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Formation Control with Collision Avoidance for First-Order Multi-Agent Systems using Bounded Vector Fields ⋆

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Abstract: In this paper, we propose a solution to the formation control problem with collision avoidance for an arbitrary number of first-order agents by using repulsive vector fields with bounded magnitude. When the agents are far enough from each other, e. i., there is no risk of collision, they move at some known velocity determined by the application of a saturation function. It is assumed that every agent is able to detect other robots or obstacles into a circular region and apply repulsive vector fields. Once an agent detects any other agent or obstacle into its sensing radius, it regards that there exists an unstable focus in the position of the surrounding object. These fields are such that the agents neither become nearer than a predefined collision radius nor exceed a maximum control input magnitude. The application of a reactive component of the control law is done by means of a continuous function which makes the whole control law to be smooth. Simulations were carried out to verify the performance of the proposed scheme.

Keywords: Multi-agent systems, formation control, collision avoidance, repulsive vector fields, saturated control.

1. INTRODUCTION

Motion coordination of multi-agent systems has been an intense research area in the last years because of its wide field of applications. The main advantage of this kind of systems is the ability to achieve goals that a single robot may not be able to reach (Ramirez-Paredes et al. (2015), Bekirov and Asanov (2017), Mouradian et al. (2017), Chikwanha et al. (2012)). One problem that has attracted too much attention in this area is the formation control problem, where a set of agents has the objective of forming a specific geometrical pattern using only information about nearing partners ((Deghat et al. (2016), Yang et al. (2018)), Yu et al. (2018), Do (2014)). An additional issue in formation control is the design of collision free transient behaviour (Hernandez-Martinez and Bricaire (2012), Do (2006), Rodríguez-Seda et al. (2016)). There exist several solutions to this problem which include the prediction of collisions from initial conditions, is case these could be selected arbitrarily. Another approach is the use of repulsive potential functions as a common tool to avoid collisions, even when this type of functions could lead to undesired equilibrium points or control signals of high magnitude if a pair of agents get close enough. Navigation functions have been proposed as an alternative to the potential functions to guarantee trajectory tracking without collisions. In Flores-Resendiz and Aranda-Bricaire (2019), repulsive vector fields were applied to ensure a collision free solution to the formation control problem. Such fields work in a discontinuous manner which could provoke the appearance of chattering phenomenon and no considerations were done about the control effort needed to, effectively, use the proposed control strategy. In this paper we propose an strategy to solve the formation control problem without collisions composed of two components, each of them to attend the convergence and the collision avoidance issue, respectively. Attractive vector fields are applied to ensure the convergence of the agents to the desired geometrical pattern regarding the most general case of communication topology, that is, the existence of a spanning tree in the communication graph. In order to avoid control actions with high magnitude, a saturation function is applied to this first component of the control law. Besides, we propose the use of a continuous switching function to determine whether the repulsive action is needed or not. We find conditions in the selection of the design parameters to guarantee that there exists a minimum safety distance among agents, and at the same time, the control law satisfy input constraints. The rest of this paper is organized as follows. In Section II, some antecedents and definitions are reviewed. In Section III we state formally the problem to be solved, while in Section IV, the main result is presented. Simulation results of interesting cases were developed and are presented in Section V. Finally, some conclusions and guidelines for future work are discussed in Section VI.

2. PRELIMINARIES

The communication among a set of agents can be modelled by a graph $G = \{V, E, C\}$ which consists of a set of vertices $V = \{R_1, \ldots, R_n\}$ corresponding to each of the agents; a set of edges $E = \{(R_j R_i) \in V \times V, i \neq j\}$, which indicates that the agent R_i receive information about R_j and also a set $C = \{c_{ji} \in \mathbb{R}^2 \mid (R_j R_i) \in V \times V, i \neq j\}$ of constant vectors that represent

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the relative desired position of agent R_i with respect to R_j. A formation graph is said to be undirected if $(R_jR_i) \in E$ implies that $(R_iR_j) \in E$, otherwise, it is called a directed graph. For an edge (R_jR_i), R_j is called the parent node and R_i is the child node, then it is said R_j is neighbour of R_i. The set of all neighbours of R_i is denoted as N_i. A directed path from R_i to R_j is a sequence of edges of the form $(R_iR_p), \dots, (R_qR_j)$. A directed tree is a directed graph in which every node has exactly one parent, except for one single node called the root. The root has no parent and has a directed path to a any other node. A directed spanning tree of a directed graph G is a directed tree involving every node in G. The Laplacian matrix associated with a formation graph G is given by

$$\mathscr{L}(G) = \Delta - \mathscr{A}_d \tag{1}$$

where $\Delta = \text{diag}\{n_1, \dots, n_N\}$ is the in-degree matrix where n_i is the cardinality of N_i and $\mathscr{A}_d = [a_{ij}] \in \mathbb{R}^{N\times N}$ is the adjacency matrix of G whose entries are defined as follows

$$a_{ij} = \begin{cases} 1, \text{ if } & (R_jR_i) \in E \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

