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Fractional Order Model and fractional order control for 1, 2 & 3 DOF robot arm in STM32L476 development board

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Abstract: The Fractional-Order Calculus is applied to the dynamic-model of robotic arms of 1, 2 and 3 Degree-of-freedom. In general the arm robots present error in steady statem, this error is explained due to nonlinear effects in friction, however this document proposes an explanation based on the dynamic-model of fractional-order, in addition the control algorithm implemented is the fractional-order proportional-derivative (PD) control, both the dynamic model and the fractional order control are implemented in an STM32L476RG development board, using the analog outputs the signals are sent to an oscilloscope and a series of points are displayed that draw the letters “H” “O” “L” “A”. The results obtained are concordant with the expected in the theoretical objective of this work.

Keywords: Fractional-Order Model, Factional-Order Control, Arm-Robot, STM32-Board.

1. INTRODUCTION

Experimentally speaking, the dynamics of the systems present a difference between the mathematical model and the experimental behaviour. On the subject of robotics, dynamic models and experimental data show certain differences, one of the purposes of this document is to propose a model using Fractional-Order Calculus (FOC) through which a possible solution can be provided.

The use of Fractional-Order Calculus applied to dynamic models is a widely researched topic. In fact, one of the most interesting applications is shown inBill Goodwine’s (2015) work, where the Fractional-Order Calculus is employed in welding dynamics.

This work is based on the works by Ceron-Morales where Fractional-Order control is applied to the manipultion of a solar system (2018a) and to the control of the dynamics of a two Degrees-of-freedom (DOF) robot (2018b).

In the most recent work by Ceron-Morales (2019) it is explained how to implement the dynamic model on a robot in order to make it write, to eventually implement this in a microcontroller (MCU). Finally in the work by Flores-Ordeñana (2017) show how to implement fractional-order control in an MCU.

2. MATH MODELS

The math models have demonstrated to be an approximation to the behavior of physical systems. When building a robotic system there is a difference between experimental data and the data obtained from the dynamic math-model. Euler-Lagrange’s methodology is used to obtain dynamic math-model of rotational link manipulator robots.

The hypothesis of this paper is: a math model based on Fractional-Order Calculus presents numerical data with less differences between the experimental data and the physical system, compared to a system based on Integer-Order Calculus.

To answer the hypothesis this work begins considering the dynamic models of robot arms of 1, 2, and 3 degrees of freedom shown in Figures 1, 2, and 3 respectively.

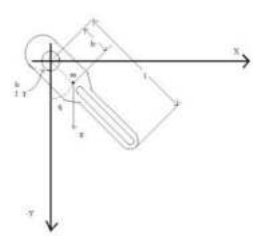

Fig. 1. Diagram of 1 DOF Arm, parameters in Table 1.

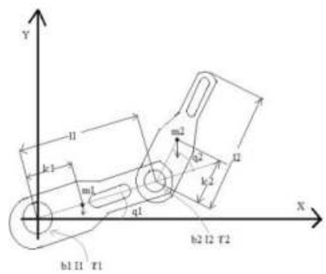

Fig. 2. Diagram of 2 DOF Arm, parameters in Table 2.

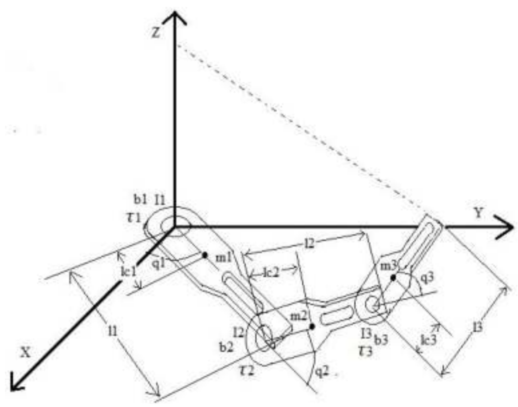

Fig. 3. Diagram of 3 DOF Arm, parameters in Table 3.

