilon < \min_{i\in\bar{N}}\left\{\sqrt{\frac{1}{2m_{2i}L_{ii}p_i}}\right\}$$

ensures that $\left(\frac{1}{2} - \epsilon^2 m_{2i}L_{ii}p_i\right) > 0$ and hence $\mathcal{W}$ is positive definite and radially unbounded with regards to $\dot{\mathbf{q}}_i, \mathbf{q}_i - \mathbf{q}_j$ and $\tilde{\boldsymbol{\theta}}_i$.

Now, the resulting closed-loop system is

$$\ddot{\mathbf{q}}_i = -\mathbf{M}_i^{-1}(\mathbf{q}_i)\left[\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i + d_i\dot{\mathbf{q}}_i + p_i\mathbf{e}_i + \mathbf{Y}_i(\mathbf{q}_i)\tilde{\boldsymbol{\theta}}_i\right] \tag{9}$$

where $\tilde{\boldsymbol{\theta}}_i := \boldsymbol{\theta}_i - \hat{\boldsymbol{\theta}}_i$. Moreover, $\dot{V}_i$ evaluated along (9), using the fact that $\dot{\tilde{\boldsymbol{\theta}}}_i = -\dot{\hat{\boldsymbol{\theta}}}_i$, yields

$$\begin{aligned}\dot{V}_i =& -\frac{d_i}{p_i}|\dot{\mathbf{q}}_i|^2 - \dot{\mathbf{q}}_i^\top\mathbf{e}_i - \tilde{\boldsymbol{\theta}}_i^\top\left[\frac{1}{p_i}\mathbf{Y}_i^\top(\mathbf{q}_i)\dot{\mathbf{q}}_i + \boldsymbol{\Gamma}_i^{-1}\dot{\hat{\boldsymbol{\theta}}}_i\right]\\ &+ \frac{1}{2}\sum_{j\in\mathcal{N}_i}a_{ij}(\dot{\mathbf{q}}_i - \dot{\mathbf{q}}_j)^\top(\mathbf{q}_i - \mathbf{q}_j),\end{aligned}$$

and using (5) it becomes

$$\begin{aligned}\dot{V}_i =& -\frac{d_i}{p_i}|\dot{\mathbf{q}}_i|^2 - \dot{\mathbf{q}}_i^\top\mathbf{e}_i + \epsilon\mathbf{th}^\top(\mathbf{e}_i)\mathbf{Y}_i(\mathbf{q}_i)\tilde{\boldsymbol{\theta}}_i\\ &+ \frac{1}{2}\sum_{j\in\mathcal{N}_i}a_{ij}(\dot{\mathbf{q}}_i - \dot{\mathbf{q}}_j)^\top(\mathbf{q}_i - \mathbf{q}_j).\end{aligned}$$

From the property that the Laplacian matrix is symmetric, we have that

$$\sum_{i\in\bar{N}}\left[\dot{\mathbf{q}}_i^\top\mathbf{e}_i - \frac{1}{2}\sum_{j\in\mathcal{N}_i}a_{ij}(\dot{\mathbf{q}}_i - \dot{\mathbf{q}}_j)^\top(\mathbf{q}_i - \mathbf{q}_j)\right] = 0.$$

Hence, defining $V = \sum_{i\in\bar{N}}V_i$ yields

$$\dot{V} = -\sum_{i\in\bar{N}}\left[\frac{d_i}{p_i}|\dot{\mathbf{q}}_i|^2 - \epsilon\mathbf{th}^\top(\mathbf{e}_i)\mathbf{Y}_i(\mathbf{q}_i)\tilde{\boldsymbol{\theta}}_i\right].$$

The time-derivative of U_i is

$$\dot{U}_i = \left(\frac{d}{dt}\mathbf{th}^\top(\mathbf{e}_i)\right)\mathbf{M}_i\dot{\mathbf{q}}_i + \mathbf{th}^\top(\mathbf{e}_i)\left[\dot{\mathbf{M}}_i\dot{\mathbf{q}}_i + \mathbf{M}_i\ddot{\mathbf{q}}_i\right].$$

Using **P1** and (9) yields

$$\begin{aligned}\dot{U}_i =& \left(\frac{d}{dt}\mathbf{th}^\top(\mathbf{e}_i)\right)\mathbf{M}_i(\mathbf{q}_i)\dot{\mathbf{q}}_i + \mathbf{th}^\top(\mathbf{e}_i)\mathbf{C}_i^\top(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i\\ &- \mathbf{th}^\top(\mathbf{e}_i)\left[d_i\dot{\mathbf{q}}_i + p_i\mathbf{e}_i + \mathbf{Y}_i(\mathbf{q}_i)\tilde{\boldsymbol{\theta}}_i\right].\end{aligned}$$

