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Time-delay Based Controller for Independent Joint Control of Robot Manipulators ⋆

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Abstract: A well-known strategy for the control of robot manipulators consists in independently controlling each joint. In this paper, we propose a class of time-delay based controllers under this scheme, which ensures practical stability of the tracking position error dynamics corresponding to each joint. In order to tune the controller, a *simple* methodology based on the feasibility of a Linear Matrix Inequality derived from a proposed Lyapunov-Krasovskii functional is provided. In contrast to classical control algorithms, the presented scheme does not require estimators nor velocity measures but only measures of position at the present and a given delayed time. Numerical simulations show the potential advantages of the proposed class of controllers when position measures are corrupted by noise.

Keywords: control of robot manipulators; time-delay based controllers; time-delay systems.

1. INTRODUCTION

We consider robot manipulators with n links described by

$$M(q)\ddot{q}(t) + C(q,\dot{q})\dot{q} + G(q) = \tau(t), \tag{1}$$

where $q^T(t) = (q_1(t) \ \dots \ q_n(t)) \in \mathbb{R}^n$ represents the position vector of the joints, $M(q)$ and $C(q,\dot{q})$ are respectively the inertia and Coriolis matrices in $\mathbb{R}^{n\times n}$, $G(q) \in \mathbb{R}^n$ is the gravity vector, and $\tau^T(t) = (\tau_1(t) \ \dots \ \tau_n(t)) \in \mathbb{R}^n$ is the joint torque vector. Each joint is considered to be actuated by a DC motor and its dynamic to be modeled by the second order differential equation

$$J_{mi}\ddot{q}_i(t) + B_{mi}\dot{q}_i(t) = r_{mi}K_{mi}v_i(t) - r_{mi}^2\tau_i(t), i = 1,\dots,n, \tag{2}$$

where J_{mi} is the total inertia of the motor and gear, B_{mi} is the damping coefficient, K_{mi} is the torque constant, r_{mi} is the gear ratio of the coupling of the motor to the link, and v_i is the armature voltage. We assume throughout this paper that all the functions $t \mapsto \tau_i(t)$, $i = 1,\dots,n$, are bounded.

A classical control scheme for robot manipulators described by (1) consists in independently controlling the actuator of each joint. Under this scheme, the control of nonlinear system (1) is reduced to the control of n linear systems of the form (2) with disturbance $r_{mi}^2\tau_i(t)$. If the gear ratio r_{mi} is small enough, then the effect of the disturbance is small and classical PD or PID controllers are able to deal with it well enough (Lewis et al., 2003). Advantages and disadvantages of this control scheme with respect to others are well known and have been extensively discussed in the literature (Lewis et al., 2003; Spong et al., 2006; Kelly and Santibáñez, 2003).

Although classical controllers might be used in order to achieve a desired performance by mitigating the effects of $r_{mi}^2\tau_i(t)$ (Lewis et al., 2003), they still face well-known problems when a derivative action is required. With no velocity measures available, the implementation of a derivative term requires state estimators or estimation of the derivative from the measured position. Using state estimators on the one hand increases the complexity of the overall system, and estimation of the derivative by using measures of the position on the other might result in the amplification of high frequency noise. An alternative to the previously mentioned approaches that has received increasing attention in the last two decades consists in introducing intentional delays in the control signal. Early results on time-delay based controllers were presented by (Suh and Bien, 1979), whereas recent results showing their potential in addressing several classes of problems including noise mitigation and fast convergence can be found in (Villafuerte et al., 2012; Ramírez et al., 2015; Sipahi et al., 2011; Niculescu and Michiels, 2004; Gomez et al., 2019). This class of controllers has also been applied to the control of a class of robot manipulators in (Ochoa-Ortega et al., 2019), but the proposed scheme there is different from the one considered in this work.