Table 1. Parameters for 1 FOD Arm

parameter	Link 1	value	Units
Mass	m1	1	Kgs
Length	l1	1	m
Length mass center	lc1	0.5	m
Rotational Inertia	I1	0.1	m
Viscous friction	b1	0	$N\cdot m\cdot s^{-1}$
Torque	$\tau 1$	___	N·m
Angular Position	q_1	___	Degrees
Angular Velocity	$\dot{q_1}$	___	degrees·s^{-1}
Angular Acceleration	$\ddot{q_1}$	___	degrees·s^{-2}

The math-model of the robot of 1 DOF is (1).

$$\tau_1 = \left(m_1 l{c_1}^2 + I_1\right)\ddot{q_1} + m_1 g l c_1 \cos(q_1) \quad (1)$$

Apply state variables

$x_1 = q_1$

$x_2 = \dot{x_1} = \dot{q_1}$

$\dot{x_2} = \ddot{q_1}$

We have (2).

$$\begin{bmatrix}\dot{x_1}\\ \dot{x_2}\end{bmatrix} = \begin{bmatrix} x_2 \\ \left(m_1 l{c_1}^2 + I_1\right)^{-1}(\tau_1 - m_1 g l c_1 \cos(x_1))\end{bmatrix} \quad (2)$$

In Laplace variable its (3).

$$\begin{bmatrix}sx_1\\ sx_2\end{bmatrix} = \begin{bmatrix} x_2 \\ \left(m_1 l{c_1}^2 + I_1\right)^{-1}(\tau_1 - m_1 g l c_1 \cos(x_1))\end{bmatrix} \quad (3)$$

We propose the fractional order model (4) according to Krishna (2011).

$$\begin{bmatrix}s^\mu x_1\\ s^\mu x_2\end{bmatrix} = \begin{bmatrix} x_2 \\ \left(m_1 l{c_1}^2 + I_1\right)^{-1}(\tau_1 - m_1 g l c_1 \cos(x_1))\end{bmatrix} \quad (4)$$

The order of the derivative is bounded (5) this is explained in Krishna's work (2011).

$$0 < \mu < 1 \quad (5)$$

An approximation is made in continous fraction of (5) is shown in (6), this approach is for a reduced bandwidth (Krishna 2011).

$$s^\mu \approx \frac{As+1}{s+A} \quad (6)$$

Where

$$A = \frac{1+\mu}{1-\mu} \quad (7)$$

We made a simplification by (8).

$$f = \left(m_1 l{c_1}^2 + I_1\right)^{-1}(\tau_1 - m_1 g l c_1 \cos(x_1)) \quad (8)$$

Substituting in the model of state-variables we get (9).

$$(As+1)\begin{bmatrix}x_1\\ x_2\end{bmatrix} = (s+A)\begin{bmatrix}x_2\\ f\end{bmatrix} \quad (9)$$

Applying algebra and the Euler-approximation (10)

$$sx_i \approx \frac{x_i - x_{it}}{T} \quad (10)$$

Finally, the fractional order model is (11)

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \left(\frac{T}{A^2}\right)\dot{f} + \frac{T}{A}f + T\left(\frac{A^2-1}{A^2}\right)x_{2t} + x_{1t}\left(1-\frac{T}{A}\right) \\ \left(\frac{T}{A}\right)\dot{f} + Tf + x_{2t}(1-\frac{T}{A}) \end{bmatrix} \quad (11)$$

Where

$$\dot{f} \approx \frac{f(t-T)-f(t-2T)}{T} \quad (12)$$

Equation (11)is in discrete format, it can be programmed in adigital device. The development card used in the STM32 NUCLEO L476RG.

The parameters used are shown in Table 1. The control algorithm is the one proposed by Takegaki and Arimoto (1981) plus gravity compensation.

The results are shown in Figure 4 for μ=0.99 and μ=0.95.