A real function $\phi(x)$ is said to be a saturation function if it satisfy: $\phi(x) = 0 \Leftrightarrow x = 0$; $-r \le \phi(x) \le r$ for some $r > 0$; $x\phi(x) > 0$, $\forall x \neq 0$ and $0 < \frac{d\phi(x)}{dx} < M_1 < \infty$. In the same way, the function $m(x)$ is a smooth switching function if the next properties hold: $m(x) = 1$ if $x \le a$; $m(x) = 0$ if $x \ge b$; $0 < m(x) < 1$ if $a < x < b$ and $-\infty < \frac{\partial m(x)}{\partial x} < 0$, where $b > a > 0$.

3. PROBLEM STATEMENT

Let N be a group of mobile robots denoted by $R_1, \dots, R_N$ which are moving on a 2D plane. The agents are modelled by a single integrator

$$\dot{z}_i = u_i, \quad i = 1, \dots, N \tag{3}$$

where $z_i(t) = [x_i(t), y_i(t)]^T \in \mathbb{R}^2$ are the position coordinates of agent R_i and $u_i \in \mathbb{R}^2$ is the control input, which are the velocities along the x and y axes. Assume that robot R_i is able to determine, at every time instant, the position of a subset of robots $N_i \subset N$ which defines its desired position z_i^* and is given by

$$z_i^* = \frac{1}{n_i} \sum_{j\in N_i} (z_j + c_{ji}), \tag{4}$$

where n_i is the cardinality of N_i and $c_{ji} = [h_{ji}, v_{ji}]^T \in \mathbb{R}^2$, $\forall j \in N_i$ are constant vectors which specify the desired spatial distribution of agents. Besides, assume that every agent is equipped to sense the surrounding area and can measure the position of any other agent within a circle of radius D_i, defining the set $M_i(t) = \{R_j \in N \mid \|z_i(t) - z_j(t)\| \le D_i\}$, where D_i is called the *sensing radius*. For simplicity, we consider the same sensing radius for all the agents, that is, $D_i = D, \forall i \in N$. On the other hand, regarding physical dimensions of agents, we define the *collision radius*, which is the minimum safety distance between any pair of agents and is denoted by d. Also assume that $0 < d < D$. The sensing and collision radii are illustrated in Fig. 1.

The control objective is to design control laws $u_i = u_i(z_i, z_i^*, N_i \cup M_i)$, $i = 1, \dots, N$ such that:

i) The agents reach a desired formation, that is, $\lim_{t\to\infty}(z_i(t) - z_i^*(t)) = 0$, $i = 1, \dots, N$,
ii) the agents avoid collisions by remaining at some distance greater than or equal to the collision radius d from each other, i.e., $\|z_i(t) - z_j(t)\| \ge d$, $\forall t \ge 0$, $i \neq j$, and

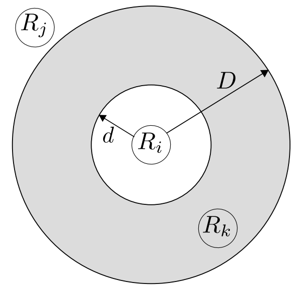

Fig. 1. Sensing and collision radii. The grey area is referred as the influence region. Agents R_i and R_k are in conflict and they can detect the position of each other. Meanwhile, robot R_j is not risk in conflict with any other agent.

iii) the control input in each agent is bounded, that is, $\|u_i\| \le u_{max}$, $i = 1, \dots, N$, for some positive constant u_{max}.

It is assumed that at the initial conditions all agents satisfy $\|z_i(0) - z_j(0)\| \ge d$, $\forall i \neq j$.

4. CONTROL STRATEGY

The proposed control scheme is designed in two steps. The first one attends the convergence to the desired formation while the second one regards the collision avoidance issue.

Theorem 1. Consider a group of N agents moving on a plane modelled by (3) along with the saturated control law

$$u_i = -\mu\varphi(\tilde{z}_i) \tag{5}$$

where $\mu > 0$, $-1 \le \varphi(\cdot) \le 1$ is a saturation function and $\tilde{z}_i = z_i - z_i^*$ is the position error of the i-th agent. If the communication graph G contains a directed spanning tree, then, in the closed-loop system the agents reach their desired formation asymptotically.

