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Towards the control of heat exchangers based on thermodynamic principles

Juan Paulo García Sandoval *

* *Departamento de Ingeniería Química, Universidad de Guadalajara. Jalisco, México, (e-mail: paulo.garcia@cucei.udg.mx)*

Abstract: In this contribution, a thermodynamics based regulation problem is proposed to control a counter flow shell-and-tube heat exchanger. To regulate the system, an output error, correlated with the total entropy production inside the heat exchanger, is proposed; this error depends on the output variable, its reference, the control variable and the measured disturbances, therefore a dynamical controller with a PID behavior is derived as a preliminary solution of the proposed regulation problem. Finally, this controller is tested via numerical simulation showing satisfactory stability properties.

Keywords: heat exchangers, entropy, distributed parameter systems, process control, temperature control

1. INTRODUCTION

In recent years, physics-based control design for the control of physical processes has attracted attention, because the main objective of this approach is to understand how the physical phenomena produces specific dynamical behaviors and this knowledge allows to improve the control strategies. In particular, it has been possible to systematize the analysis of mechanical and electrical systems using energy and power based port-hamiltonian formulation (Maschke et al., 1998; Ortega et al., 2002); however, for chemical and thermal processes the use this approach has not been completely fruitful, because not only energy and power come into play to describe the dynamics of this class of systems. On these cases, thermodynamics seems to be a more genuine candidate. In particular, the use of variables associated to entropy, for instance the entropy itself, the availability and the entropy production seems to be promising (Alonso and Ydstie, 1996; Ydstie, 2002). For instance, it has been shown that the entropy production is a Lyapunov function for closed thermodynamic consistent systems and a storage function for open thermodynamic consistent systems (García-Sandoval et al., 2015; García-Sandoval et al., 2016). Therefore, the use of entropy production for control design is appealing, because it is related to the cost of irreversibility in the processes (Dammers and Tels, 1974; Alonso and Ydstie, 2001; Alonso et al., 2002). However, this approach may only be successful if the model is thermodynamic consistent, adding more complexity to the analysis, and this problem increases when the system has a distributed-parameter model, as is the case with shell-and-tube heat exchangers.

Heat exchangers transfer thermal energy between fluids usually separated by a physical barrier. Although the selection of materials in the physical barrier allows heat transfer to be efficient, controlling the temperature of the fluid being cooled or heated at a specific and stable setpoint can be challenging (Lienhard and Lienhard, 2019). Many classical control strategies, such as feedback and cascade PID controllers, have been applied to regulate heat exchangers Padhee et al. (2011); Ahn et al. (2014); Kumar et al. (2021). However, fulfilling the efficiency requirements to avoid wasting energy and control effort adds more complexity to the control design. Therefore, the use of thermodynamics principles may improve the efficiency of the control of heat exchangers to decrease irreversibility (Jin et al., 2017). In this work, a thermodynamics based regulation problem is proposed to control a counter flow shell-and-tube heat exchanger, defining an output error correlated with the total entropy production inside the heat exchanger. A preliminary solution of the regulation problem is propose and tested via numerical simulation. The present document is organized as follows: in section 2 the model of the heat exchanger and its thermodynamic behavior is presented. Then, in section 3 the thermodynamics based regulation problem and a preliminary solution are proposed and evaluated via numerical simulation.

2. THE HEAT EXCHANGER

Consider the counter flow shell-and-tube heat exchanger shown in Figure 1 with length L, where two fluids exchange heat through a wall. Fluid 1 enters at $z = 0$ with a temperature $T_{1,in}$ and volumetric flow rate F_1, while the fluid 2 enters at $z = L$ with temperature $T_{2,in}$ and volumetric flow rate F_2. The linear velocities inside the

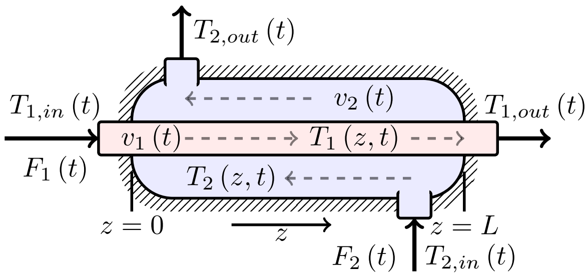

Fig. 1. Heat exchanger scheme.

heat exchanger are v_1 and v_2, with $v_i = F_i/A_i \in \mathbb{R}^+$, for $i = 1, 2$, where A_i is the constant cross-section areas of each fluid. In the present work it is assumed that, due to the magnitudes of fluid linear velocities inside the heat exchanger, heat convection has a bigger order of magnitude in comparison with heat conduction, therefore conduction can be neglected in the model proposed below.

