Master - slave configuration of fractional - order Lorenz chaotic oscillators embedded in FPGA

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ResumenIn this paper, the FPGA implementation for a chaotic communication system using chaos is presented. Fractional-order nonlinear systems are used as chaos generators. Specially, fractional-order Lorenz chaotic oscillators are arranged as network in master-slave configuration, where the synchronization is achieved based on the coupling matrix from complex systems theory.

Keywords: Fractional-order chaotic oscillators, complex networks, master-slave synchronization, chaotic encryption.

1. INTRODUCTION

In recent years, chaotic synchronization has received an increasing attention. Since L.M. Pecora and T.L. Carroll synchronized two chaotic oscillators [M. and L., 1990], it has been established a considerably body ok knowledge, mainly on synchronization. Several methods have been proposed to achieve chaotic synchronization between chaotic oscillators, e.g., sliding mode [Rajagopal et al., 2017], adaptive control [Xiong et al., 2016], Pecora & Carroll and Wang & Chen methods [Li et al., 2003].

The last decade chaotic signals have been successfully used as a mean of encryption due to their potential application in secure communication to veil information. [Kassim et al., 2017]. Researchers have showed interest to chaotic oscillators, these have been studied and implemented in the field of communications [Tlelo-Cuautle et al., 2016, Méndez-Ramírez et al., 2018], this because have properties of unpredictability as quasi-randomness and high sensibility on initial conditions [Soriano-Sánchez et al., 2016]. Many systems can exhibit the chaos phenomenon, as fractional-order chaotic oscillators, chaotic maps of one or two dimensions, three-dimensional chaotic oscillators, for instance, and new kinds of chaos generators can be find in the literature, e.g., systems without equilibrium points[Rajagopal et al., 2017], multiple scrolls attractor [Tlelo-Cuautle et al., 2016], piecewise-linear functions [Echenausía-Monroy et al., 2018], and so on.

Chaos is presented for different parameters of the system with different order in the derivatives. About the use of chaotic oscillators of fractional order in the transmission of information relapses in that besides expanding the space of keys [Angulo-Guzman et al., 2016], at the same time the level of security of the encrypted information is increased because besides ignoring the initial conditions of the oscillator, the order of the derived one is also unknown.

Due to the previously reported works in Soriano-Sánchez et al. [2015, 2016] about the energy magnitude that chaotic oscillator signal presents, it is suggested to attenuate the information that is desired to mask, with the aim of keeping the chaos phenomenon. Attenuate the information can generate an unfavorable effect when decrypting information, largely due to which can be presented loss of information or in case to perform a physical implementation, because it is unknown the final state of electronic components. Based on the above considerations, we propose an alternative to secure communication systems. Regarding the contribution of the study carried out, details are provided considering the implementation of encrypting audio without losing the phenomenon of chaos.

The main goal of this document is the FPGA implementation for a chaotic communication system using chaos. We obtain the chaotic synchronization of a network based on the coupling matrix from complex systems theory. The nodes are modeled by fractional-order chaotic oscillators.

This paper is organized as follow: Section 2 gives some necessary definitions and notations of fractional calculus; the model of the fractional-order Lorenz chaotic oscillator is presented, which will be used as fundamental oscillator of the master-slave configuration; in this section a brief review on synchronization of complex dynamical networks is also included. Synchronization of a pair of nodes in master-slave configuration of two chaotic oscillators of fractional-order Lorenz via coupling matrix is exposed in section 3; the corresponding simulation results are provided also in this section. Section 4 corresponds to the description of the NI myRIO-1900 system for the FPGA implementation of the secure communication system as well as provides considerations for transmitting encrypted audio. Physical implementation of the fractional-order chaotic synchronization in the master - slave configuration using the additive encryption method, in this case the chaotic system is the fractional-order Lorenz's oscillator corresponds to Section 5. Finally, some conclusions are given in Section 6.