Proof. Taking the error dynamics we have

$$\dot{\tilde{z}}_i = \dot{z}_i - \frac{1}{n_i} \sum_{j\in N_i} \dot{z}_j, \tag{6}$$

which in closed-loop with (3) becomes

$$\dot{\tilde{z}}_i = -\mu \frac{1}{n_i} (\mathscr{L}_i(G) \otimes I_2)\, \phi(\tilde{z}) \tag{7}$$

where $\mathscr{L}_i(G)$ is the $i-$th row of the Laplacian matrix and $\otimes$ denotes the Kronecker matrix product, Zhang and Ding (2013). In vector form, the whole system is represented as

$$\dot{\tilde{z}} = -\mu \left(\Delta^{-1}\mathscr{L}(G) \otimes I_2\right) \phi(\tilde{z}), \tag{8}$$

with Δ defined previously, $\tilde{z} = [\tilde{z}_1^T, \tilde{z}_2^T, \dots, \tilde{z}_N^T]^T$ and, correspondingly, $\phi(\tilde{z}) = [\phi(\tilde{z}_1)^T, \phi(\tilde{z}_2)^T, \dots, \phi(\tilde{z}_N)^T]^T$.

Taking the Lyapunov function candidate

$$V(\tilde{z}) = \sum_{i=1}^{N} \int_0^{\tilde{z}} \phi(\tau) d\tau = \sum_{i=1}^{N} \left(\int_0^{\tilde{z}_i} \phi_x(\tau) d\tau + \int_0^{\tilde{z}_i} \phi_y(\tau) d\tau \right) \tag{9}$$

where, $\phi(\tilde{z}) = [\phi_x(\tilde{z}_i), \phi_y(\tilde{z}_i)]^T$. The time-derivative along the trajectories of the closed-loop system is

$$\dot{V}(\tilde{z}) = \sum_{i=1}^{N} \left(\phi_x(\tilde{z}_i)\dot{\tilde{x}}_i + \phi_y(\tilde{z}_i)\dot{\tilde{y}}_i\right) = \sum_{i=1}^{N} \left(\phi(\tilde{z}_i)\dot{\tilde{z}}_i\right), \tag{10}$$

which in vector form becomes

$$\dot{V}(\tilde{z}) = -\mu\phi^T(\tilde{z}) \left(\Delta^{-1}\mathscr{L}(G) \otimes I_2\right) \phi(\tilde{z}). \tag{11}$$

If every agent in the group defines its desired position with respect to, at least, any other robot, that is, the scheme is leaderless, the matrix $\Delta^{-1} > 0$. Even more, as the communication graph contains a spanning tree, the Laplacian matrix have exactly one zero eigenvalue, say $\lambda_n = 0$, that implies, $\text{rank}(\mathscr{L}(G)) = n-1$, while the rest of the eigenvalues have positive real parts, that is, $\text{Re}(\lambda_i) > 0$, $1 = 1, \ldots, n-1$. Then, we can only ensure that

$$\dot{V}(\tilde{z}) \leq 0. \tag{12}$$

Now, to prove asymptotic convergence, consider the eigenvector associated to the zero eigenvalue which is $1_n = [1, 1, \ldots, 1]$, Ren and Beard (2008). Then, $\mathscr{L}(G)1_n = 0$, which implies that $\phi(\tilde{z}_1) = \phi(\tilde{z}_2) = \cdots = \phi(\tilde{z}_n)$ and because of the properties of saturation functions, $\tilde{z}_1 = \tilde{z}_2 = \cdots = \tilde{z}_n = z^*$. On the other hand, there exists a left eigenvector such that

$$[\alpha_1, \alpha_2, \ldots, \alpha_n]\mathscr{L}(G) = 0, \tag{13}$$

or, in other words, there exists a linear combination of the position errors in such a way that

$$\sum_{i=1}^{N} \alpha_i \tilde{z}_i = 0, \tag{14}$$

with coefficients α_i not all equal to zero, Ren and Beard (2008). Moreover, the coefficients α_i satisfy that $\sum_{i=1}^{N} \alpha_i = 1$. Finally,

$$\sum_{i=1}^{N} \alpha_i \tilde{z}_i = \sum_{i=1}^{N} \alpha_i z^* = 0 \tag{15}$$

implies that $z^* = 0$ which means that the largest invariant set within the Lyapunov function is zero is the set where all the position errors are identically zero, then, $\tilde{z}_i = 0$ is asymptotically stable. Since the Lyapunov function is radially unbounded, the convergence to the desired formation is global. □