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In line with the above ideas, in this paper, we propose an independent joint time-delay based control scheme aiming at solving the tracking problem in robot manipulators whose dynamics are described by (1). Clearly, under this scheme, solving the tracking problem for (1) requires addressing the tracking problem of each joint actuator. The target of the proposed controller is thus to perform the independent control of n systems of the form (2). The main feature of the proposed control algorithm is that it is able to mimic derivative-based controllers without requiring the implementation of state estimators or derivative computation from position measures, but only requiring measures at present and delayed times. Since a delay is induced in the control signal, the resulting system in closed-loop corresponds to a perturbed time-delay system.

We show that the proposed time-delay based controller ensures the practical stability of the tracking position error dynamics of the joints. This is carried out by proposing a simple Lyapunov-Krasovskii functional that, as long as a Linear Matrix Inequality (LMI) is satisfied, fulfills the sufficient conditions for practical stability presented in (Villafuerte et al., 2011). A distinctive feature of the proposed functional with respect to the class proposed by (Villafuerte et al., 2011) is that it enables a simple tunning of the gains of the controller, despite the induced complexity by the delayed term. Specifically, it allows proving that, provided a delay-free system is stable, it is always possible to find a controller that practically stabilizes the system under study.

The manuscript is organized as follows: In Section 2, we introduce results on practical stability for a class of second order systems. There, the proposed Lyapunov-Krasovskii functional is introduced and its main differences with respect to others reported in the literature are pointed out. In Section 3, the theoretical results of Section 2 are applied to solve the tracking problem of joint actuators described by (2). We illustrate the proposed approach by some numerical simulations and compare its performance with the performance of a classical PD when position measures are corrupted by noise. We close the paper with some concluding remarks in Section 4.

We adopt the following notation: The space of piecewise continuous differentiable vector functions is denoted by PC. For vector and matrices we use the Euclidian norm, which is represented by $\|\cdot\|$. The space of piecewise continuous functions is equipped with the supremum norm defined as

$$\|\varphi\|_h := \sup_{\theta\in[-h,0]} \|\varphi(\theta)\|.$$

The solution $x(t,\varphi)$ of a retarded type system with delay $h>0$ restricted to the interval $[t-h,t]$ is denoted by

$$x_t : \theta \mapsto x(t+\theta,\varphi).$$

Finally, the notation $A > 0$ means that matrix A is symmetric and positive definite.

2. PRACTICAL STABILITY FOR SECOND ORDER SYSTEMS

In this section, we consider the representation in state space of second order systems of the form

$$\dot{x}(t) = Ax(t) + Bu(t) + \eta(t), \tag{3}$$

where $x^T(t) = (x_1(t)\ x_2(t)) = (x_1(t)\ \dot{x}_1(t)) \in \mathbb{R}^2$, $u \in \mathbb{R}$,

$$A = \begin{pmatrix} 0 & 1 \\ a_1 & a_2 \end{pmatrix},\ B = \begin{pmatrix} 0 \\ b_1 \end{pmatrix},$$

with a_1, a_2 and b_1 real numbers, and $\eta : \mathbb{R}_{\geq 0} \to \mathbb{R}$ satisfying $\|\eta(t)\| \leq m$ for all $t \geq 0$ and for some positive real number m. As we shall see in Section 3, the position error dynamics of the ith joint under study can be written as a system of the form (3).

Let us consider the control law

$$u(t) = k_1 x_1(t) + k_2 x_1(t-h), \tag{4}$$

where $h > 0$ represents an intentionally introduced delay. Notice that the proposed control law does not require the state x_2 but only x_1 at the present and delayed time. Closed-loop system (3),(4) is then a time-delay system described by

$$\begin{aligned} \dot{x}(t) &= A_0 x(t) + A_1 x(t-h) + \eta(t),\ t \geq 0, \\ x(t) &= \varphi(t) \in PC,\ t \in [-h, 0), \end{aligned} \tag{5}$$

with

$$A_0 = \begin{pmatrix} 0 & 1 \\ a_1 + b_1 k_1 & a_2 \end{pmatrix} \text{ and } A_1 = \begin{pmatrix} 0 & 0 \\ b_1 k_2 & 0 \end{pmatrix}.$$

Before we continue, let us recall some basic definitions and instrumental results. We start with the notion of practical stability for system (5) formally defined in (Villafuerte et al., 2011).