2.1 The model

The dynamical behavior of the heat exchanger, associated to the internal energies per unit of mass (extensive properties) and temperatures (intensive properties) vectors of the two fluids, i.e. $\mathbf{u}(z,t) = \text{col}\{u_1(z,t), u_2(z,t)\}$ and $\mathbf{T}(z,t) = \text{col}\{T_1(z,t), T_2(z,t)\}$, respectively, is described by the following first-order partial differential equations (PDEs) :

$$\rho_1 \frac{\partial u_1}{\partial t} = -\rho_1 v_1 \frac{\partial u_1}{\partial z} + \frac{Q}{A_1 L} \tag{1a}$$

$$\rho_2 \frac{\partial u_2}{\partial t} = \rho_2 v_2 \frac{\partial u_2}{\partial z} - \frac{Q}{A_2 L} \tag{1b}$$

with the boundary conditions

$$T_1(0,t) = T_{1,in}(t) \tag{2a}$$

$$T_2(L,t) = T_{2,in}(t) \tag{2b}$$

and initial conditions

$$\mathbf{T}(z,0) = \mathbf{T}_0(z) \tag{3}$$

where $\mathbf{u}(z,t) \in \mathcal{H}^2\left[(0,L), \mathbb{R}^2\right]$ and $\mathbf{T}(z,t) \in \mathcal{H}^2\left[(0,L), \mathbb{R}^2\right]$, being $\mathcal{H}^2\left[(0,L), \mathbb{R}^2\right]$ the infinite-dimension Hilbert space on 2-dimensional like vector functions defined on the interval $[0,L]$, the axis position and time being denoted by $z \in [0,L] \subset \mathbb{R}$ and $t \in [0,\infty)$, respectively. ρ_i, for $i = 1, 2$, are the densities. Finally, $Q(z,t) = hA\left[T_2(z,t) - T_1(z,t)\right]$ is the heat transfer rate, with h as the heat transfer coefficient and $A = pL$ as the total heat transfer area between the two fluids with L as the total heat exchanger length and p the perimeter.

2.2 Thermodynamics

Under the assumption of incompressible fluids, according to thermodynamics, the correlation between the internal energy per unit of mass and temperature of each fluid is $du_i = c_{v,i} dT_i$, where $c_{v,i}$ is the heat capacity per unit of mass. Therefore (1) as a function of temperature becomes

$$\frac{\partial T_1}{\partial t} = -v_1 \frac{\partial T_1}{\partial z} + H_1(T_2 - T_1) \tag{4a}$$

$$\frac{\partial T_2}{\partial t} = v_2 \frac{\partial T_2}{\partial z} - H_2(T_2 - T_1) \tag{4b}$$

where $H_i = hp/\rho_i c_{v,i} A_i$, for $i = 1, 2$.

According to the second law of thermodynamics, the entropy function of each fluid, $s_i(z,t)$, is a real valued function at least two times differentiable, reaching a maximum in isolated systems. Under the classical non equilibrium thermodynamics assumption of local equilibrium (de Groot and Manzur, 1984), the Gibbs relation for an isochoric system without mass variations is $ds_i = du_i/T_i$. Therefore, considering (1), the entropy balances of each fluid are

$$\rho_1 \frac{\partial s_1}{\partial t} = -\rho_1 v_1 \frac{\partial s_1}{\partial z} + \frac{hp}{A_1} \frac{(T_2 - T_1)}{T_1} \tag{5a}$$

$$\rho_2 \frac{\partial s_2}{\partial t} = \rho_2 v_2 \frac{\partial s_2}{\partial z} - \frac{hp}{A_2} \frac{(T_2 - T_1)}{T_2} \tag{5b}$$

while the total entropy per unit of length define as $\hat{s}(z,t) = A_1 \rho_1 s_1(z,t) + A_2 \rho_2 s_2(z,t)$ has the dynamic behavior

$$\frac{\mathrm{d}\hat{s}}{\mathrm{d}t} = hp \frac{(T_2 - T_1)^2}{T_1 T_2} =: \hat{\sigma} \tag{6}$$