In order to develop a smooth strategy to solve the formation control problem for first order agents, we start by recalling the collision avoidance strategy proposed in Flores-Resendiz and Aranda-Bricaire (2014), Flores-Resendiz et al. (2015) and Flores-Resendiz and Aranda-Bricaire (2019). There, every agent considered any other robots or obstacles in its sensing region as an unstable focus in such a way that a repulsive vector field appeared between them. Then, the whole control law was given by

$$u_i = \gamma_i + \beta_i = -\mu\varphi(\tilde{z}_i) - \varepsilon \sum_{j=1, j\neq i}^{n} \delta_{ij} \begin{bmatrix} p_{ij} - q_{ij} \\ p_{ij} + q_{ij} \end{bmatrix}, \tag{16}$$

where $\varepsilon > 0$ and we have introduced the relative position variables $p_{ij} = x_j - x_i$ and $q_{ij} = y_j - y_i$ which make up the unstable focus. Distance-based parameters δ_{ij} are defined as

$$\delta_{ij} = \begin{cases} 1, & \text{if} \quad \|z_i - z_j\| \leq d, \\ 0, & \text{if} \quad \|z_i - z_j\| > d. \end{cases} \tag{17}$$

Furthermore, According to Flores-Resendiz and Aranda-Bricaire (2019), the control law (16)-(17) ensures collision avoidance among agents as long as

$$\varepsilon > \frac{2\mu}{d}. \tag{18}$$

It is important to notice that the unstable focus structure of the repulsive vector fields is useful to prevent the agents to get stuck at undesired equilibrium points in the formation problem. However, they should be scaled by the parameter ε to avoid the agents to get nearer than a safety distance. In Fig. 2 the magnitude of the repulsive fields is shown. As it can be easily seen, the vector fields vanishes when the relative position between a pair of agents becomes zero. In the same way, Fig. 3 illustrates the behaviour of the discontinuous component of the control

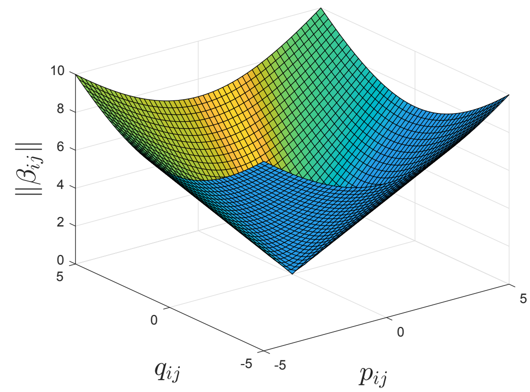

Fig. 2. Magnitude of the repulsive vector fields with unstable focus structure.

law (16), where the repulsive vector field is turned on when the relative distance between a pair of agents reaches the minimum one allowed. As it has been reported, the distance-based collision avoidance mechanism working in a discontinuous manner could lead to the appearance of chattering phenomenon. In this work, we propose a modified strategy, given by

$$\beta_i = -\varepsilon \sum_{j=1, j\neq i}^{n} m(d_{ij}) \begin{bmatrix} p_{ij} - q_{ij} \\ p_{ij} + q_{ij} \end{bmatrix}, \tag{19}$$

where $d_{ij} = \|z_i - z_j\|$ is the distance between the $i-$th and the $j-$th agent and $m(\cdot)$ is a smooth switching function. The basic idea is to apply the repulsive vector fields gradually, regarding the sensing radius D which indicates the agents are getting into the influence region of any other and ensuring that agents never get nearer than the collision radius. Alternative switching functions have been proposed in Do (2014), Li and Yang (2017), and others. Of course, when $d_{ij} > D$ the repulsive vector field are deactivated and when $d < d_{ij} < D$ the repulsive fields increase their magnitude but the distance between them could still decrease. Finally, when $d_{ij} \leq d$ corresponds to the condition presented in Flores-Resendiz and Aranda-Bricaire (2019) which utilizes a discontinuous vector fields activation/deactivation mechanism, wherein it was proved that the agents remain at a distance greater than or equal to the collision radius. Moreover, the conflicts between agents in risk of collision were solved in finite-time.

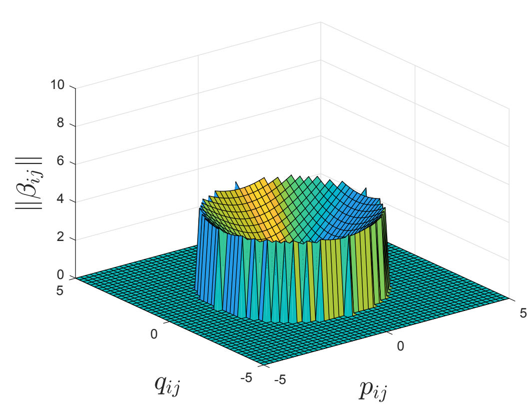

Fig. 3. Magnitude of the distance-based discontinuous vector fields. Turned on when the agents reach the minimum allowed distance.