Definition 1. (Villafuerte et al., 2011) System (5) is $\mu-$practically stable if for some $\mu > 0$ there exists $T = T(\mu, \varphi)$ such that $\|x(t,\varphi)\| \leq \mu\ \forall t \geq T$.

A sufficient condition for μ-practical stability based on the existence of a Lyapunov-Krasovskii functional is also presented in (Villafuerte et al., 2011):

Lemma 2. System (5) is μ-practically stable if there exists a functional $v(x_t)$ satisfying

$$\alpha_1 \|x(t)\|^2 \leq v(x_t) \leq \alpha_2 \|x_t\|_h^2 \tag{6}$$

and

$$\frac{d}{dt} v(x_t) \leq -2\sigma v(x_t) + \kappa\sqrt{v(x_t)}, \tag{7}$$

where α_1, α_2, κ and σ are positive real numbers. Moreover, $\mu > \dfrac{\kappa}{2\sigma\sqrt{\alpha_1}}$ and

$$T = \begin{cases} 0, & \text{if } \|\varphi\|_h \leq \dfrac{\kappa}{2\sigma\sqrt{\alpha_2}}, \\ \dfrac{1}{\sigma} \ln\left(\dfrac{2\sigma\sqrt{\alpha_2}\|\varphi\|_h - \kappa}{2\sigma\sqrt{\alpha_1}\mu - \kappa} \right), & \text{otherwise.} \end{cases}$$

We consider the functional $v : x_t \to \mathbb{R}$,

$$v(x_t) = x^T(t)Px(t) + h\int_{t-h}^{t}(\theta - t + h)x_2^2(\theta)k_2^2 r e^{2\sigma(\theta-t)}d\theta, \quad (8)$$

where $\mathbb{R}^{2\times2} \ni P > 0$, and σ and r are positive real numbers. A particular feature of functional (8) is that the integral term does not depend on neither the whole state as in (Villafuerte et al., 2011) (but only on x_2) nor $\dot{x}_2$ as in the recent work by (Fridman and Shaikhet, 2017). This feature enables a simple tuning methodology of control parameters (k_1, k_2, h).

The main result of this section, Theorem 4, is presented after the following instrumental lemma.

Lemma 3. (Solomon and Fridman, 2013) Let $f : [a, b] \to [0, \infty)$ and $x \in \mathbb{R}^n$. Then, the following inequality holds for any positive definite matrix $R \in \mathbb{R}^{n\times n}$:

$$\left(\int_a^b f(\theta)x(\theta)d\theta\right)^T R\left(\int_a^b f(\theta)x(\theta)d\theta\right) \le \int_a^b f(\theta)d\theta \int_a^b f(\theta)x^T(\theta)Rx(\theta)d\theta.$$

Theorem 4. Let us define the matrices

$$M(P, r) := \begin{pmatrix} E^TP + PE + R & -PB \\ -B^TP & -re^{-2\sigma h}\end{pmatrix},$$

with $E := A_0 + A_1$ and

$$R := \begin{pmatrix} 0 & 0 \\ 0 & h^2rk_2^2\end{pmatrix}.$$

If there exist a positive definite matrix $P \in \mathbb{R}^{2\times2}$ and a real number $r > 0$ such that, for given k_1, k_2, σ and delay $h > 0$, the LMI

$$M(P, r) + 2\sigma\begin{pmatrix} P & 0 \\ 0 & 0\end{pmatrix} < 0 \quad (9)$$

is satisfied, then system (5) is μ-practically stable with $\mu > \dfrac{m\|P\|}{\sigma\alpha_1}$. Moreover, the LMI (9) is always feasible for a sufficiently small delay $h > 0$ provided that the matrix $E + \sigma I$ is Hurwitz stable for some $\sigma > 0$.