Table 2. Parameters for 2 FOD Arm

Parameter	Link 1	Link 2	Units
Mass	m1	m2	Kgs
	0.1	0.05	
Length	l1	l2	m
	0.1	0.1	
Length-mass center	lc1	lc2	m
	0.05	0.05	
Inertia rotational	I1	I2	m
	0.2	0.09	
Viscous friction	b1	b2	$N\cdot m\cdot s^{-1}$
	0.2	0.17	
Torque	$\tau 1$	$\tau 2$	N·m
Angular position	q_1	q_2	Degrees
Velocity angular	$\dot{q_1}$	$\dot{q_2}$	$degrees\cdot s^{-1}$
Acceleration angular	$\ddot{q_1}$	$\ddot{q_2}$	$degrees\cdot s^{-2}$

The math-model for 2 FOD Arm Robot is (13).

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} m_{11}(q) & m_{12}(q) \\ m_{21}(q) & m_{22}(q) \end{bmatrix}\begin{bmatrix} \ddot{q_1} \\ \ddot{q_2} \end{bmatrix} + \begin{bmatrix} b_1 & 0 \\ 0 & b_2 \end{bmatrix}\begin{bmatrix} \dot{q_1} \\ \dot{q_2} \end{bmatrix} + \begin{bmatrix} c_{11}(q,\dot{q}) & c_{12}(q,\dot{q}) \\ c_{21}(q,\dot{q}) & c_{22}(q,\dot{q}) \end{bmatrix}\begin{bmatrix} \dot{q_1} \\ \dot{q_2} \end{bmatrix} + \begin{bmatrix} g_1(q) \\ g_2(q) \end{bmatrix} \quad (13)$$

The physics parameter of 2 DOF is shown in Table 2. In (14) to (23) we have tie explicit equation.

$$m_{11}(q) = m_1 lc_1{}^2 + m_2 lc_2{}^2 + I_1 + I_2 + 2m_2 l_1 lc_2 \cos(q_2) \quad (14)$$
$$m_{12}(q) = m_2 lc_2{}^2 + I_2 + m_2 l_1 lc_2 \cos(q_2) \quad (15)$$
$$m_{21}(q) = m_2 lc_2{}^2 + I_2 + m_2 l_1 lc_2 \cos(q_2) \quad (16)$$
$$m_{22}(q) = m_2 lc_2{}^2 + I_2 \quad (17)$$
$$c_{11}(q) = -2m_2 l_1 lc_2 \dot{q_2} \sin(q_2) \quad (18)$$
$$c_{12}(q) = -m_2 l_1 lc_2 \dot{q_2} \sin(q_2) \quad (19)$$
$$c_{21}(q) = m_2 l_1 lc_2 \dot{q_1} \sin(q_2) \quad (20)$$
$$c_{22}(q) = 0 \quad (21)$$

$$g_1(q) = m_1 g lc_1 sin(q_1) + m_2 g l_1 sin(q_1) + m_2 g lc_2 sin(q_1 + q_2) \quad (22)$$
$$g_2(q) = m_2 g lc_2 sin(q_1 + q_2) \quad (23)$$

Apply the same procedure in 1 DOF Arm-Robot we have using the state-variables (24) to (27).

$$q_1 = x_1 \quad (24)$$
$$q_2 = x_3 \quad (25)$$
$$\dot{q_1} = x_2 = \dot{x_1} \quad (26)$$
$$\dot{q_2} = x_4 = \dot{x_2} \quad (27)$$

The state-variable model is (28) and (29)

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_2 \\ x_4 \end{bmatrix} \quad (28)$$

$$\begin{bmatrix} \dot{x_2} \\ \dot{x_4} \end{bmatrix} = \begin{bmatrix} m_{11}{}^{-1}(x) & m_{12}{}^{-1}(x) \\ m_{21}{}^{-1}(x) & m_{22}{}^{-1}(x) \end{bmatrix}\left\{\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} - \begin{bmatrix} c_{11}(x) & c_{12}(x) \\ c_{21}(x) & c_{22}(x) \end{bmatrix}\begin{bmatrix} x_3 \\ x_4 \end{bmatrix} - \begin{bmatrix} b_1 & 0 \\ 0 & b_2 \end{bmatrix}\begin{bmatrix} x_3 \\ x_4 \end{bmatrix} - \begin{bmatrix} g_1(x) \\ g_2(x) \end{bmatrix}\right\} \quad (29)$$

