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Position Control of a DC Motor based on Block Control and a Sliding Mode Observer

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Abstract: This paper proposes a position controller for a DC motor based on block control and sliding modes. The load torque of the motor is considered as an unknown bounded disturbance; hence the characteristic robustness of the sliding modes is used to minimize its effect on the tracking performance. In addition, a sliding mode observer is used to retrieve the state variables that are not measured directly. The stability analysis is presented to demonstrate de convergence of the angular position of the motor to a desired time-varying reference despite the unknown load torque. Simulations results are presented to show the effectiveness of the proposed control and estimation scheme. Finally, the conclusions and considered future work are outlined.

Keywords: DC motor; Block Control; Nested Sliding Mode; Sliding Mode Observer.

1. INTRODUCTION

Every control scheme requires an actuator to modify the behaviour of the target system. Amongst all the actuators, the electric motor is one of the most used in real-time implementations of a controller due to its simple structure, relative low cost, diversity in available size, weight, power consumption, and output torque.

On the other hand, the DC motor is one of the simpler types of electric motor and, due to that, has been applied in a lot of engineering and research areas as robotics, vehicles, home appliances, electronic toys, etcetera. Consequently, a lot of research has been done to model and control the angular position, angular velocity or output torque of this kind of motors.

For instance, Bacac et al. (2014) analyses and compares three different control schemes for DC motors: PID control, cascade control and state space controller. The main advantages of the PID controller is that it does not need to know the mathematical model of the plant and its simple structure. These are key features in a real time implementation of any controller. However, PID control is not robust against time variant external disturbances or unknown parameters. In Maung et al. (2018) a PID controller is proposed considering a friction compensator to improve the closed loop performance.

More complex control schemes have been applied to the position control of a DC motor. Saikumar and Dinesh (2012) proposes an experience mapping based prediction controller for the DC motor obtaining good results. Castañeda et al. (2010) proposed a high order neural network controller for the position of a DC motor, Furthermore, another option of the so-called intelligent control was used in De Azevedo et al. (1993) and Hendijanizadeh et al. (2006) which explores the application of fuzzy controllers in the DC motors. The latter includes a sliding mode term to increase the robustness of the proposed control scheme against external disturbances and model uncertainties. These are factors usually presented in a real time application of a controller, as it is virtually impossible to know exactly the mathematical model of any real system. In addition, any environment includes external disturbances that affects the plant.

In that regard, sliding modes have proved to be one of the best options to control a system with external disturbances and model uncertainties. Moreover, it assures finite time convergence in the closed loop system and it is very simple to implement. Mamani et al. (2010) proposes a sliding mode controller and shows the robustness of the control scheme.

Another usual characteristic of a real time plant is the lack of sensors for all the state variables of the system. This is normally compensated by means of state observers. In Keswani1 and Verma (2017) a sliding mode controller was based on a Luenberger linear observer for the position control of a DC motor. The sliding mode approach has been also applied to the design of robust observers as in Yong et al. (2013) and Drakunov and Utkin (1995). In the former a hybrid sliding mode observer has been designed, and in the latter a standard sliding mode observer is proposed for linear and nonlinear systems. The advantage of the second approach is the simplicity of the observer's structure which is very important in real time or embedded applications as in Mondala et al. (2013).

In this paper, we present a block control scheme for the angular position of a DC motor. The load torque is regarded as an unknown bounded disturbance, so the nested sliding mode approach is implemented as well to attenuate the effect of this unmatched disturbance term. Furthermore, a sliding mode observer is proposed to retrieve the value of the states of the system that are not measured by sensors. Also, the stability analysis of the closed loop system is presented to demonstrate the convergence of the error variables to zero. Simulation results are presented to validate the effectiveness of the proposed control scheme. Finally, the conclusions of the work and future work are outlined.

2. BLOCK CONTROLLER FOR A DC MOTOR

The proposed controller is based on the block control approach in combination with nested sliding modes. In addition, a sliding mode observer is designed as two states (ω and I) are not directly measured. The first part of this section is dedicated to define the dynamical model of the system.