Proof. Let us first prove that, if (9) holds, then functional (8) satisfies the conditions of Lemma 2.

It is straightforward to see that, since $P > 0$ and $r > 0$, functional (8) satisfies (6) with $\alpha_1 = \lambda_{\min}(P)$ and $\alpha_2 = \lambda_{\max}(P) + h^2rk_2^2$. Let us focus on proving that it satisfies the derivative condition (7). In order to do this, we consider the transformation (Niculescu, 2001)

$$x_1(t-h) = x_1(t) - \int_{t-h}^{t} x_2(\theta)d\theta,$$

which, by direct substitution, transforms system (5) into a system of the form

$$\dot{x}(t) = Ex(t) - k_2B\int_{t-h}^{t} x_2(\theta)d\theta + \eta(t). \quad (10)$$

By the step-by-step method it follows that any solution of system (5) is also a solution of (10) (Niculescu, 2001).

The derivative of the functional along the solutions of (10) yields

$$\frac{d}{dt}v(x_t) = x^T(t)\left(E^TP + PE\right)x(t) - 2x^T(t)PBk_2\int_{t-h}^{t} x_2(\theta)d\theta + 2x^T(t)P\eta(t) + h^2x_2^2(t)k_2^2r - h\int_{t-h}^{t} x_2^2(\theta)k_2^2re^{2\sigma(\theta-t)}d\theta - 2\sigma v_2(x_t),$$

where

$$v_2(x_t) = h\int_{t-h}^{t}(\theta - t + h)x_2^2(\theta)k_2^2re^{2\sigma(\theta-t)}d\theta.$$

Observe that

$$-h\int_{t-h}^{t} x_2^2(\theta)k_2^2re^{2\sigma(\theta-t)}d\theta = -h\int_{-h}^{0} x_2^2(t+\theta)k_2^2re^{2\sigma\theta}d\theta \le -h\int_{-h}^{0} x_2^2(t+\theta)k_2^2re^{-2\sigma h}d\theta \le -\left(\int_{t-h}^{t} x_2(\theta)d\theta\right)^2 k_2^2re^{-2\sigma h},$$

where the last inequality follows from Lemma 3. Then, by collecting terms with the same factors and writing them in matrix form, we arrive at

$$\frac{d}{dt}v(x_t) \le \left(x^T(t)\ \xi(t)\right)M(P, r)\begin{pmatrix} x(t) \\ \xi(t)\end{pmatrix} + 2x^T(t)P\eta(t) - 2\sigma v_2(x_t),$$

or, equivalently,

$$\frac{d}{dt}v(x_t) \le \left(x^T(t)\ \xi(t)\right)\left(M(P, r) + 2\sigma\begin{pmatrix} P & 0 \\ 0 & 0\end{pmatrix}\right)\begin{pmatrix} x(t) \\ \xi(t)\end{pmatrix} + 2x^T(t)P\eta(t) - 2\sigma v(x_t),$$

where $\xi(t) = \int_{t-h}^{t} x_2(\theta)d\theta k_2$. Thus, as

$$2x^T(t)P\eta(t) \le 2\|P\|\|x(t)\|m \le \frac{2}{\sqrt{\alpha_1}}m\|P\|\sqrt{v(x_t)},$$

fulfillment of (9) implies

$$\frac{d}{dt}v(x_t) \le -2\sigma v(x_t) + \kappa\sqrt{v(x_t)}$$

with $\kappa = \dfrac{2}{\sqrt{\alpha_1}}m\|P\|$. By Lemma 2 we conclude that system (3) is μ-practically stable with $\mu > \dfrac{m\|P\|}{\sigma\alpha_1}$.