Simplification by (30) and (31).

$$f_1 = \tau_1 - c_{11}x_2 - c_{12}x_4 - g_1 - b_1 x_2 \quad (30)$$
$$f_2 = \tau_2 - c_{21}x_2 - c_{22}x_4 - g_2 - b_1 x_2 \quad (31)$$

More simplification by (32) and (33).

$$G_1 = m_{11}^{-1} f_1 + m_{12}^{-1} f_2 \quad (32)$$
$$G_2 = m_{21}^{-1} f_1 + m_{22}^{-1} f_2 \quad (33)$$

Apply fractional order approximation (34).

$$\begin{bmatrix} s^\mu x_1 \\ s^\mu x_2 \\ s^\mu x_3 \\ s^\mu x_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ G_1 \\ x_4 \\ G_2 \end{bmatrix} \quad (34)$$

In discrete equation we have (35).

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \left(\frac{T}{A^2}\right)\dot{G}_1 + \frac{T}{A}G_1 + T\left(\frac{A^2-1}{A^2}\right)x_{2t} + x_{1t}\left(1-\frac{T}{A}\right) \\ \left(\frac{T}{A}\right)\dot{G}_1 + TG_1 + x_{2t}(1-\frac{T}{A}) \\ \left(\frac{T}{A^2}\right)\dot{G}_2 + \frac{T}{A}G_2 + T\left(\frac{A^2-1}{A^2}\right)x_{4t} + x_{3t}\left(1-\frac{T}{A}\right) \\ \left(\frac{T}{A}\right)\dot{G}_2 + TG_2 + x_{4t}(1-\frac{T}{A}) \end{bmatrix}$$
(35)

Where $i = 1$ to $i = 2$

$$\dot{G}_i \approx \frac{G_i(t-T)-G_i(t-2T)}{T} \qquad (36)$$

The results are shown in Figure 5 where the fractional-order model in applied for μ= 0.99 and μ=0.95.

The model for 3 DOF Arm-Robot is (37).

$$\begin{bmatrix} \ddot{q_1} \\ \ddot{q_2} \\ \ddot{q_3} \end{bmatrix} = \begin{bmatrix} i_{11} & i_{12} & i_{13} \\ i_{21} & i_{22} & i_{23} \\ i_{31} & i_{32} & i_{33} \end{bmatrix} \left\{ \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} - \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \begin{bmatrix} \dot{q_1} \\ \dot{q_2} \\ \dot{q_3} \end{bmatrix} - \begin{bmatrix} b_1\dot{q_1} \\ b_2\dot{q_2} \\ b_3\dot{q_3} \end{bmatrix} - \begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix} \right\} \qquad (37)$$

Where the matrix is the inverse of matrix m (38).

$$\begin{bmatrix} i_{11} & i_{12} & i_{13} \\ i_{21} & i_{22} & i_{23} \\ i_{31} & i_{32} & i_{33} \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}^{-1} \qquad (38)$$

In (39) to (59) we have tie explicit equation.

$$m_{11} = m_1 {lc_1}^2 + I_1 + m_2 {l_1}^2 + 2m_3 l_1 lc_2 \cos(q_2) + m_2 {lc_2}^2 cos^2(q_2) + I_2 + m_3 {l_1}^2 + 2m_3 l_1 l_2 \cos(q_2) + 2m_3 l_1 lc_3 \cos(q_2+q_3) + m_3 {l_2}^2 cos^2(q_2) + 2m_3 l_2 lc_3 \cos(q_2)\cos(q_2+q_3) + m_3 {lc_3}^2 cos^2(q_2+q_3) + I_3 \qquad (39)$$

$$m_{12} = I_2 + I_3 \qquad (40)$$

$$m_{13} = I_3 \qquad (41)$$

$$m_{21} = I_2 + I_3 \qquad (42)$$

$$m_{22} = m_2 {lc_2}^2 + m_3 {l_2}^2 + m_3 {lc_1}^2 + I_2 + I_3 + 2m_3 l_2 lc_3 \cos(q_3) \qquad (43)$$