where $\frac{\mathrm{d}\hat{s}}{\mathrm{d}t} = \frac{\partial \hat{s}}{\partial t} + F_1 \rho_1 \frac{\partial s_1}{\partial z} - F_2 \rho_2 \frac{\partial s_2}{\partial z}$ is the comoving time derivative and $\hat{\sigma}(z,t)$ is the entropy production per unit of length inside the hear exchanger (Kjelstrup et al., 2010). Therefore, the total entropy production in the heat exchanger is

$$\Sigma(t) := \int_0^L \hat{\sigma}(z,t)\, dz \tag{7}$$

2.3 Steady state

At steady state (4) becomes

$$v_1 \frac{d\tilde{T}_1}{dz} = H_1\left(\tilde{T}_2 - \tilde{T}_1\right)$$

$$v_2 \frac{d\tilde{T}_2}{dz} = H_2\left(\tilde{T}_2 - \tilde{T}_1\right)$$

where $\tilde{[\cdot]}$ denotes steady state values. Notice from these equations that

$$\frac{d\tilde{T}_1}{dz} = \alpha \frac{d\tilde{T}_2}{dz}$$

where

$$\alpha = \frac{v_2 A_2 \rho_2 c_{v,2}}{v_1 A_1 \rho_1 c_{v,1}}. \tag{8}$$

Given the boundary conditions (2) and assuming that the heat capacities are constants, it holds that

$$\tilde{T}_{2,out} = \tilde{T}_{2,in} - \left(\tilde{T}_{1,out} - \tilde{T}_{1,in}\right)/\alpha. \tag{9}$$

On the other hand, according to (6) the steady state of the entropy production per unit of length is

$$hp\frac{\left(\tilde{T}_2-\tilde{T}_1\right)^2}{\tilde{T}_1\tilde{T}_2}=v_1A_1\frac{\rho_1c_{v,1}}{\tilde{T}_1}\frac{d\tilde{T}_1}{dz}-v_2A_2\frac{\rho_2c_{v,2}}{\tilde{T}_2}\frac{d\tilde{T}_2}{dz}$$

while the total entropy production according to (7) must be

$$\tilde{\Sigma}=v_1A_1\rho_1c_{v,1}\ln\left(\frac{\tilde{T}_{1,out}}{\tilde{T}_{1,in}}\right)+v_2A_2\rho_2c_{v,2}\ln\left(\frac{\tilde{T}_{2,out}}{\tilde{T}_{2,in}}\right)$$

or equivalently, with (9), a dimensionless total entropy production with respect to fluid 1 is

$$\frac{\tilde{\Sigma}}{F_1\rho_1c_{v,1}}=\ln\left[\frac{\hat{T}_{1,out}}{\hat{T}_{1,in}}\left(1-\frac{\hat{T}_{1,out}-\hat{T}_{1,in}}{\alpha\hat{T}_{2,in}}\right)^{\alpha}\right]. \quad (10)$$

Equation (10) represents a key variable associated with the irreversibility of the heat transfer that can be used to optimize the operation of the heat exchanger, therefore it will be used in the following section to define the thermodynamic based regulation problem.

3. THERMODYNAMIC BASED REGULATION PROBLEM

3.1 Problem statement

Consider the heat exchanger analyzed in section 2, the typical control objective is stated in the following regulation problem (RP):

RP To regulate the outlet temperature of fluid 1, $T_{1,out}(t)$ around the reference $T_{1,r}$ manipulating the linear velocity of fluid 2, $v_2(t)$, while $v_1(t)$, $T_{1,in}(t)$, and $T_{2,in}(t)$ are considered as measured disturbances with slow or step-like variations.

Many approaches can be applied to solve the regulation problem **RP**, typically considering a control error of the form $e=T_{1,out}(t)-T_{1,r}$, which is linear regarding the fluid 1 outlet temperature, however on this work it is proposed a nonlinear error related with the entropy production. The use of the entropy production for control design is appealing because it is related to the cost of irreversibility in the heat exchanger associate to the waste of energy. Thus, given the reference $T_{1,r}$, the dimensionless total entropy production with respect to fluid 1 described in (10) at the reference value is

$$\frac{\tilde{\Sigma}_r}{v_1A_1\rho_1c_{v,1}}=\ln\left[\frac{T_{1,r}}{T_{1,in}}\left(1-\frac{T_{1,r}-T_{1,in}}{\alpha_rT_{2,in}}\right)^{\alpha_r}\right]$$