2.1 DC Motor Dynamic Model

Consider the following differential equations of a DC motor

$$\begin{aligned} \dot{\theta}(t) &= \omega(t) \\ \dot{\omega}(t) &= -\frac{b}{J}\omega(t) + \frac{k_T}{J}I(t) - \tau_L(t) \\ \dot{I}(t) &= -\frac{k_b}{L}\omega(t) - \frac{R}{L}I(t) + \frac{1}{L}V(t) \end{aligned} \tag{1}$$

where $\theta(t), \omega(t)$ are the angular position and velocity of the DC motor shaft, respectively; $I(t)$ is the motor's current; $V(t)$ is the applied voltage; J is the rotor inertia; R, L are the resistance and inductance of the armature; k_T, k_b are the torque and back-emf constants, respectively; and b is the friction constant. In addition, $\tau_L(t)$ is the load torque, which is unknown, but it is assumed bounded by

$$|\tau_L(t)| < \beta, \qquad \forall t > t_0. \tag{2}$$

Defining the state of the system as $x(t) = [\theta(t), \omega(t), I(t)]^T$, the input as $u = V(t)$, and the output as $y = \theta(t)$, a state-space model of the motor can be obtained as

$$\dot{x}(t) = Ax(t) + Bu - \begin{bmatrix} 0 \\ \tau_L(t) \\ 0 \end{bmatrix}, \qquad y = Cx(t) \tag{3}$$

with

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{b}{J} & \frac{k_T}{J} \\ 0 & -\frac{k_b}{L} & -\frac{R}{L} \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix}, C = [1 \quad 0 \quad 0].$$

It is worth to note, that the output y of the model was defined considering sensors attached to the rotor such that its relative angular position and velocity w.r.t. the motor can be retrieved.

2.2 Block Control Design

Assume a known output reference y_r and its derivatives $\dot{y}_r$, $\ddot{y}_r$ and $\dddot{y}_r$. Then, the error variable for the first block of the system is given by $e_1 = y_r - y(t)$ and, by means of (3), its dynamics are obtained as

$$\begin{aligned} \dot{e}_1 &= \dot{y}_r - \dot{y}(t) \\ &= \dot{y}_r - \omega(t). \end{aligned} \tag{4}$$

Equation (4) represents the first block of the system which will be controlled by the term $\omega(t)$. This pseudo-control term permits to define the reference term ω_r for the next block of the system. Defining the desired dynamics for e_1 as

$$\dot{e}_1 = -k_1 e_1, \qquad k_1 > 0 \tag{5}$$

and using (4), the reference for the next block yields

$$\omega_r = \dot{y}_r + k_1 e_1. \tag{6}$$

Now, the error variable for the second block can be defined as $e_2 = \omega_r - \omega(t)$ and its dynamics can be obtained using (3) as

$$\begin{aligned} \dot{e}_2 &= \dot{\omega}_r - \dot{\omega}(t) \\ &= \dot{\omega}_r + \frac{b}{J}\omega(t) - \frac{k_T}{J}I(t) + \tau_L(t) \end{aligned} \tag{7}$$

where $\dot{\omega}_r = \ddot{y}_r - k_1(\dot{y}_r - \omega(t))$.

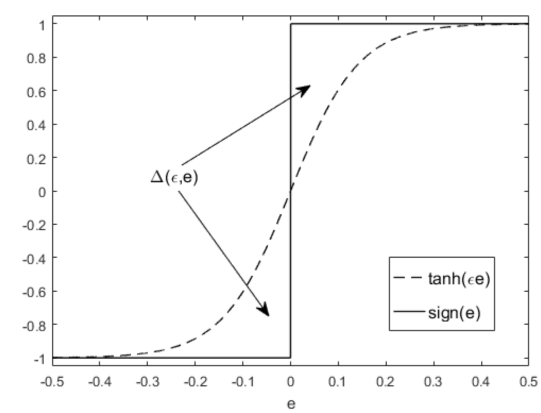

Fig.1. The relation of $\Delta(\epsilon, e)$ with sign, sigmoidal functions.