Regarding smooth functions $m(\cdot)$ to mediate the application of the repulsive vector fields, the reactive component of the control law looks like the one illustrated in Fig. 4. Once the repulsive vector fields are modified as in (19), each element of the sum could be bounded as follows

$$\beta_i = -\varepsilon \sum_{j=1, j\neq i}^{n} \beta_{ij} \leq -\varepsilon \sum_{j=1, j\neq i}^{n} \beta_{ij}(d^*), \tag{20}$$

where $d^* = \arg\max_{d_{ij}}(\beta_{ij})$ depends on the selection of the function $m(d_{ij})$ and can be estimated by solving the equation

$$d^* = -\frac{m(d^*)}{m'(d^*)}, \tag{21}$$

which is easily stated by fundamental calculus theorem. Although the last equation could imply the use of numerical methods, this operation is done only once. Now we can state our main result.

Theorem 2. Consider the N agents modelled by (3) along with the control law (16) with β_i defined as in (19). Suppose there exists a maximum available control input $\|u_i\| < u_{max}$ and that every agent could get in risk of collision with, at most, p other agents. If the communication graph G contains a directed spanning tree and the design parameters are selected in such a way that

$$\frac{2\mu}{d} < \varepsilon \leq \frac{u_{max} - \mu}{\sqrt{2}pd^* m(d^*)} \tag{22}$$

holds, then the agents reach the desired formation without collisions and no pair of agents get at a distance smaller than the predefined collision radius.

Proof. If there exists a constraint in the control input magnitude, that is,

$$\|u_i\| < u_{max}, \tag{23}$$

we have

$$\|u_i\| = \|\gamma_i + \beta_i\| \leq \|\gamma_i\| + \|\beta_i\|. \tag{24}$$

By bounding each component of the control law,

$$\|\gamma_i\| \leq \mu, \tag{25}$$

and, for the repulsive vector fields

$$\beta_i = -\varepsilon \sum_{j=1, j\neq i}^{n} m(d_{ij}) \begin{bmatrix} p_{ij} - q_{ij} \\ p_{ij} + q_{ij} \end{bmatrix} = -\varepsilon \sum_{j=1, j\neq i}^{n} \beta_{ij} \tag{26}$$

where

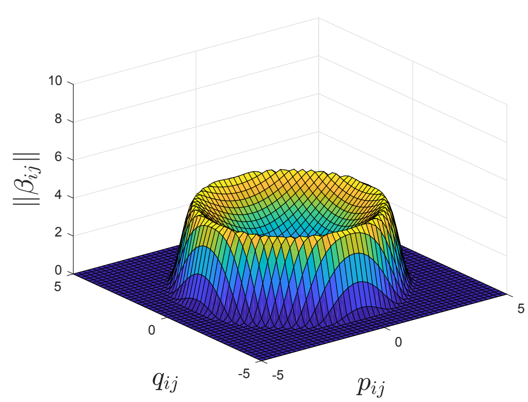

Fig. 4. Discontinuous vector fields activated by a smooth function

$$\beta_{ij} = m(d_{ij}) \begin{bmatrix} p_{ij} - q_{ij} \\ p_{ij} + q_{ij} \end{bmatrix} \tag{27}$$

$$= m(d_{ij}) \left(\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \otimes I_2 \right) \begin{bmatrix} p_{ij} \\ q_{ij} \end{bmatrix}. \tag{28}$$

Now, bounding each of the repulsive vector fields

$$\|\beta_{ij}\| \leq \left(\sqrt{\lambda_{max}(F^T F)} \right) \left\| \begin{bmatrix} p_{ij} \\ q_{ij} \end{bmatrix} \right\|, \tag{29}$$

where

$$F = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}. \tag{30}$$

When the i-th robot is in risk of collision with exactly p other agents, the same number of repulsive vector fields are activated, and each of them is bounded as above, then

$$\|\beta_i\| \leq \varepsilon \sum_{j=1, j\neq i}^{p} \beta_{ij} \leq \sqrt{2}p\varepsilon d^* m(d^*). \tag{31}$$

Finally, in order to achieve the control goal regarding the constraint in the input we have

$$\mu + \sqrt{2}p\varepsilon d^* m(d^*) \leq u_{max}, \tag{32}$$

and then,

$$\varepsilon \leq \frac{u_{max} - \mu}{\sqrt{2}pd^* m(d^*)}. \tag{33}$$

The lower bound for ε is obtained in the same way than in Flores-Resendiz and Aranda-Bricaire (2019). This completes the proof. □