It is left to prove that LMI (9) is always feasible for a sufficiently small delay h. For some positive definite matrix $Q \in \mathbb{R}^{2\times2}$, let $P \in \mathbb{R}^{2\times2}$ be a positive definite matrix, solution of the Lyapunov equation

$$(E + \sigma I)^TP + P(E + \sigma I) = -Q.$$

Since $E + \sigma I$ is Hurwitz stable for a given $\sigma > 0$, there always exists such a matrix P. Then, by Schur complement, LMI (9) is satisfied if and only if there exists a real number $r > 0$ and

$$-Q + R + \frac{1}{r}PBB^TPe^{2\sigma h} < 0.$$

By choosing $r = \dfrac{1}{h}$ it is clear that the above condition is satisfied for sufficiently small values of h. □

Remark 5. According to Theorem 4, time $T = T(\mu, \varphi)$ of Definition 1 depends on the given gain k_1, the found matrix P and the bound on the uncertainty term m. Furthermore, knowledge of κ enables establishing $T = 0$ by choosing a suitable initial condition.

3. DELAY-BASED CONTROLLER APPLIED TO THE ROBOT JOINTS

As already mentioned in the introductory part, to tackle the tracking control problem of (1) under the independent joint control scheme, one requires to address the tracking control problem of the actuators of each joint described by (2). In this section, we use Theorem 4 in order to do so.

In the forthcoming analysis, we do not index the states x_1 and x_2 nor matrices A, B and η according to the ith joint to avoid proliferation of subscripts. Needless to say that different coefficients in the system matrices are obtained depending on the joint actuator under study.

The desired trajectory of the ith joint is represented by $q_{di}(t)$. Let $e_i(t)$ be the tracking position error and the signal control $v(t)$ in (2) respectively defined as

$$e_i(t) := q_{di}(t) - q_i(t)$$

and

$$v(t) := u_{pr}(t) + u(t), \tag{11}$$

where

$$u_{pr}(t) = \frac{J_{mi}}{r_{mi}K_{mi}}\ddot{q}_{di}(t) + \frac{B_{mi}}{r_{mi}K_{mi}}\dot{q}_{di}(t).$$

Then, by setting $x_1(t) = e_i(t)$ and $x_2(t) = \dot{e}_i(t)$, straightforward calculations lead to the state space representation (3) with

$$A = \begin{pmatrix} 0 & 1 \\ 0 & -\dfrac{B_{mi}}{J_{mi}} \end{pmatrix},\ B = \begin{pmatrix} 0 \\ -\dfrac{K_{mi}r_{mi}}{J_{mi}} \end{pmatrix},\ \eta = \begin{pmatrix} 0 \\ \dfrac{r_{mi}^2}{J_{mi}} \end{pmatrix}\tau_i(t).$$

Considering the control signal u in (11) as in (4), namely

$$u(t) = k_1 e_i(t) + k_2 e_i(t-h), \tag{12}$$

closed-loop system takes the form (5) with matrices

$$A_0 = \begin{pmatrix} 0 & 1 \\ -\dfrac{K_{mi}r_{mi}}{J_{mi}}k_1 & -\dfrac{B_{mi}}{J_{mi}} \end{pmatrix} \text{and} A_1 = \begin{pmatrix} 0 & 0 \\ -\dfrac{K_{mi}r_{mi}}{J_{mi}}k_2 & 0 \end{pmatrix}.$$

In view of Theorem 4, we adopt the following methodology in order to tune the control law (11) corresponding to each robot joint.

(1) Set $\sigma > 0$ and find gains k_1 and k_2 such that the matrix $E + \sigma I = A_0 + A_1 + \sigma I$ is Hurwitz stable.
(2) Set a small delay h and the gains k_1 and k_2 from the previous step in matrix $M(P, r)$.
(3) Solve the LMI (9) for P and r.

If a solution to the LMI (9) is found in the last step, then control (12) with parameters (k_1, k_2, h) ensures that $\|e_i(t)\| \leq \mu\ \forall t \geq T$, with μ and T defined as in Theorem 4 and as in Definition 1, respectively.

Numerical experiment. The above methodology is applied next to solve the tracking problem in two joints of a PUMA robot. Notice that the term $\tau_i(t)$ is time-varying and depends on the states of the system. However, as it is bounded and is factor of a small gear ratio r_{mi}, for illustration purposes of the presented approach we simply consider that $\tau_i(t) = \sin(10t)$.