In order to define the reference for the last block, is necessary to determine the desired dynamics for e_2. In addition, as the term $\tau_L(t)$ is unknown but bounded by (2), we can use the nested sliding modes approach in this block to minimize its effect, as in González-Jiménez et al. (2011). Hence, the desired dynamics for this block results in

$$\dot{e}_2 = -k_2 \text{sigm}(\epsilon_2 e_2) + \tau_L(t), \qquad k_2 > 0 \tag{8}$$

where $\text{sigm}(\cdot)$ is a sigmoidal function with slope $\epsilon_2 > 0$, and it is related to the $\text{sign}(\cdot)$ function by the following equation

$$\Delta(\epsilon_2, e_2) = \text{sign}(e_2) - \text{sigm}(\epsilon_2 e_2) \tag{9}$$

with $\Delta(\epsilon_2, e_2)$ representing the positive bounded difference of the sign and sigmoidal function. A graphical depiction of (9) can be appreciated in Figure 1. Nonetheless, it is important to remark that this approximation affects the disturbance rejection and finite time convergence properties of the standard sliding modes and achieves only convergence to a vicinity of $e_2 = 0$ which is determined by the parameters ϵ_2, k_2 and β. These features will be taken into consideration during the stability analysis of the closed loop system, which is presented in a latter section.

Then, the reference I_r for the third block is obtained as

$$I_r = \frac{J}{k_T}\dot{\omega}_r + \frac{b}{k_T}\omega(t) + \frac{Jk2}{k_T}\mathrm{sigm}(\epsilon_2 e_2) \tag{10}$$

where $\mathrm{sigm}(\epsilon_2 e_2)$ was implemented by the $\tanh(\epsilon_2 e_2)$ function.

Finally, the error variable for the last block of the system results in $e_3 = I_r - I(t)$. By differentiation, the dynamics of e_3 is obtained of the form

$$\begin{aligned} \dot{e}_3 &= \dot{I}_r - \dot{I}(t) \\ &= \dot{I}_r + \frac{k_b}{L}\omega(t) + \frac{R}{L}I(t) - \frac{1}{L}u \end{aligned} \tag{11}$$

where, by direct differentiation, $\dot{I}_r = \frac{J}{k_T}\ddot{\omega}_r + \frac{b}{k_T}\dot{\omega}(t) + \frac{Jk_2}{k_T}\epsilon_2(1 - \tanh^2(\epsilon_2 e_2))$. It is worth to note that it could be computed using a robust sliding mode differentiator as in Levant (2001). It depends on the plant dynamics which approach would offer a lower computational cost.

Then, if the desired dynamics for the last block is designed as

$$\dot{e}_3 = -k_3\mathrm{sign}(e_3), \qquad k_3 > 0 \tag{12}$$

the control input of the system can be designed as

$$u = L\dot{I}_r + k_b\omega(t) + RI_o + k_3L\mathrm{sign}(e_3) \tag{13}$$

which finalizes the control design procedure.

2.3 Current Sliding Mode Observer

In addition to the calculation of I_r, the overall control scheme requires the knowledge of the current of the motor. However, the system only accounts with sensors to retrieve $\theta(t)$ and $\omega(t)$, so an observer must be used to estimate the remaining state $I(t)$. The proposed observer is based on Drakunov et al. (1995) and a necessary condition for its design is that the pair (A, C) of the system is observable i.e. the observability matrix is full rank. Using the matrices of (3), the observability matrix results

$$\mathcal{O} = [C \quad CA \quad CA^2]^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{b}{J} & \frac{k_T}{J} \end{bmatrix} \tag{14}$$

which is full rank as b, k_T and J are positive constants. Hence, the following full order sliding mode observer is proposed

$$\dot{x}_o = Ax_o + Bu + L_o\mathrm{sign}(\tilde{y}), \qquad \tilde{y} = y - Cx_o \tag{15}$$

where $x_o = [\theta_o \quad \omega_o \quad I_o]^T$ is the observed state vector, and $L_o \epsilon \mathbb{R}^{3\times 1}$. Defining the observation error as $e_o = x(t) - x_o$, its dynamics are defined as

$$\dot{e}_o = Ae_o - L_o\mathrm{sign}(\tilde{y}) + \lambda_o \tag{16}$$

with $\lambda_o = [0 \quad -\tau_L(t) \quad 0]^T$.