Remark 1. As a design consideration, in a reduced group of agents the parameter p in (22) provides a degree of freedom to select the gain ε. Regarding geometrical conditions, it has been shown in (Flores-Resendiz and Aranda-Bricaire, 2019) that a agent could not get in risk of collision with more than six agents at the same time, that is, we can choose $p = 6$ to cover all possible scenarios. This selection reduces (22) to

$$\frac{2\mu}{d} < \varepsilon \leq \frac{u_{max} - \mu}{6\sqrt{2}d^* m(d^*)}. \tag{34}$$

5. SIMULATION RESULTS

In this Section, we present results of two different simulation examples. In the first case, we deal with a reduced number of agents with the objective of illustrating in detail the performance of the proposed scheme. In the second simulation, a system with five agents is presented and the main objective of this example is to show the performance of the proposed control law in a complex environment. In both these examples, the function $m(d_{ij})$ is defined as,

$$m(d_{ij}) = \frac{1}{1 + e^{a(d_{ij} - b)}} \tag{35}$$

where the parameters have been selected as $a = 10$ and $b = 2.4$ which give a good approximation to $D = 2.8$ and $d = 2$. This function is depicted in Fig. 5. By solving (21), it could be found out that the repulsive vector field between a pair of agents has its maximum magnitude when they are at a distance

$$d^* \approx 2.10041, \tag{36}$$

using proper units.

Example 1: Two agents.

Consider the case of two agents which are located at $z_1(0) = [3,3]^T$ and $z_2(0) = [-3,-3]^T$ with bidirectional communication, $c_{21} = [-3,-3]^T$ and $c_{12} = -c_{21}$. This configuration is

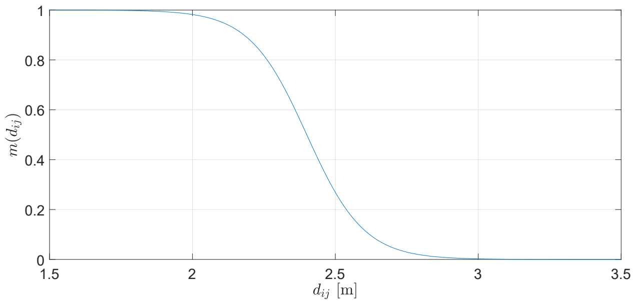

Fig. 5. Transition function

such that the agents will move straightforward each other to exchange relative positions. Under this conditions, the use of single potential functions could lead to undesired equilibrium points. By using (16), regarding (19) and (22), with $\mu = 1$, $\varepsilon = 1.1$, $d = 2$, $D = 2.8$ and $u_{max} = 4.5$, the agents reached their desired position while avoiding each other, as shown in Fig. 6. It is clear that the distance between them does not become smaller than the collision radius. Both, the sensing and the collision radius are depicted with dash-dotted lines. Even

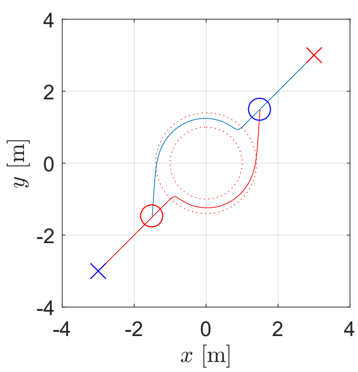

Fig. 6. Agents' behaviour under the smooth control law. The dotted lines indicate the sensing and collision radius. "X": initial position, "O": final position.

more, the control inputs are illustrated in Fig. 7 and Fig. 8 and is clear that the magnitude of this signal is much smaller than the maximum value u_{max}. This case illustrates the ability

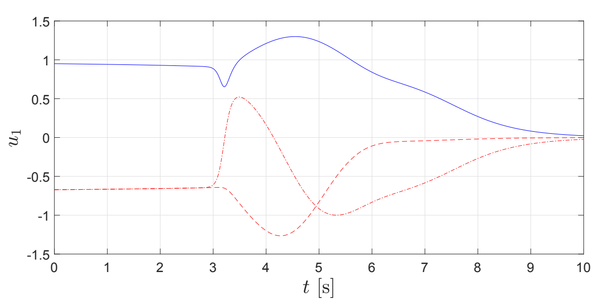

Fig. 7. Control input for agent 1. The solid line represents the magnitude of the signal. "–" x-axis velocity, "-." y-axis velocity.

of our proposed scheme to avoid getting stuck at undesired equilibriums points while is clear that the control objective is reached.