where $\alpha_r=v_{2r}A_2\rho_2c_{v,2}/(v_1A_1\rho_1c_1)$, with v_{2r} as the linear velocity of fluid 2 required to reach the temperature $T_{1,r}$ at the outlet of fluid 1, given the inflow temperatures $T_{1,in}$ and $T_{2,in}$ and the linear velocity of fluid 1, v_1. From here, it is possible to define an error of the form

$$\varepsilon(t)=\ln\left[\frac{T_{1,out}(t)}{T_{1,r}}\frac{\left(\frac{\hat{T}_{2,out}}{T_{2,in}}\right)^{\alpha(t)}}{\left(\frac{\hat{T}_{2,out,r}}{T_{2,in}}\right)^{\alpha_r}}\right] \quad (11)$$

where $\hat{T}_{2,out}=T_{2,in}-(T_{1,out}-T_{1,in})/\alpha$ and $\hat{T}_{2,out,r}=T_{2,in}-(T_{1,r}-T_{1,in})/\alpha_r$. Notice that $\alpha(t)$ depends on time because, according to (8), α is proportional to $v_2(t)$, therefore the error (11) in addition to depend on the output variable and its reference, $T_{1,out}(t)$ and $T_{1,r}$, respectively, it also depends on the control variable and measured disturbances, $v_2(t)$ and v_1, $T_{1,in}$, and $T_{2,in}$, respectively. Then, the regulation problem **RP** can be specified as follows:

TBRP Given the outlet temperature of fluid 1, $T_{1,out}(t)$ and its reference $T_{1,r}$, the thermodynamics based regulation problem consists in guaranteeing that the steady state dimensionless total entropy production, given in (11), satisfies that $\lim_{t\to\infty}\varepsilon(t)=0$ by manipulating the linear velocity of fluid 2, $v_2(t)$, while $v_1(t)$, $T_{1,in}(t)$, and $T_{2,in}(t)$ are considered as measured disturbances with slow or step-like variations.

3.2 Proposed controller

Let consider an ideal PID controller of the form

$$v_2(t)=-K_p\left[\varepsilon(t)+\frac{1}{\tau_i}\int_0^t\varepsilon(\tau)\,d\tau+\tau_d\frac{d\varepsilon(t)}{dt}\right], \quad (12)$$

where K_p is the controller gain, τ_i is the integration time, and τ_d is the derivation time. Notice in (11) that $\varepsilon(t)$ depends on $\alpha(t)$ and $\alpha(t)$ depends on $v_2(t)$, therefore (12) contains $v_2(t)$ in both sides of the equality, thus it can not be easily applied to compute $v_2(t)$. However, from (12) it holds that

$$\frac{d\varepsilon(t)}{dt}=-\frac{\frac{v_2(t)}{K_p}+\varepsilon(t)+\frac{1}{\tau_i}\int_0^t\varepsilon(\tau)\,d\tau}{\tau_d}. \quad (13)$$

In addition, assuming that v_1, $T_{1,in}$, and $T_{2,in}$ are approximately constant, the time derivative of (11) is

$$\frac{d\varepsilon}{dt}=\left(\frac{1}{T_{1,out}}-\frac{1}{\hat{T}_{2,out}}\right)\frac{dT_{1,out}}{dt}+\left[\ln\left(\frac{\hat{T}_{2,out}}{T_{2,in}}\right)+\frac{T_{1,out}-T_{1,in}}{\alpha\hat{T}_{2,out}}\right]\frac{d\alpha}{dt} \quad (14)$$

where $\frac{d\alpha}{dt}=\frac{A_2\rho_2c_{v,2}}{v_1A_1\rho_1c_{v,1}}\frac{dv_2}{dt}$. Combining (13) and (14) the time derivative of the linear velocity of fluid 2 is

$$\frac{dv_2}{dt}=-\frac{\frac{v_2}{K_p\tau_d}+\frac{\varepsilon}{\tau_d}+\frac{\zeta}{\tau_d\tau_i}+\left(\frac{1}{T_{1,out}}-\frac{1}{\hat{T}_{2,out}}\right)\frac{dT_{1,out}}{dt}}{\frac{T_{1,out}-T_{1,in}}{v_2\hat{T}_{2,out}}+\frac{1}{v_1}\frac{A_2\rho_2c_{v,2}}{A_1\rho_1c_{v,1}}\ln\left(\frac{\hat{T}_{2,out}}{T_{2,in}}\right)} \quad (15)$$

where $\zeta(t)$ allows to compute the integral $\int_0^t\varepsilon d\tau$ through the dynamics

$$\frac{d\zeta}{dt}=\varepsilon. \quad (16)$$

Given an initial condition for v_2, (15) and (16) can be integrated to give $v_2(t)$, allowing to compute $\varepsilon(t)$ with (11), and guaranteeing that the control variable, $v_2(t)$, behaves similar to an ideal PID controller with respect to error $\varepsilon(t)$. Then the adjustable control parameters are those of the PID controller, i.e. K_p, τ_i, and τ_d.