2. FRACTIONAL-ORDER CHAOTIC OSCILLATORS

2.1 Fractional Calculus

Fractional calculus is a generalization of integration and differentiation to non - integer order fundamental operator ${}_aD_t^{\alpha}$, where a and t are the limits of the operation and $\alpha \in \Re$ and is defined as [Petráš, 2011]:

$$ {}_aD_t^{\alpha} = \begin{cases} \frac{d^{\alpha}}{dt^{\alpha}}, & \alpha > 0, \\ 1, & \alpha = 0, \\ \int_a^t (d\tau)^{-\alpha}, & \alpha < 0. \end{cases} \qquad (1)$$

Three of the most commonly used definitions for the numerical solution of differential - integral operator are generally the definition Grünwald - Letnikov (GL), the Riemann - Liouville (RL) definition and the definition of Caputo [Rajagopal et al., 2017, Jiang et al., 2016].

For the numerical calculation of the derivative of fractional-order the following relationship derived from the definition of Grünwald - Letnikov can be used. This approach is based on the fact that for a large class of functions, three definitions GL, RL and Caputo are equivalent [Angulo-Guzman et al., 2016, Tolba et al., 2018]. The relation for explicit numerical approximation the α derived in kh points where $(k = 1, 2, \dots)$ has the form:

$$ {}_{k-L_m/h}D_{t_k}^{\alpha} f(t) \approx h^{-\alpha} \sum_{j=0}^{k} (-1)^j \binom{\alpha}{j} f(t_k - j), \qquad (2)$$

where L_m is the "memory length", h is the time step of the calculation and $(-1)^j \binom{\alpha}{j}$ are the binomial coefficients $c_j^{(q)} (j = 0, 1, \dots)$. For their calculation the following expression can be used:

$$ c_0^{(q)} = 1, \quad c_j^{(q)} = \left(1 - \frac{1+q}{j}\right) c_{j-1}^{(q)}, \qquad (3)$$

where $q = \alpha =$ order.

The general solution of the fractional differential equation

$$ {}_aD_t^{q} y(t) = f(y(t), t), \qquad (4)$$

can be expressed as:

$$ y(t_k) = f(y(t_k), t_k)h^q - \sum_{j=v}^{k} c_j^{(q)} y(t_k - j). \qquad (5)$$

2.2 Fractional-Order Lorenz chaotic oscillator

Fractional-order Lorenz's oscillator is described as [Angulo-Guzman et al., 2016]:

$$ \begin{aligned} {}_0D_t^{q_1} x_1(t) &= \sigma(x_2(t) - x_1(t)), \\ {}_0D_t^{q_2} x_2(t) &= x_1(t)(\rho - x_3(t)) - x_2(t), \\ {}_0D_t^{q_3} x_3(t) &= x_1(t)x_2(t) - \beta x_3(t), \end{aligned} \qquad (6)$$

where σ is called the Prandtl number and ρ is called the Rayleigh number. All $\sigma, \rho, \beta > 0$, but usually $\sigma = 10$, $\beta = 8/3$ and ρ is varied. The system exhibits chaotic behavior for $\rho = 28$ and displays orbits for other values. The minimal order of $q > 0.9941$ is obtained according to [Tolba et al., 2018, Rajagopal et al., 2017]. From equation (2), by means of the relation $N = T_{sim}/h$, where $n = k$ is the number of binomial coefficients, T_{sim} is the simulation time and h is the integration step. $h = 0.005$ is set to decrease the prediction error.

Using equations (6), $q = 0.995$ and the previously mentioned parameters, the dynamics on Figure 1 are obtained.

2.3 Complex networks

Consider a dynamic network interconnected with N oscillators, each oscillator is a fundamental unit, with its dynamic depending on the nature of the network [Anzo-Hernández et al., 2018a, Arellano-Delgado et al., 2018]. Each oscillator is defined as follows:

$$ {}_aD_t^{q} x_i = f(x_i) + u_i, \; x_i(0) \neq 0, \; i = 1, 2, \dots, N, \qquad (7)$$

where N is the size of the network,
$x_i = [x_{i1} \; x_{i2} \; x_{i3}, \dots, x_{in-1}, \; x_{in}] \in \Re^{\mathrm{n}}$ represents the state variables of the oscillator i, $x_i(0) \in \Re^{\mathrm{n}}$ are the initial conditions of oscillator i, $u_i \in \Re^{\mathrm{n}}$ is the control law and establishes synchronization between at least two oscillators and is defined as follows [López-Mancilla et al., 2019]

$$ u_i = c \sum_{j=1}^{N} a_{ij} \Gamma x_j, \quad i = 1, 2, \dots, N. \qquad (8)$$