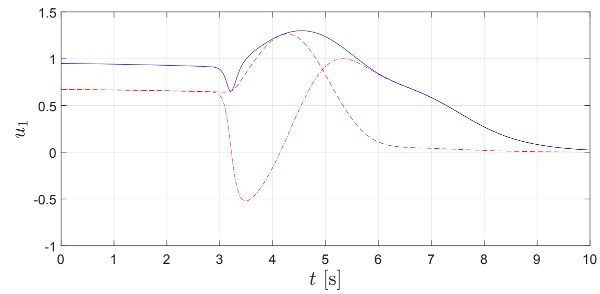

Fig. 8. Control input for agent 2. The solid line represents the magnitude of the signal. "–" x-axis velocity, "-." y-axis velocity.

Example 2: Five agents.

Although the previous case shows a good performance when dealing with undesired equilibria, we now present a result simulation regarding a more complex environment with 5 agents. Taking the initial conditions $z_1(0) = [8,8]^T$, $z_2(0) = [8,-8]^T$, $z_3(0) = [0,0]^T$, $z_4(0) = [-8,8]^T$ and $z_5(0) = [-8,-8]^T$, the objective is to reach the desired formation shown in Fig. 9, where not only the communication among agents is provided but also the spatial distribution. The trajectories followed by the agents are depicted in Fig. 10

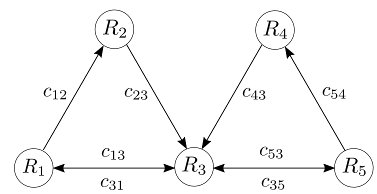

Fig. 9. Desired formation for example 2.

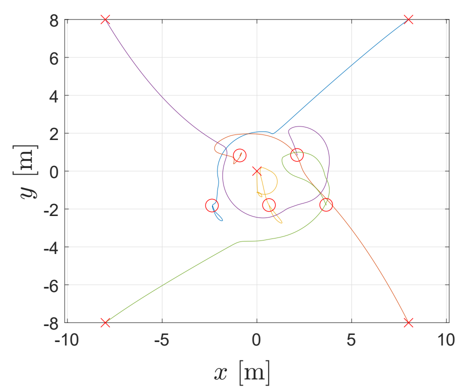

Fig. 10. Agents' behaviour in example 2. "X": initial position, "O": final position..

In Fig. 11, are shown the distances between any pair of agents as well as the distances corresponding to the sensing and collision radii. It is clear that the agents are always at a distance greater than collision radius. Finally, the control effort needed by each agent are shown in Fig. 12.

6. CONCLUSIONS

In this paper, we proposed a solution for the formation control problem by using repulsive vector fields with unstable focus structure and activated by a smooth function. We ensured that

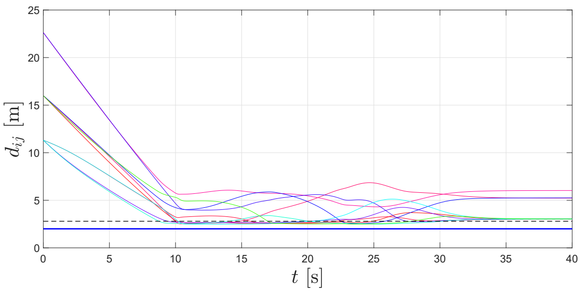

Fig. 11. Distance between any pair of agents in example 2. The horizontal dashes lines represent the sensing and collision radii.

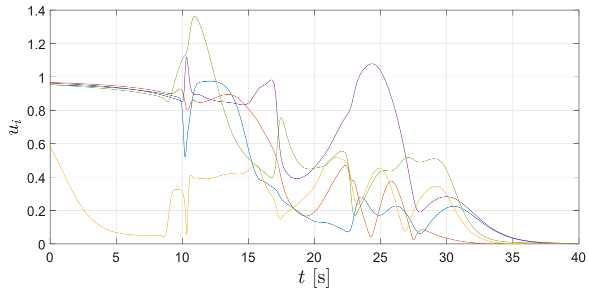

Fig. 12. Control input for agents in example 2.

the agents avoid collisions among them while there exist an input constraint given as a maximum value. Conditions for the selection of design parameters were found and the performance of the scheme was shown by numerical simulations. This approach could be applied to systems with an arbitrary number of agents while ensuring there will not exist undesired equilibrium the agents could get stuck at. The use of continuous function to mediate the application of the repulsive bounded component encourage the study of second order systems using the well-known backstepping technique.

REFERENCES

Bekirov, E.A. and Asanov, M.M. (2017). Digital temperature measuring device for use on unmanned aerial vehicles. In *2017 IEEE 4th International Conference Actual Problems of Unmanned Aerial Vehicles Developments (APUAVD)*, 171–174. doi:10.1109/APUAVD.2017.8308802.