3.3 Simulation results

The behavior of the proposed controller composed of the dynamic (15) and (16) and the error (11) was tested using the parameters shown in table 1. The heat exchanger's PDEs (4) together with the boundary and initial conditions (2) and (3) and controller's dynamic (15) and error (11) were numerically solved with Python using a partial discretization method with 26 spatial nodes (including boundaries), with backward approximations of order $(\Delta z)^2$ for the convective term $-v_1 \frac{\partial T_1}{\partial z}$ and forward approximation, also of order $(\Delta z)^2$, for the convective term $v_2 \frac{\partial T_2}{\partial z}$. Thus, the PDEs and boundary conditions were reduce to a set of 50 ordinary differential equations that, together with (15), were solve using a Adams/BDF method with automatic stiffness detection and switching (Hindmarsh, 1983; Petzold, 1983). It is important to remark that, as a preliminary test of the proposed regulation problem, the time derivative $dT_{1,out}/dt$ is directly computed with the ODE of T_1 at $z = L$ from the discretization.

To test controller performance, the system underwent a set-point change and disturbances. The resulting closed-loop dynamical behavior is shown in Figs. 2, 3, and 4. In particular, Fig. 2 shows the time evolution of control associated variables; Fig. 2a shows the control variable, v_2, Fig. 2b shows the output variable, $T_{1,out}$, and Fig. 2c shows the error (11), ε. On the other hand, Figs. 3a and 3b show the 3D temperature profiles of T_1 and T_2, respectively, while Fig. 4 depicts the time evolution of the total heat exchanger entropy production (7).

Initially, the temperatures of both fluids were 298 K throughout the domain of z, the inflow temperature of fluid 1 was $T_{1,in} = 298\,\mathrm{K}$ and the set-point for fluid 1 was $T_{1,r} = 320\,\mathrm{K}$, while the fluid 2 linear velocity and inflow temperature were $v_1 = 0.05\,\mathrm{m/s}$ and $T_{2,in} = 400\,\mathrm{K}$, respectively (see Figs. 2b and 3); Immediately, the controller increased v_2 to decrease the initial error and after a transitory time, $T_{1,out}$ reaches the set-point (see Figs. 2a and 2c). Then, at $t = 15\,\mathrm{min}$ the set-point was

Table 1. Parameters of the heat exchanger and controller.

Parameter	Value	Parameter	Value
A_1	$0.00196\,\mathrm{m^2}$	A_2	$0.02937\,\mathrm{m^2}$
ρ_1	$800\,\frac{\mathrm{kg}}{\mathrm{m^3}}$	ρ_2	$1000\,\frac{\mathrm{kg}}{\mathrm{m^3}}$
$c_{v,1}$	$2.306\,\frac{\mathrm{kJ}}{\mathrm{kg\ K}}$	$c_{v,2}$	$4.186\,\frac{\mathrm{kJ}}{\mathrm{kg\ K}}$
p	$0.15865\,\mathrm{m}$	L	$1\,\mathrm{m}$
h	$2.5\,\frac{\mathrm{kW}}{\mathrm{m^2 K}}$	K_p	$2\,\frac{\mathrm{m}}{\mathrm{s}}$
τ_d	$1\,\mathrm{s}$	τ_i	$25\,\mathrm{s}$