3. CLOSED LOOP STABILITY ANALYSIS

Using equations (4), (6), (7), (10), (11) and (13), the closed loop dynamics of the system can be obtained as

$$\begin{aligned} \dot{e}_1 &= -k_1e_1 + e_2 \\ \dot{e}_2 &= -k_2 sigm(\epsilon_2 e_2) + \frac{k_T}{J}e_3 + \tau_L(t) \\ \dot{e}_3 &= \frac{R}{L}e_o(3) - k_3 sign(e_3) \\ \dot{e}_o &= Ae_o - L_o sign(\tilde{y}) + \lambda_o. \end{aligned} \tag{17}$$

The stability of the observation error system (16) has been demonstrated in Drakunov et al. (1995) and in Utkin et al. (2009). The necessary stability condition is that $L_o(2) > \beta$ which assures finite time convergence, after the sliding mode occurrence, of the observation errors e_o to a vicinity of its origin which bound is directly proportional to the value of L_o and inversely proportional to β.

According to the previous conclusion, let us assume that the bound of $e_o(3)$ is defined as $|e_o(3)| < \alpha_o$. Then, the dynamics of e_3 results

$$\dot{e}_3 = -k_3\mathrm{sign}(e_3) + \frac{R}{L}e_o(3). \tag{18}$$

Defining the Lyapunov function $V_3 = \frac{1}{2}e_3^2$ which derivative is given by $\dot{V}_3 = e_3\dot{e}_3 = -k_3|e_3| + \frac{R}{L}e_3e_o(3)$. Using the bound of $e_o(3)$ the derivative of V_3 fulfils

$$\begin{aligned} \dot{V}_3 &\leq -k_3|e_3| + \tfrac{R}{L}|e_3|\alpha_o \\ &\leq -|e_3|\left(k_3 - \tfrac{R}{L}\alpha_o\right). \end{aligned} \tag{19}$$

Hence, if the condition $k_3 > \frac{R}{L}\alpha_o$ then $\dot{V}_3 \leq 0$ and e_3 converges to zero in finite time.

After that, the dynamics of e_2 is given by

$$\dot{e}_2 = -k_2 sigm(\epsilon_2 e_2) + \tau_L(t).$$

Now, repeating the approach used for e_3, let us define a Lyapunov function $V_2 = \frac{1}{2}e_2^2$ which derivative is given by $\dot{V}_2 = e_2\dot{e}_2 = -k_2e_2 sigm(\epsilon_2 e_2) + e_2\tau_L(t)$. Using (2) and (9), $\dot{V}_2$ fulfils the following inequality

$$\dot{V}_2 \leq -|e_2|(k_2 - k_2\Delta(\epsilon_2, e_2) - \beta) \tag{20}$$

and if the condition $k_2 > \frac{\beta}{1-\Delta(\epsilon_2, e_2)}$ is met then $\dot{V}_2 \leq 0$ and e_2 converges to a vicinity of zero in finite time. It is worth to note that the term $1 - \Delta(\epsilon_2, e_2)$ is always positive as depicted in Fig. 1, but this affects the robustness of the sliding mode approach as it only achieves the convergence to a vicinity of zero, which bound is directly proportional to β and $\Delta(\epsilon_2, e_2)$ and inversely proportional to k_2.

Henceforth, the dynamics of e_1 is given by

$$\dot{e}_2 = -k_2 sigm(\epsilon_2 e_2) + \tau_L(t).$$

Now, let us assume that the bound of e_2 is defined as $|e_2| < \alpha_2$ and defining a Lyapunov function $V_1 = \frac{1}{2}e_1^2$ which derivative is given by $\dot{V}_1 = e_1\dot{e}_1 \leq -k_1|e_1|^2 + |e_1|\alpha_2 \leq -|e_1|(k_1|e_1| - \alpha_2)$. Therefore, if $k_1|e_1| > \alpha_2$ then $\dot{V}_2 \leq 0$ which can be interpreted as e_1 converging to a vicinity of the origin bounded by $\frac{\alpha_2}{k_1}$.