In what follows we find an upper bound of $\dot{U}_i$. For, note that the term $\left(\frac{d}{dt}\mathbf{th}^\top(\mathbf{e}_i)\right)\mathbf{M}_i(\mathbf{q}_i)\dot{\mathbf{q}}_i$ can be manipulated as

$$\sum_{i\in\bar{N}}\left(\frac{d}{dt}\mathbf{th}^\top(\mathbf{e}_i)\right)\mathbf{M}_i(\mathbf{q}_i)\dot{\mathbf{q}}_i = \left(\frac{d}{dt}\mathbf{th}^\top(\mathbf{e})\right)\mathbf{M}(\mathbf{q})\dot{\mathbf{q}},$$

where $\dot{\mathbf{q}}$ is defined pilling-up their N elements and $\mathbf{M}(\mathbf{q}) := \text{blockdiag}\{\mathbf{M}_i(\mathbf{q}_i)\}$. The fact that

$$\left|\frac{d}{dt}\mathbf{th}(\mathbf{e}_i)\right| \le |\dot{\mathbf{e}}| = |(\mathbf{L}\otimes\mathbf{I}_n)\dot{\mathbf{q}}| \le \lambda_M\{\mathbf{L}\}|\dot{\mathbf{q}}|$$

is employed to obtain the following inequality

$$\left(\frac{d}{dt}\mathbf{th}^{\top}(\mathbf{e})\right)\mathbf{M}(\mathbf{q})\dot{\mathbf{q}} \le m_2^{\max}\lambda_M\{\mathbf{L}\}|\dot{\mathbf{q}}|^2.$$

Moreover, using **P2** and $|\mathbf{th}(\mathbf{e}_i)|^2 \le n$, yields

$$\mathbf{th}^{\top}(\mathbf{e}_i)\mathbf{C}_i^{\top}(\mathbf{q}_i,\dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i \le \sqrt{n}k_{ci}|\dot{\mathbf{q}}_i|^2.$$

For any $\mu_i > 0$ it holds that

$$-\mathbf{th}^{\top}(\mathbf{e}_i)\dot{\mathbf{q}}_i \le \frac{\mu_i}{2}|\mathbf{th}(\mathbf{e}_i)|^2 + \frac{1}{2\mu_i}|\dot{\mathbf{q}}_i|^2.$$

Hence

$$\dot{U}_i \le \left(m_2^{\max}\lambda_M\{\mathbf{L}\} + \sqrt{n}k_{ci} + \frac{d_i}{2\mu_i}\right)|\dot{\mathbf{q}}_i|^2 - \left(p_i - \frac{\mu_i d_i}{2}\right)|\mathbf{th}(\mathbf{e}_i)|^2 - \mathbf{th}^{\top}(\mathbf{e}_i)\mathbf{Y}_i(\mathbf{q}_i)\tilde{\boldsymbol{\theta}}_i,$$

where we have also employed the fact that

$$\mathbf{th}^{\top}(\mathbf{e}_i)\mathbf{e}_i \ge |\mathbf{th}(\mathbf{e}_i)|^2.$$

Thus $\dot{\mathcal{W}}$ can be bounded as

$$\dot{\mathcal{W}} \le -\sum_{i\in\bar{N}}\left[c_{1i}|\dot{\mathbf{q}}_i|^2 + c_{2i}\epsilon|\mathbf{th}(\mathbf{e}_i)|^2\right],$$

where

$$c_{1i} := \frac{d_i}{p_i} - \epsilon\left(m_2^{\max}\lambda_M\{\mathbf{L}\} + \sqrt{n}k_{ci} + \frac{d_i}{2\mu_i}\right),$$

and

$$c_{2i} := p_i - \frac{\mu_i d_i}{2}.$$

Clearly, setting ϵ and μ_i satisfying (6) ensures that $\dot{\mathcal{W}}$ is negative definite with regards to $\dot{\mathbf{q}}_i$ and $\mathbf{e}_i$. Invoking the LaSalle-Krasovskii Invariance Theorem (Krasovskiĭ, 1963), together with the properties of the Laplacian matrix, we finish the proof. ◁

The following remarks are in order:

Remark 1. The adaptive P+d controller is inspired in the control law reported in (Nuño et al., 2013b), for networks of EL-agents, and in the adaptive scheme reported in (Tomei, 1991; Kelly, 1993). In fact, instead of the hyperbolic tangent function, other bounded functions of the error can be employed in the adaptation law, e.g., the function $\mathbf{h}_i(\mathbf{e}_i) := \frac{1}{1+|\mathbf{e}_i|^2}\mathbf{e}_i$ that is used in (Tomei, 1991), or function $\mathbf{f}_i(\mathbf{e}_i) := \frac{1}{1+|\mathbf{e}_i|}\mathbf{e}_i$, that is applied in (Kelly, 1993). Refer to (Ortega et al., 1998) for the generalization of these bounded functions.