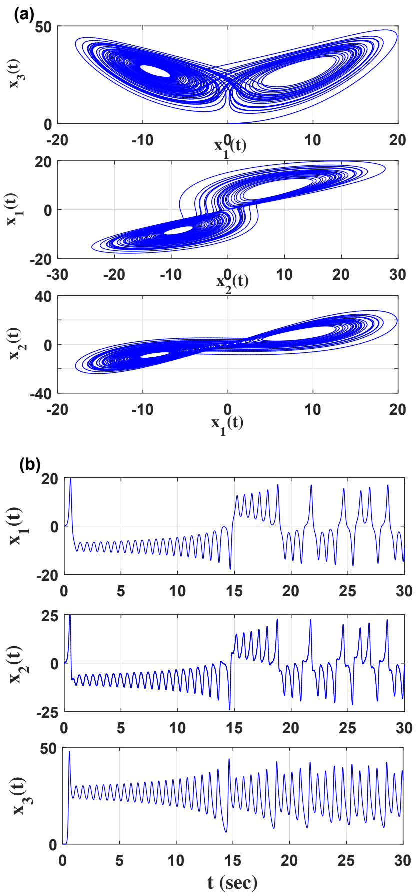


Figura 1. **(a)** Simulation result of the Lorenz system in $x_1 - x_3$ (up), $x_2 - x_1$ (middle) and $x_1 - x_2$ (down) planes for parameters: $\sigma = 10$, $\beta = 8/3$ and $\rho = 28$, orders $q_1 = q_2 = q_3 = 0.995$, and initial conditions $x(0) = [0.1, -0.1, 0.1]^T$, **(b)** $x_1(t)$, $x_2(t)$ and $x_3(t)$ states variables obtained with $x(0) = [0.1, -0.1, 0.1]^T$.

The term $c > 0$ represents the coupling strength, $\Gamma \in \Re^{n\times n}$ is a constant matrix to determine the variable state coupled of each oscillator. Defining $\Gamma = diag[r_1, \ r_2, \ldots, r_n]$ as a diagonal matrix, and if two oscillators are linked through their kth state variables, the diagonal element with $r_k = 1$ for a particular k and $r_j = 0$ for $j \neq k$.

The matrix $A \in \Re^{N\times N}$ is the coupling matrix which shows connections between oscillators, if the oscillator i is connected to the oscillator j, then $a_{ij} = 1$, otherwise $a_{ij} = 0$ for $i \neq j$. The diagonal elements of the matrix A are defined as:

$$a_{ii} = -\sum_{j=1\ j\neq i}^{N} a_{ij} = -\sum_{j=1\ j\neq i}^{N} a_{ji}, \quad i = 1, 2, \ldots, N. \quad (9)$$

The dynamical complex network (7) and (8) is said to achieve identical synchronization if

$$x_1(t) = x_2(t) = \cdots = x_n(t), \quad \text{as} \quad t \to \infty. \quad (10)$$

3. MASTER - SLAVE SYNCHRONIZATION

Respect to the dynamics of an isolated node and the inner linking structure, given a specific coupling configuration the synchronization property of the network is said to be strong if the network can synchronize with a small coupling strength, c [Li et al., 2003].

Based on the stability theory of Lyapunov, Theorem 3.1 and Lemma 3.1 are presented below proposed by Wang & Chen, with a modification due to fractional order systems, where $\alpha = 1$ in the case of integer order systems Soriano-Sánchez et al. [2016, 2018].