$$m_{23} = I_3 + m_3 l_2 lc_3 \cos(q_3) + m_3 {lc_3}^2 \qquad (44)$$

$$m_{31} = I_3 \qquad (45)$$

$$m_{32} = I_3 + m_3 l_2 lc_3 \cos(q_3) + m_3 {lc_3}^2 \qquad (46)$$

$$m_{33} = I_3 + m_3 {lc_3}^2 \qquad (47)$$

$$c_{11} = -2m_2 l_1 lc_2 \dot{q_2}\,\mathrm{sen}(q_2) - 2m_2 {lc_2}^2 \dot{q_2} \cos(q_2)\,\mathrm{sen}(q_2) - 2m_3 l_1 l_2 \dot{q_2}\,\mathrm{sen}(q_2) - 2m_3 l_1 lc_3 \dot{q_2}\,\mathrm{sen}(q_2+q_3) \qquad (48)$$

$$c_{12} = -2m_3 {l_2}^2 \dot{q_1} \cos(q_2)\,\mathrm{sen}(q_2) - 2m_3 l_2 lc_3 \dot{q_1} \cos(q_2)\,\mathrm{sen}(q_2+q_3) - 2m_3 {lc_3}^2 \dot{q_1} \cos(q_2+q_3)\, sen(q_2+q_3) \qquad (49)$$

Table 3. Parameters for 3 FOD Arm

parameter	Link 1	Link 2	Link 3	Units
Mass	m1	m2	m3	Kgs
	19.5	1.3	1.1	
Length	l1	l2	l3	m
	1.2	1.1	1.1	
Length-mass center	lc1	lc2	lc3	m
	0.5	0.5	0.5	
Inertia rotational	I1	I2	I3	m
	4.15	0.37	0.271	
Viscous friction	b1	b2	b3	N·m·s^{-1}
	1.8	1.8	1.8	
Torque	$\tau 1$	$\tau 2$	$\tau 3$	N
Position angular	q_1	q_2	q_3	Degrees
Velocity angular	$\dot{q_1}$	$\dot{q_3}$	$\dot{q_3}$	Degrees·s^{-1}
Acceleration angular	$\ddot{q_1}$	$\ddot{q_2}$	$\ddot{q_3}$	Degrees·s^{-2}

$$c_{13} = -2m_3 l_1 lc_3 \dot{q_1}\,\mathrm{sen}(q_2+q_3) - 2\, m_3 l_2 lc_3 \dot{q_1} \mathrm{sen}(q_2+q_3)\cos(q_2) - 2m_3 {lc_3}^2 \dot{q_1} \cos(q_2+q_3)\, sen(q_2+q_3) \qquad (50)$$

$$c_{21} = m_3 l_1 l_2 \dot{q_1}\,\mathrm{sen}(q_2) + m_3 l_1 lc_3 \dot{q_1}\,\mathrm{sen}(q_2+q_3) + m_3 {l_2}^2 \dot{q_1} \cos(q_2)\,\mathrm{sen}(q_2) + 2m_3 l_2 lc_3 \dot{q_1} \cos(q_2)\,\mathrm{sen}(q_2+q_3) + 2m_3 l_2 lc_3 \dot{q_1}\,\mathrm{sen}(q_2)\cos(q_2+q_3) + 2m_3 {lc_3}^2 \dot{q_1} \cos(q_2+q_3)\,\mathrm{sen}(q_2+q_3) \qquad (51)$$

$$c_{22} = -2m_3 l_2 lc_3 \dot{q}_3 \operatorname{sen}(q_3) \tag{52}$$
$$c_{23} = -m_3 l_2 lc_3 \dot{q}_3 \operatorname{sen}(q_3) \tag{53}$$