Remark 2. Controller (4)-(5) does not require the neighbours velocities and the regressor matrix $\mathbf{Y}_i(\mathbf{q}_i)$ is only related to the gravity vector in (1).

Remark 3. Although the controller gains are chosen to be positive scalars, the analysis can be extended to diagonal matrix gains with strictly positive elements.

Remark 4. The bounds of the inertia matrices are required to satisfy the stability condition (6). Therefore, some physical information about the agents is required.

4. SIMULATION

Consider a network of ten 2-DOF planar manipulators. Physical parameters of each manipulator is determined with a vector $\overline{P}_i = [m_{1i}, m_{2i}, l_{1i}, l_{c1i}, l_{2i}, l_{c2i}, I_{1i}, I_{2i}]$, where m_{ki}, l_{ki}, and l_{cki} are the mass, the length and the distance to the center of mass of link k. Also, I_{1i} and I_{2i} are the diagonal elements of the inertia matrix. Three different groups of manipulators are considered. The physical parameters of these manipulators are given in Table 1. The graph Laplacian matrix is given by $L = 0.1L_g$, where

$$L_g = \begin{bmatrix} 14 & 0 & -3 & 0 & 0 & 0 & 0 & -4 & 0 & -7 \\ 0 & 9 & 0 & -8 & 0 & 0 & 0 & 0 & 0 & -1 \\ -3 & 0 & 5 & 0 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & -8 & 0 & 10 & 0 & 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & 8 & 0 & -5 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 & 0 & -4 & -2 & 0 \\ 0 & 0 & 0 & 0 & -5 & 0 & 14 & 0 & 0 & -9 \\ -4 & 0 & -2 & 0 & 0 & -4 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & -2 & -3 & -2 & 0 & 0 & 7 & 0 \\ -7 & -1 & 0 & 0 & 0 & 0 & -9 & 0 & 0 & 17 \end{bmatrix}.$$

The initial joint velocities are assumed to be zero while the initial joint positions are set as $\mathbf{q}(0) = [2, 6, -7, 3, 1, 8, 0, 1, -6, 9, -5, 0, -4, 5, -3, 4, -2, 7, -8, 1]^{\top}$.

The simulation scenario is the same as in (Nuño and Ortega, 2018) with the sole difference being that the distance to the center of mass is distinct from the length of the link. The initial parameter estimates are set as $\hat{\boldsymbol{\theta}}(0) = 10\mathbf{1}_2$. The controller gains are chosen as $p_i = 4$, $d_i = 8$, $\boldsymbol{\Gamma}_i = 80\mathbf{I}_2$, $\epsilon = 0.25$.

The simulation results for the leaderless consensus controller without using the neighbours velocities are shown in Figs. 1-4. Fig. 1 depicts the joint positions agreement in a consensus point. Fig. 2 shows the joint velocities of manipulators. Fig. 3 shows the evolution of the estimated parameters. Input torques of the robot manipulators are drawn in Fig. 4.

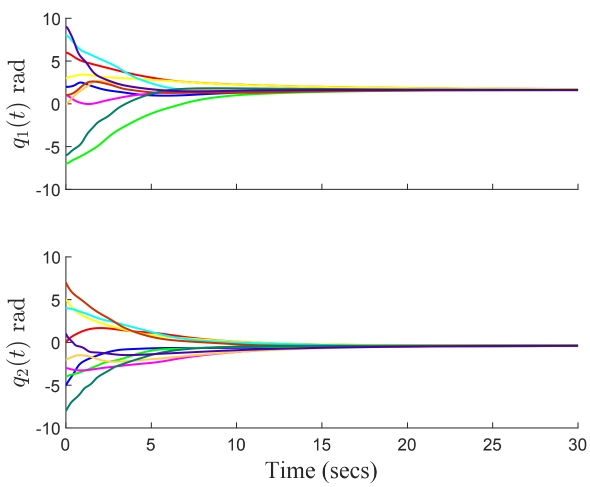

Fig. 1. Position consensus using the adaptive P+d controller.