Theorem 1. Consider the dynamical network (7), be the eigenvalues of its coupling matrix A on $0 = \lambda_1 > \lambda_2, \ldots \geq, \lambda_N$. Suppose that there exist an $n\times n$ diagonal matrix $D > 0$ and two constants $\bar{d} < 0$ and $\tau > 0$ such that:

$$[{}_aD_t^\alpha f(s(t)) + d\Gamma]D + [{}_aD_t^\alpha f(s(t)) + d\Gamma]^T D \leq -\tau I_n, \quad (11)$$

for all $d \leq \bar{d}$, where $I_n \in \Re^{n\times n}$ is an unitary matrix. If moreover,

$$c\lambda_2 \leq \bar{d}, \quad (12)$$

then the synchronization state (10) is exponentially stable.

Lemma 2. Consider the dynamical network given by (7). Let λ_2 be the largest nonzero eigenvalue of the coupling matrix A of the network. The synchronization state of network (7) defined by $x_1 = x_2 = \cdots = x_n$ is asymptotically stable, if the coupling strength meets the restriction

$$c \geq \left| -\frac{T}{\lambda_2} \right|, \quad (13)$$

where $c > 0$ is the coupling strength of the network and $T > 0$ is a positive constant such that zero is an exponentially stable point of the n-dimensional system

$$\begin{aligned} {}_aD_t^\alpha z_1 &= f_1(z), \\ {}_aD_t{}^\alpha z_2 &= f_2(z) - Tz_2, \\ &\vdots \\ {}_aD_t{}^\alpha z_n &= f_n(z). \end{aligned} \quad (14)$$

Note that system (14) is actually a single oscillator with self-feedback $-Tz_2$. Condition (13) means that the entire network will synchronize provided that T is a constant so that the self-feedback term Tz_2 could stabilize an isolated oscillator and means that the entire network will synchronize provided that λ_2 is negative enough.

Performing tests to determine not only the value of T but also the state to ensure the asynthic stability of the oscillator, it was found that both the x_1 and x_2 states meet this objective. This also made the identical synchronization. Although in the implementation only the synchrony was obtained in the state x_2. For this state the value of minimum T was 1, but a more favorable result was obtained by setting $T = 5$, even with $T = 10$.

$$A = \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix}. \quad (15)$$

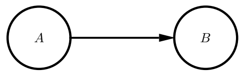


Figura 2. Master-slave configuration, the information flows in one direction.

3.1 Chaotic synchronization in master - slave configuration

Master-slave chaotic synchronization is normally performed in pairs of oscillator using control schemes [Pena-Ramírez et al., 2018], besides when coupled oscillators are seen as a network, synchronization is achieved through different techniques [Xiong et al., 2016, López-Mancilla et al., 2019, Rajagopal et al., 2017] where the oscillators and the topology are take into account, both play a key role in this process [Anzo-Hernández et al., 2018b, Arellano-Delgado et al., 2018].

Consider a complex network of two fractional-order Lorenz chaotic oscillators in master-slave configuration, Figure 2. The calculation of the coupling matrix A for the network of the Figure 2 is presented on (15), where the first row corresponds to the master and the last row to the slave.

Due to the oscillators are coupled through the second state, the diagonal matrix $\Gamma = diag[0, 1, 0]$. Then, the control law is applied to the state $x_2(t)$ of the network. Therefore, the fractional-order Lorenz chaotic oscillators presented in the complex network of Figure 2 by means of equations (7), (8) and (9) is described as

$$\begin{aligned} {}_0D_t^{q_1} x_{1M}(t) &= \sigma(x_{2M}(t) - x_{1M}(t)), \\ {}_0D_t^{q_2} x_{2M}(t) &= x_{1M}(t)(\rho - x_{3M}(t)) - x_{2M}(t), \\ {}_0D_t^{q_3} x_{3M}(t) &= x_{1M}(t)x_{2M}(t) - \beta x_{3M}(t), \end{aligned} \quad (16)$$

$$\begin{aligned} {}_0D_t^{q_1} x_{1S}(t) &= \sigma(x_{2S}(t) - x_{1S}(t)), \\ {}_0D_t^{q_2} x_{2S}(t) &= x_{1S}(t)(\rho - x_{3S}(t)) - x_{2S}(t) + u(t), \\ {}_0D_t^{q_3} x_{3S}(t) &= x_{1S}(t)x_{2S}(t) - \beta x_{3S}(t). \end{aligned} \quad (17)$$

The control law to obtain synchronization in the slave master configuration, according to the coupling matrix is:

$$u(t) = c(x_{2M}(t) - x_{2S}(t)), \quad (18)$$

where the subscript M corresponds to the master and S to the slave.