$$c_{31} = m_3 l_1 lc_3 \dot{q}_1 \operatorname{sen}(q_2 + q_3) + m_3 l_2 lc_3 \dot{q}_1 \cos(\mathrm{q}_2)\operatorname{sen}(q_2 + q_3) + m_3 {lc_3}^2 \dot{q}_1 \cos(q_2 + q_3) \operatorname{sen}(q_2 + q_3) \tag{54}$$
$$c_{32} = -m_3 l_2 lc_3 \dot{q}_3 \operatorname{sen}(q_3) \tag{55}$$
$$c_{33} = m_3 l_2 lc_3 \dot{q}_2 \operatorname{sen}(q_3) \tag{56}$$
$$g_1 = 0 \tag{57}$$
$$g_2 = m_2 g lc_2 \cos(q_2) + m_3 g l_2 \cos(q_2) + m_3 g lc_3 \cos(q_2 + q_3) \tag{58}$$
$$g_3 = m_3 g lc_3 \cos(q_2 + q_3) \tag{59}$$

Apply the same method we have the fractional order model for 3 DOF in (60). The results are shown in Figure 6 where the fractional-order model in applied for μ= 0.99 and μ=0.95.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} \left(\frac{T}{A^2}\right)\dot{G}_1 + \frac{T}{A}G_1 + T\left(\frac{A^2-1}{A^2}\right)x_{4t} + x_{1t}\left(1-\frac{T}{A}\right) \\ \left(\frac{T}{A^2}\right)\dot{G}_2 + \frac{T}{A}G_2 + T\left(\frac{A^2-1}{A^2}\right)x_{5t} + x_{2t}\left(1-\frac{T}{A}\right) \\ \left(\frac{T}{A^2}\right)\dot{G}_3 + \frac{T}{A}G_3 + T\left(\frac{A^2-1}{A^2}\right)x_{6t} + x_{3t}\left(1-\frac{T}{A}\right) \\ \left(\frac{T}{A}\right)\dot{G}_1 + TG_1 + x_{4t}\left(1-\frac{T}{A}\right) \\ \left(\frac{T}{A}\right)\dot{G}_2 + TG_2 + x_{5t}\left(1-\frac{T}{A}\right) \\ \left(\frac{T}{A}\right)\dot{G}_3 + TG_3 + x_{6t}\left(1-\frac{T}{A}\right) \end{bmatrix} \tag{60}$$

3. RESULTS

The fractional order control for 1, 2 and 3 FOD is (61), (62) and (63).

$$\tau = k_p(qd - q1) - k_v s^{\mu} q_1 + g \tag{61}$$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} kp_1 & 0 \\ 0 & kp_2 \end{bmatrix}\begin{bmatrix} qd_1 - q_1 \\ qd_2 - q_2 \end{bmatrix} - \begin{bmatrix} kv_1 & 0 \\ 0 & kv_2 \end{bmatrix}\begin{bmatrix} s^{\mu}q_1 \\ s^{\mu}q_2 \end{bmatrix} + \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} \tag{62}$$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} kp_1 & 0 & 0 \\ 0 & kp_1 & 0 \\ 0 & 0 & kp_1 \end{bmatrix}\begin{bmatrix} qd_1 - q_1 \\ qd_2 - q_2 \\ qd_3 - q_3 \end{bmatrix} - \begin{bmatrix} kv_1 & 0 & 0 \\ 0 & kv_2 & 0 \\ 0 & 0 & kv_3 \end{bmatrix}\begin{bmatrix} s^{\mu}q_1 \\ s^{\mu}q_2 \\ s^{\mu}q_3 \end{bmatrix} + \begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix} \tag{63}$$

The math procedure for writing the fractional-order control is shown by Ceron-Morales (2018b). Finally this work is used to write the letters “H” “O” “L” “A” using the 2 DOF model and analog outputs from STM32 development board Fig.7.