Chikwanha, A., Motepe, S., and Stopforth, R. (2012). Survey and requirements for search and rescue ground and air vehicles for mining applications. In *2012 19th International Conference on Mechatronics and Machine Vision in Practice (M2VIP)*, 105–109.

Deghat, M., Anderson, B.D.O., and Lin, Z. (2016). Combined flocking and distance-based shape control of multi-agent formations. *IEEE Transactions on Automatic Control*, 61(7), 1824–1837. doi:10.1109/TAC.2015.2480217.

Do, K.D. (2006). Formation control of mobile agents using local potential functions. In *2006 American Control Conference*, 6 pp.–. doi:10.1109/ACC.2006.1656537.

Do, K.D. (2014). Bounded assignment formation control of second-order dynamic agents. *IEEE/ASME Transactions on Mechatronics*, 19(2), 477–489. doi:10.1109/TMECH.2013.2243744.

Flores-Resendiz, J.F. and Aranda-Bricaire, E. (2014). Cyclic pursuit formation control without collisions in multi-agent systems using discontinuous vector fields. In *Memorias del XVI Congreso Latinoamericano de Control Automático, CLCA 2014*, 941–946. Cancún, México. URL `http://amca.mx/memorias/amca2014/articulos/0110.pdf`.

Flores-Resendiz, J.F. and Aranda-Bricaire, E. (2019). A general solution to the formation control problem without collisions for first order multi-agent systems. *Robotica*, 1–17.

Flores-Resendiz, J.F., Aranda-Bricaire, E., González-Sierra, J., and Santiaguillo-Salinas, J. (2015). Finite-Time Formation Control without Collisions for Multiagent Systems with Communication Graphs Composed of Cyclic Paths. *Mathematical Problems in Engineering*, 2015, 1–17. doi:10.1155/2015/948086. URL `https://www.hindawi.com/journals/mpe/2015/948086/`.

Hernandez-Martinez, E. and Bricaire, E.A. (2012). Non-collision conditions in multi-agent virtual leader-based formation control. *International Journal of Advanced Robotic Systems*, 9(4), 100. doi:10.5772/50722. URL `https://doi.org/10.5772/50722`.

Li, X.J. and Yang, G.H. (2017). Adaptive decentralized control for a class of interconnected nonlinear systems via backstepping approach and graph theory. *Automatica*, 76, 87 – 95. doi:https://doi.org/10.1016/j.automatica.2016.10.019. URL `http://www.sciencedirect.com/science/article/pii/S000510981630423X`.

Mouradian, C., Sahoo, J., Glitho, R.H., Morrow, M.J., and Polakos, P.A. (2017). A coalition formation algorithm for multi-robot task allocation in large-scale natural disasters. In *2017 13th International Wireless Communications and Mobile Computing Conference (IWCMC)*, 1909–1914. doi:10.1109/IWCMC.2017.7986575.

Ramirez-Paredes, J.., Doucette, E.A., Curtis, J.W., and Gans, N.R. (2015). Urban target search and tracking using a uav and unattended ground sensors. In *2015 American Control Conference (ACC)*, 2401–2407. doi:10.1109/ACC.2015.7171092.

Ren, W. and Beard, R.W. (2008). *Distributed Consensus in Multi-vehicle Cooperative Control - Theory and Applications*. Communications and Control Engineering. Springer. doi:10.1007/978-1-84800-015-5. URL `https://doi.org/10.1007/978-1-84800-015-5`.

Rodríguez-Seda, E.J., Stipanović, D.M., and Spong, M.W. (2016). Guaranteed collision avoidance for autonomous systems with acceleration constraints and sensing uncertainties. *Journal of Optimization Theory and Applications*, 168(3), 1014–1038. doi:10.1007/s10957-015-0824-7. URL `https://doi.org/10.1007/s10957-015-0824-7`.

Yang, Q., Cao, M., de Marina, H.G., Fang, H., and Chen, J. (2018). Distributed formation tracking using local coordinate systems. *Systems & Control Letters*, 111, 70 – 78. doi:https://doi.org/10.1016/j.sysconle.2017.11.004. URL `http://www.sciencedirect.com/science/article/pii/S0167691117302128`.

Yu, X., Xu, X., Liu, L., and Feng, G. (2018). Circular formation of networked dynamic unicycles by a distributed dynamic control law. *Automatica*, 89, 1 – 7. doi:https://doi.org/10.1016/j.automatica.2017.11.021. URL `http://www.sciencedirect.com/science/article/pii/S0005109817305629`.

Zhang, H. and Ding, F. (2013). On the kronecker products and their applications. *Journal of Applied Mathematics*, 2013. doi:10.1155/2013/296185.