We compare the performance of the delay based controller (12) with a classical PD controller of the form

$$u(t) = k_p e_i(t) + k_d \dot{e}_i(t), \tag{13}$$

where the derivative of the error is directly computed from measures of the position. Noisy position measures are emulated by adding a sine-wave chirp signal with increasing frequency from 1Hz to 100Hz and scaled by a factor of 0.03 on some time intervals during the simulation.

The parameters corresponding to the two joints actuators considered for simulation are taken from (Corke and Armstrong-Helouvry, 1994; Armstrong et al., 1986) and are shown in Table 1, with J_{mi} in $kg-m^2$, coefficient B_{mi} in Nms/rad and K_{mi} in Nm/A.

Table 1. Parameters corresponding to actuators of two joints of a PUMA robot.

Joint	J_{mi}	B_{mi}	K_{mi}	r_{mi}
1	1.1407	3.45	23.125	0.01597
2	4.7142	8.53	23.125	0.0093

For the tunning of the controller corresponding to actuators of both Joint 1 and Joint 2, we set $\sigma = 0.2$ and $h = 0.1$. Gains $(k_1, k_2) = (3, 2)$ and $(k_1, k_2) = (215, 53)$ ensures that the corresponding matrix $E + \sigma I$ is Hurwitz stable for Joint 1 and Joint 2, respectively. With the given parameters (σ, h, k_1, k_2), the LMI (9) is found to be feasible with

$$P = \begin{pmatrix} 1.550 & 0.5096 \\ 0.5096 & 0.4584 \end{pmatrix},\ \text{and}\ r = 1.0665.$$

for the motor parameters of Joint 1, and with

$$P = \begin{pmatrix} 4.3420 & 0.1300 \\ 0.1300 & 0.4258 \end{pmatrix},\ \text{and}\ r = 0.0265.$$

for the motor parameters of Joint 2.

A trapezoidal velocity profile trajectory is set as the desired one. Figure 1 illustrates the simulation results of position (left panel), velocity (central panel) and control signal (right panel) corresponding to Joint 1. At the top, the performance of (11) with the PD controller (13) with $k_p = k_1$ and $k_d = k_2$ is shown. One observes that the PD controller induces a not desired behavior in the velocity of the joint, which directly induces an over-amplified control signal on the time-intervals where the chirp signal is

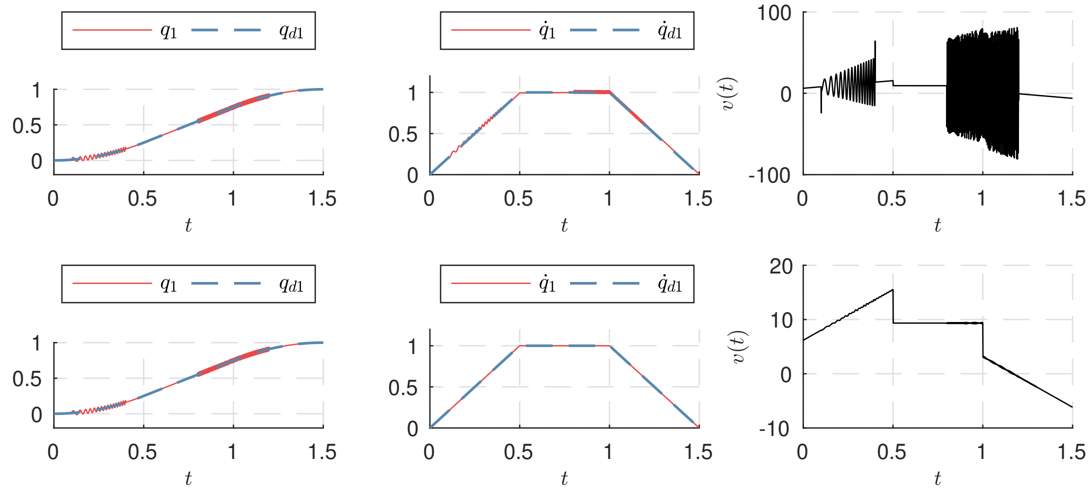

Fig. 1. Simlulation results for Joint 1 with PD controller (13) with $k_p = k_1$ and $k_d = k_2$ (top) and with controller (11) (bottom). Position, velocity and signal control are respectively shown from left to right.