5. CONCLUSIONS

In this paper we report a novel decentralized adaptive controllers to solve the leaderless consensus in networks of uncertain EL-systems. The controller estimates only the gravitational term of the dynamics. Therefore, it does not require the computational burden of estimating the

Table 1. Physical Parameters of the Robot Manipulators

	Manipulators 1-3		Manipulators 4-6		Manipulators 7-10	
	Link 1	Link 2	Link 1	Link 2	Link 1	Link 2
Link length l_{ki} (m)	0.4	0.4	0.3	0.5	0.5	0.2
Link mass m_{ki} (kg)	4	2	2.5	3	3	2.5
Link inertia I_{ki} (kg.m^2)	0.478	0.044	0.678	0.144	0.03	0.01
Center of mass l_{cki} (m)	0.2	0.2	0.15	0.25	0.25	0.1

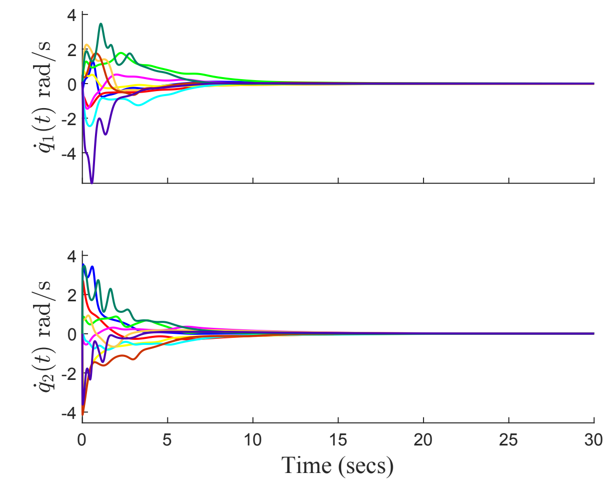

Fig. 2. Joint velocities using the adaptive P+d controller.

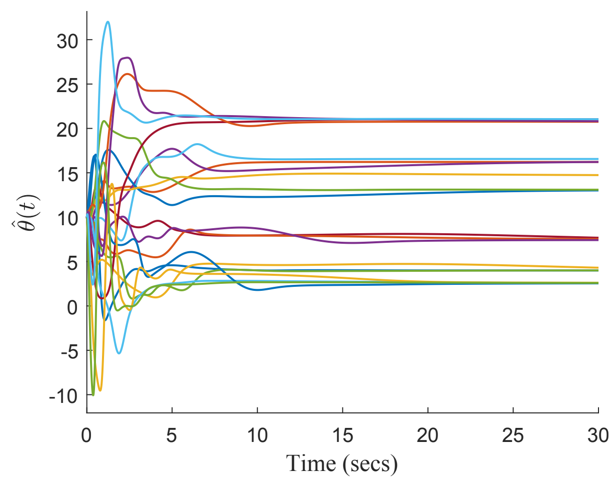

Fig. 3. Estimated parameters for the leaderless consensus with the adaptive P+d controller.

whole EL-dynamics. The proposals is a simple Proportional plus damping scheme and it does not require to exchange velocity information between the agents. Simulations show the performance of the proposed controller using a network composed of ten EL-agents.

Future research goes along two different avenues: 1) including time-delays in the communications of the agents; and 2) extending the solution to directed graphs. None of these problems can be, however, trivially solved. For the delayed case one has to design a proper Lyapunov function such that its time-derivative is negative definite with regards to the delayed error, which is rather difficult —to our knowledge, the first strict Lyapunov function for the delayed case has been designed in (Nuño et al., 2018) for the P+d with gravity cancellation, i.e., without parametric uncertainty. As for the directed case, we are analyzing the solution of Ye et al. (2017). However the Lyapunov function employed in (Ye et al., 2017) can only be employed to conclude a local stability result.

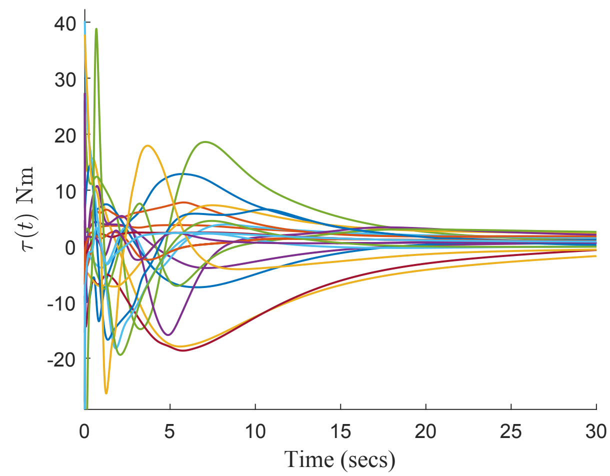

Fig. 4. Input torques for the leaderless consensus with the adaptive P+d controller.

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