The synchronization between the states of the network according to the equation (10), is illustrated in Figure 3a. Synchrony is obtained with a $c = 5$ by using equation(13) with initial conditions $x_{iM}(0) = [2.3071, -2.3071, 2.3071]^T$ and
$x_{iS}(0) = [0.5268, -0.5268, 0.5268]^T$. In order to show synchrony in Figure 3b, the phase plane is displayed, where the synchrony between x_{1M} and x_{1S} states from the oscillators present in the master-slave configuration of Figure 2 can be observed.

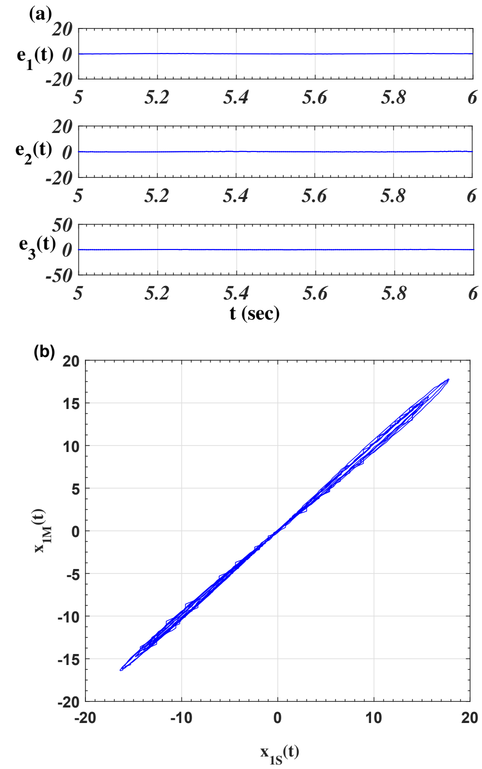


Figura 3. **(a)** Evolution of the corresponding synchronization error variables $e_1(t)$, $e_2(t)$ and $e_3(t)$ of the master-slave configuration for a coupling strength $c = 5$. **(b)** Phase portraits to corroborate synchronization between x_{iM} versus x_{iS}, where the subscript M corresponds to the master and S to the slave.

3.2 Additive encryption method

The encryption process is carried out by means of additive encryption. In this case, the fractional - order Lorenz chaotic oscillators are arranged in master - slave configuration.

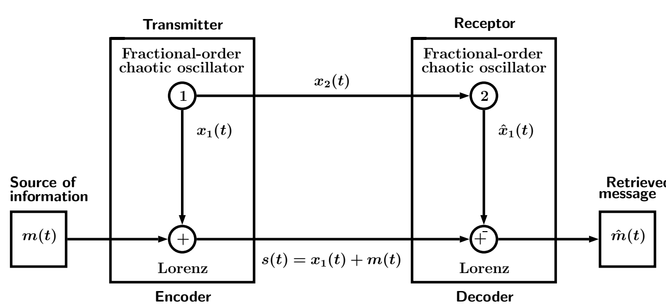


Figura 4. Additive encryption diagram of two-channel communication.

Figure 4 represents the additive encryption method [Soriano-Sánchez et al., 2015, 2016]. It consists on transmitting a encrypted signal through a communication channel, whose signal is the result of adding the information to be encrypted with the chaotic signal. Also another signal is transmitted of the encoder and used to synchronize with the decoder to create an equivalent chaotic signal, then subtracting the equivalent signal from the encrypted signal, the information can be recovered.

4. PHYSICAL IMPLEMENTATION: CASE STUDY SUGGESTED TO AUDIO ENCRYPTION AND HARDWARE USED

It is necessary to mention that nowadays, the encryption process is carried out through trial and error, that means, this process lacks a selection criterion to choose a suitable cloaking chaotic signal to accomplish message qualifications Soriano-Sánchez et al. [2015, 2016].