The next section shows the results obtained from the simulations of the closed loop system.

4. SIMULATION RESULTS

The simulation considered the parameters, control, and observer gains defined in Table 1. The parameters were obtained from a real DC motor in the ITESO University electronics laboratory. A standard experimental procedure for retrieving the parameters was carried out.

Table 1. Simulations parameters

Parameter	Value	Unit	Parameter	Value
b	2.01×10^{-6}	$\frac{\mathrm{N \cdot m \cdot s}}{\mathrm{rad}}$	k_1	5
J	9.7×10^{-7}	N	k_2	20
L	6.07	mH	k_3	40
R	17.88	Ω	L_0	$[3,20,60]^T$
k_b	11.8×10^{-3}	V · s/rad	β	5
k_T	11.8×10^{-3}	N · m/A	ϵ_2	20

Additionally, the output's reference was defined as $y_r(t) = 3 + 2\cos(2t)$; the load torque as set to $\tau_L(t) = 5\cos(3t)$ and the initial conditions of the system were $x(0) = [1 \quad 2 \quad 0]^T$. The simulation results are shown in the following figures. The output and its reference are presented in Fig. 2. It can be noted the smooth and fast convergence of the angular position of the motor to its reference. No overshoot is presented in its response which demonstrates the robustness of the proposal to the load torque disturbance term. In addition, the output tracking error is depicted in the same figure. The three states of the system are presented in Fig. 3. Their stability can be appreciated and their boundedness to real and feasible values. The performance of the sliding mode observer can be evaluated using Fig. 4 where the three estimation errors are presented. The stability and robustness of the sliding mode observer can be noted as a fast convergence of the estimation errors is depicted.

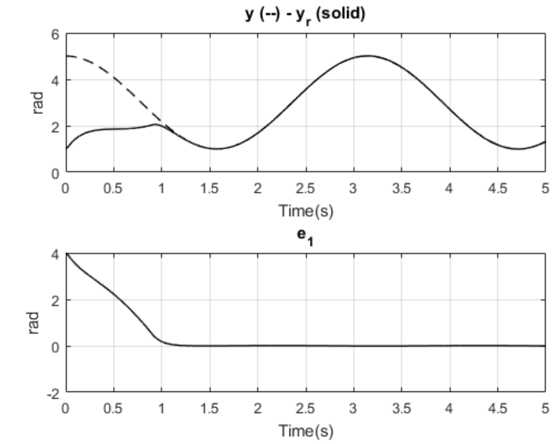

Fig. 2. The output of the system y and its reference y_r (top) and the output tracking error $e_1 = y_r - y$ (bottom).

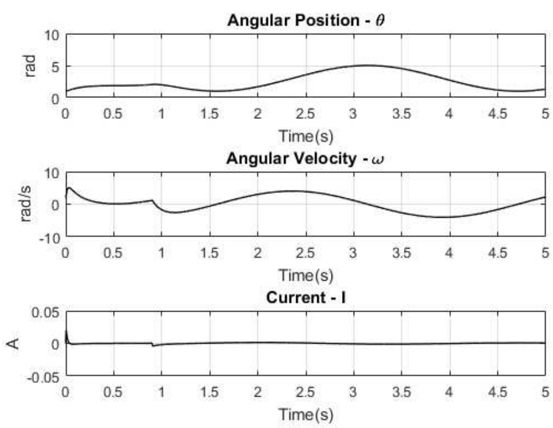

Fig. 3. Evolution of the three states of the system $\theta(t)$, $\omega(t)$ and $I(t)$.