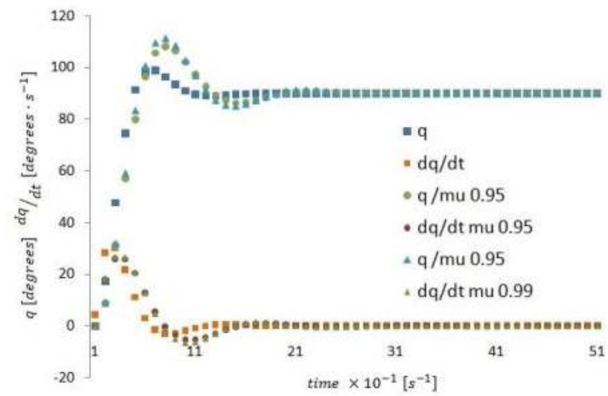

Fig. 4. Graphs of the Fractional-Order model of the 1 DOF.

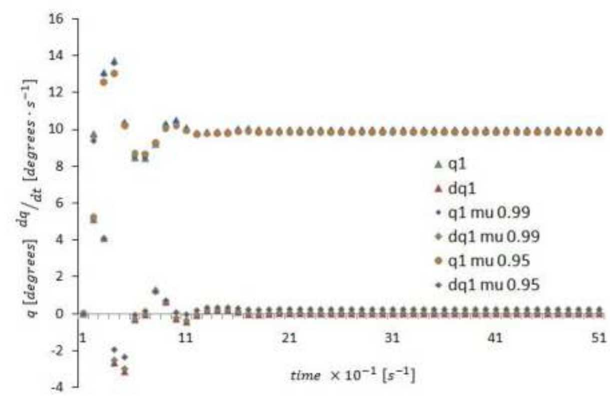

Fig. 5. Graphs of the Fractional-Order model of the 2 DOF.

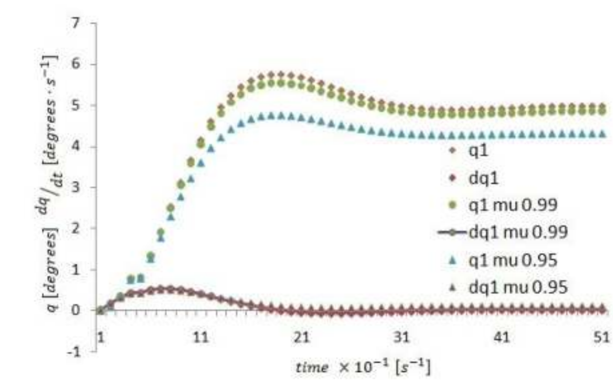

Fig. 5. Graphs of the Fractional-Order model of the 3 DOF.

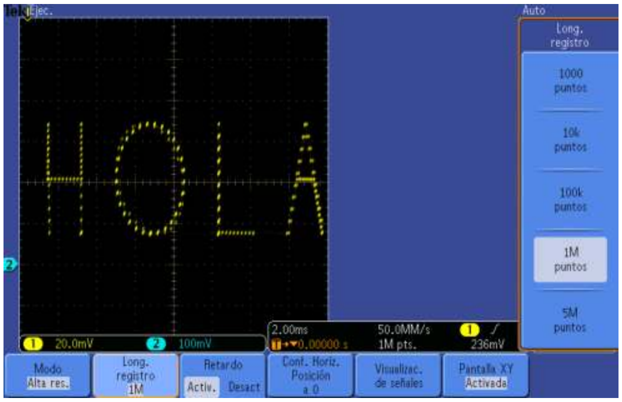

Fig. 7. Plots of the fractional model of the 2DOF robot writing “H” “O” “L” ”A”.

4. CONCLUSIONS

This work aims to apply the Fractional-Order Calculus to real-life situations as shown by HongGuang Sun (2018). This work was applied to robotic arms of up to 3 deggres of freedom, Ines Tejado (2013) applied it to a human arm and Rosario J.M. (2006) to the axes of a robotic arm. It can also be applied to situations such as inverted pendulum (Raafat Shalaby 2019) or estímate the charge of a capacitor (Lei Zhang 2016).

The proposed model presents an error in steady-state due to the Fractional-Order model that is we wanted to demostrate, in addition the importance of a fractional order control allows attenuating disturbances of the dynamic of robot (xinxin Shi 2018).

Finally, the experimental validation was carried out by programming the models and control on an STM32L476 development board.

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