There exists a criterion to select the chaotic signal by the signal result with the highest energy magnitude, to better mask the information. It is described as follow [Soriano-Sánchez et al., 2016]:

$$\sum_{n=0}^{N-1} |x_c(n)|^2 \gg \sum_{n=0}^{N-1} |x_m(n)|^2, \tag{19}$$

where $x_c(n)$ is the samples set of the chaotic signal and $x_m(n)$ is the samples set of the message. The criterion, J_1 is defined as the ratio between the right and the left parts of (19), J_1 yields the times the chaotic signal energy exceeds the message energy, therefore $J_1 \gg 1$, results in a good encryption. The energy levels magnitude of fractional-order Lorenz chaotic oscillator states $x_1(t)$, $x_2(t)$ y x_3 are 2.294×10^6, 2.9057×10^6, 22.786×10^6.

In the case of performing a physical implementation, surely the magnitude of Lorenz chaotic oscillator states would exhibit saturation, because the magnitude is very high. It is advisable to multiply by a scaling factor when transmitting the chaotic signal and after receiving the signal multiply it by the reverse of the scaling factor, since if it presents a saturation, it would not be recover information and could get lost of chaos.

4.1 Important features of the NI myRIO-1900 device

It is convenient to know the characteristics of the hardware to be used, which is the NI myRIO-1900 system. Some of its characteristics are:

- Processor: Xilinx Zynq-7010 (MCU-FPGA), to 667 MHz, 256 MB of non-volatile storage, 512 MB DRAM for determining control and analysis.
- 4 analog inputs 12-bit, 2 outputs 12-bit analog.
- Support for LabView and Xilinx ISE software.
- Xilinx FPGA (about logic cells) for reconfigurable online processing and custom control.

5. DETAILS AND RESULTS OF THE ELECTRONIC IMPLEMENTATION

It was decided to use the data type fixed point because it takes up less memory space. With the purpose to evade rounding mistakes to handle the registers, it was used word length of 32 bits where 11 bits are determined for the decimal part and to have uniformity we were used all functions are signed (I32).

Four different methods of the GL method with the aim of optimizing the solution of fractional order systems, were proposed however [Su et al., 2016], they involve including more variables in the GL method in addition to the fact that we are limited in memory by the high-level language used for implementation. To save FPGA resources, we use only eight binomial coefficients, this means, a memory length of $L_m = 8$.

For transmitting the $x_2(t)$ state in order to achieve synchrony with the system that decodes the message, the $x_{2M}(t)$ state is multiplied by a scaling factor, which is of magnitude 50 and its output is transmitted.

When encrypting audio, the sampling frequency $F_s =$ 44.1 kHz, the most used frequency in the audio system, as well as the lowest frequency capable of reproducing the entire frequency spectrum to which the human ear is sensitive: from about 20 Hz to 20 kHz. The sampling time $T_s = 22$ microseconds (μsec) is obtained from the ratio $T_s = 1/F_s$ and is used in the encoder and decoder.

So far, two things must be accomplished. The Lorenz attractor should be formed, the result of plotting the $x_2(t)$ versus $x_1(t)$ state multiplied by a scaling factor, supported by an oscilloscope. The Figure 5(a), shows the Lorenz chaotic attractor and the dynamics of $x_{1M}(t)$ at the top and $x_{2M}(t)$ at the bottom are represented in Figure 5(b).

The control law for the synchronization between the chaotic system of the transmitter system and the receptor, is of

$$u(t) = c(x_{2M}(t) - x_{2S}(t)), \tag{20}$$

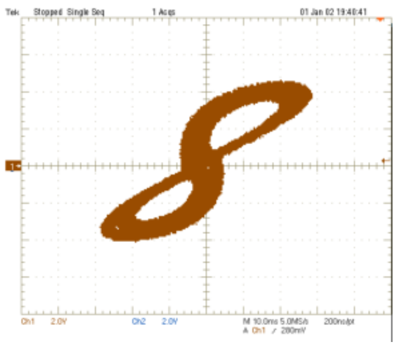
where the subscript M corresponds to the master and S to the slave. First multiply x_{2M} by the reverse of the scaling factor, which is 0.02. The coupling strength to obtain the synchrony is of $c = 5$ minimum and $c = 25$ as maximum value, with it is preserved the characteristic dynamics of the fractional Lorenz system.