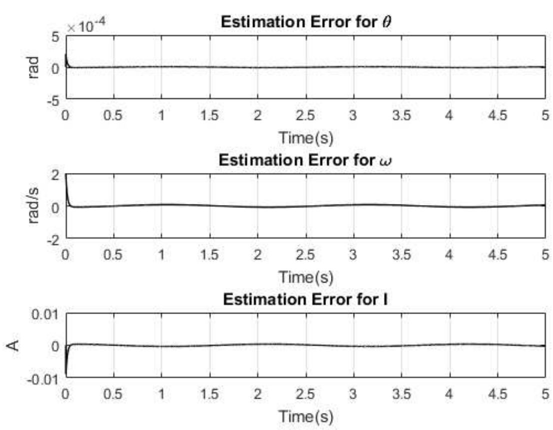

Fig. 4. The three components of the estimation error e_o.

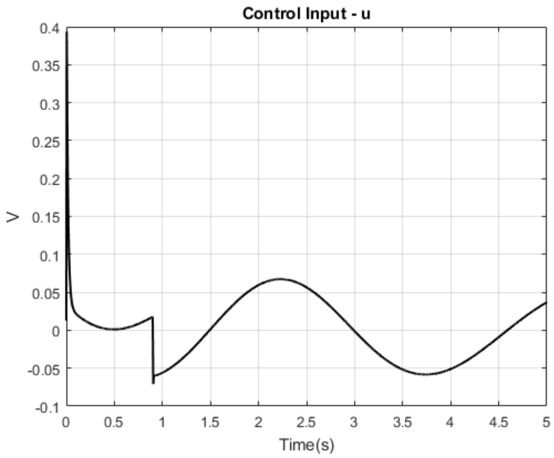

Fig. 5. Control signal behaviour during the simulation experiment.

The control signal, in this case the input voltage of the motor, is shown in Fig. 5. An initial high frequency overshoot is present at the beginning of the experiment but with values and bandwidth achievable for an electric signal as the input voltage. Furthermore, this high frequency excitation is not appreciated in the output of the system, angular position, as the mechanical part of the motor acts as a second order filter for the input voltage. Finally, the load torque is shown in Fig. 6 which is considered an unknown disturbance term of the system.

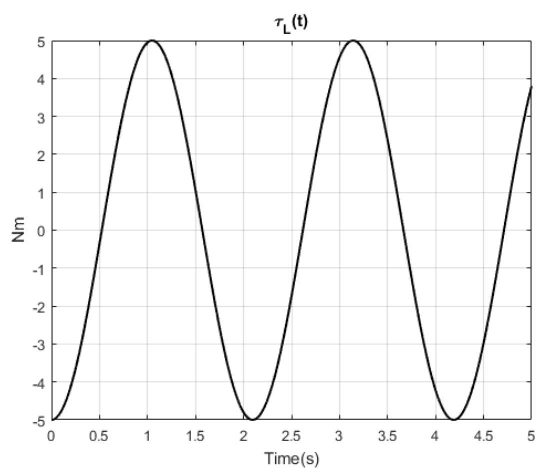

Fig. 6. Load torque used during the simulation experiment.

6. CONCLUSIONS AND FUTURE WORK

A controller for the reference tracking for the position of a DC motor was presented. The controller was designed using the block control approach in combination with nested sliding modes. Moreover, a sliding mode observer was designed as only the position and velocity of the motor are sensed and all the states are necessary for the designed control scheme. The overall scheme showed a satisfactory performance as the reference tracking response had no overshoot and low settling time. The simulation results also demonstrated the convergence of all the estimation errors; hence, validating the election of the sliding mode controller for the proposal. The input voltage signal features feasible magnitude and bandwidth so the control scheme is candidate for a real time implementation. It is worth to note, that the same scheme can be used for controlling the velocity and output torque of the DC motor by only changing the number of blocks considered for the controller design.

The proposed control scheme was designed to be implemented in an embedded system. The ongoing work in our laboratory is related to this implementation. In addition, a comparison with other control techniques is an interesting future work, Finally, the implementation of this control scheme to other types of electric motors as brushless dc motors or induction motors is left for future research.

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