added. The latter is expected as the derivative term is obtained from differentiation of the position error. At the bottom, the performance of (11) with the delay-based controller (12) is depicted. The performance of the controller in tracking the desired trajectory is similar to the PD controller with the noticeable difference that the control signal is not over-amplified despite the emulated noisy position measures.

The simulation results corresponding to the Joint 2 are shown in Figure 2 for the sake of completeness. There, the effect of the noisy position measures in the PD controller is sharpest. This can be appreciated in the amplitude of the control signal.

4. CONCLUDING REMARKS

A delay-based controller is proposed for the joint control of robot manipulators. Based on a new proposed functional, it is proved that with the class of controller (12) the dynamics of each joint can be practically stabilized in the sense of Definition 1. Simulation results show the benefit of the proposed scheme in comparison with a classical PD control when there is no access to velocity of the joint and the position measures might be corrupted by noise.

Control law (12) requires measures of position of each joint at present and delayed states. For the digital implementation of the control signal it is convenient to choose the value of the delay h as a multiple of the discretization step. Notice also that obtaining the delayed measure at the given (small) delay demands few memory allocation, which ease the implementation with commercial low-cost hardware.

Finally, it is important to mention that functional (8) belongs to the class of functionals that are at the basis of the very recent results presented in (Gomez et al., 2021). This enables taking the analysis performed in this paper as the starting point of application of state of the art techniques based on Sliding Mode Control for systems with delays in order to achieve asymptotic stability of the error dynamics. The latter is part of ongoing research work.

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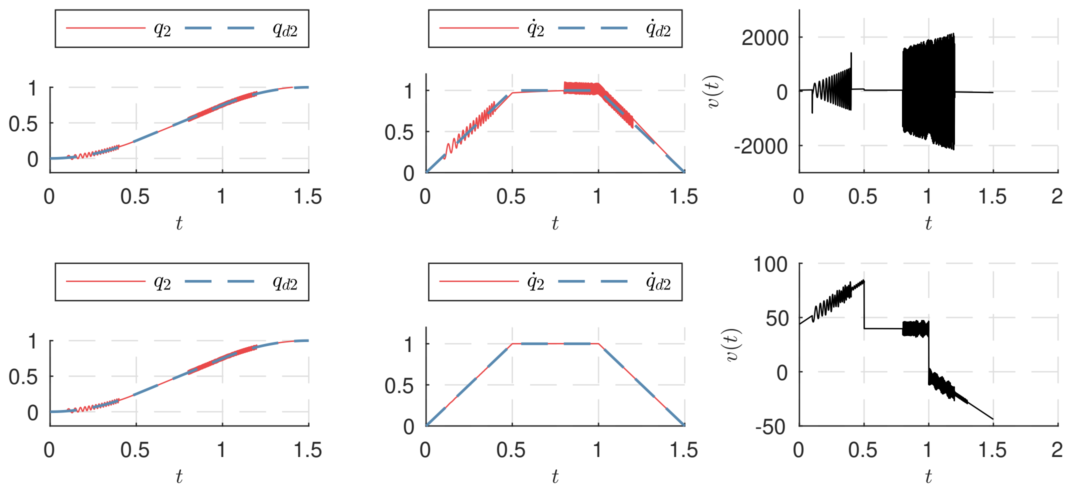

Fig. 2. Simlulation results for Joint 2 with PD controller (13) with $k_p = k_1$ and $k_d = k_2$ (top) and with controller (11) (bottom). Position, velocity and signal control are respectively shown from left to right.

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