Once the previous step is performed, the synchronization among the chaotic system that encodes the message with the system decodes is obtained. The $x_1(t)$ state is selected.

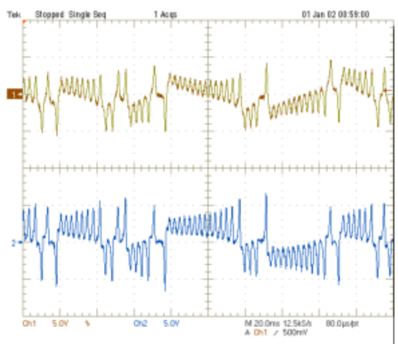
In order to achieve the synchrony between the chaotic systems present on the both NI myRIO-1900 systems, the $x_{1S}(t)$ state multiplied by the same scaling factor, which is 50 and its output is transmitted.

Now we have a chaotic communication system, using the additive encryption method, using two NI myRIO-1900 systems, one for the transmitter part and one for the receiver part, for security purposes.

It can be concluded that using the $x_1(t)$ and $x_2(t)$ state present in both systems, the fractional-order Lorenz chaotic attractor, supported by an oscilloscope, can be



(a)



(b)

Figura 5. Implementation results obtained with the support of a digital oscilloscope. **(a)** View (x_2, x_1)-plane for the fractional-order Lorenz chaotic attractor. **(b)** Time evolution of state variables .

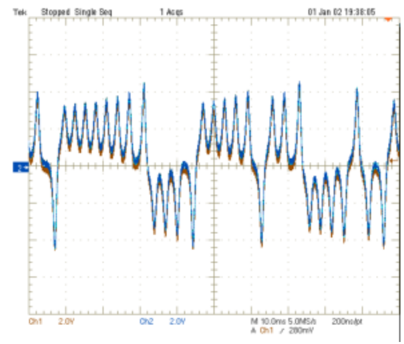
visualized by graphing one state against another, as well as observing the present synchrony between the states $x_M(t)$ and $x_S(t)$. The synchrony between $x_1(t)$ state is seen in Figure 6(b) and the dynamics in Figure 6(a).

6. CONCLUSIONS

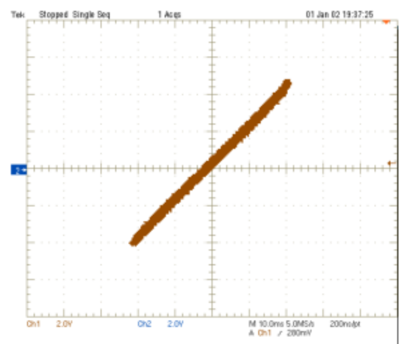
FPGA implementation for a system of secure communication using chaos was presented using the additive encryption method. A complex network in master-slave configuration was considered, where fractional-order chaotic oscillators were used.

Synchronization of a network in master-slave configuration via coupling matrix was addressed. It is important to mention, that this synchronization method was proposed to synchronize integer-order systems. Only in one state of the chaotic oscillators was used the coupling signal.

The fact of only use eight binomial coefficients has two interpretations. First one, the number of operations on convolution is reduced, we use a buffer, it is important to mention that not with eight samples is possible to get the dynamics, all dynamics is acquired and processed, but the buffer is updated every eight samples. On the other hand,



(a)



(b)

Figura 6. Implementation results obtained with the support of a digital oscilloscope. **(a)** Fractional-order Lorenz oscillator dynamics to corroborate synchronization between x_{1M} with x_{1S} . **(b)** Phase portrait to corroborate synchronization between x_{1M} versus x_{1S}, where the subscript M corresponds to the master and S to the slave.

due to the FPGA resources of NI myRIO-1900 system, we use limited to use only eight binomial coefficients, mainly due to the software implemented is an high-level language.

Although it is highlighted that the results obtained of FPGA were consistent with the numerical simulations, it is desirable to extend this study using complex networks formed by more of two nodes inside them. In a forthcoming work we will be concerned on a physical implementation to encrypt audio.

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