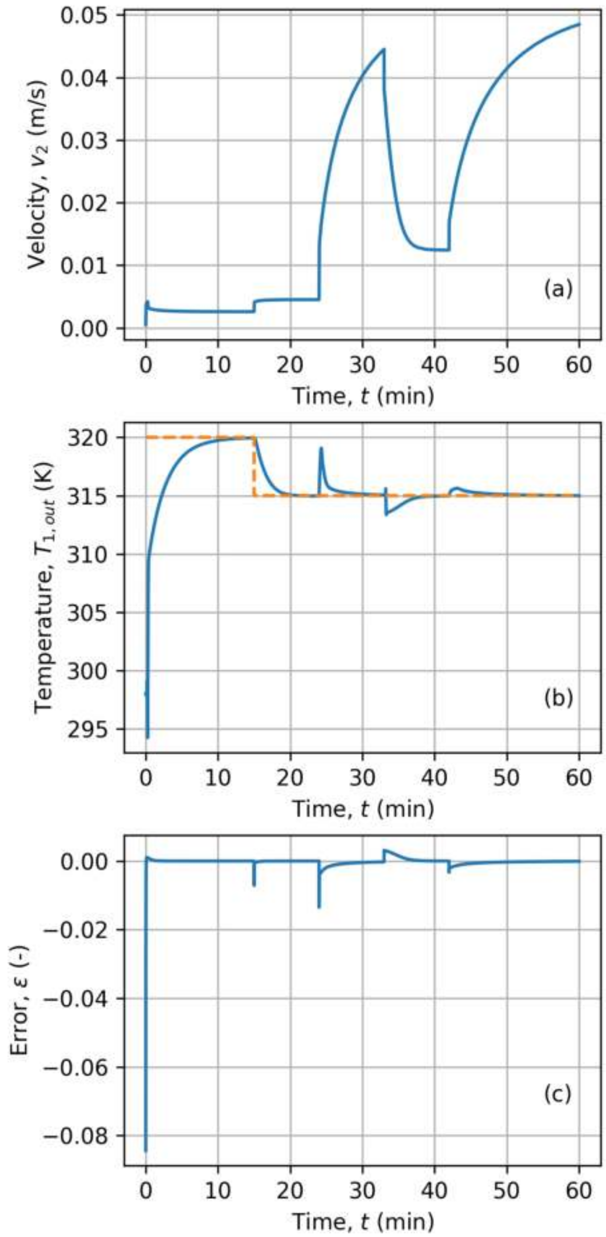

Fig. 2. Closed loop heat exchanger dynamic response. (a) Control variable: linear velocity of fluid 2, v_2. (b) Output variable: Outlet temperature of fluid 1, $T_{1,out}$. (c) Error, ε.

changed to $T_{1,r} = 315\,\mathrm{K}$ and the controller increased v_2 to decrease the error. At $t = 24\,\mathrm{min}$ a 20% perturbation was induced in the linear velocity of fluid 1, with $v_1 = 0.06\,\mathrm{m/s}$, therefore the controller increased drastically v_2 to compensate the disturbance. Then, at $t = 33\,\mathrm{min}$ and $t = 42\,\mathrm{min}$ the inflow temperatures were perturbed to $T_{1,in} = 390\,\mathrm{K}$ and $T_{2,in} = 300\,\mathrm{K}$, respectively, and the controller first decreased and later increased v_2 to rapidly recover from these disturbances (see Figs. 2 and 3). The

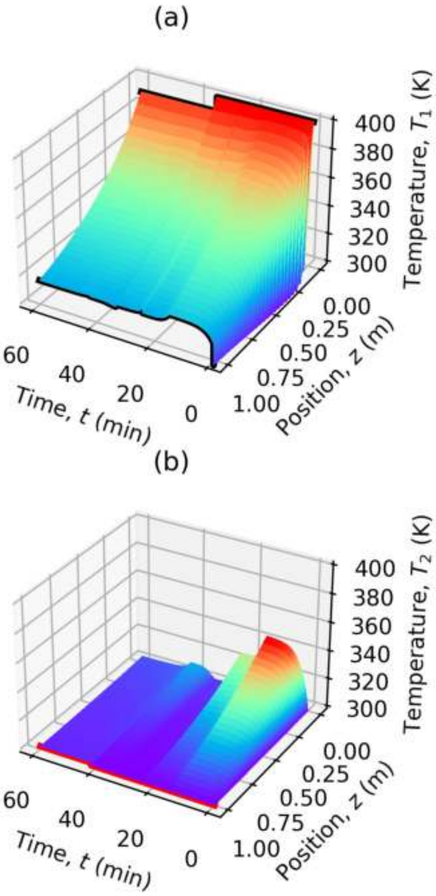

Fig. 3. Closed loop temperature profile inside the heat exchanger. (a) Temperature of fluid 1, $T_1(z,t)$. (b) Temperature of fluid 2, $T_2(z,t)$.

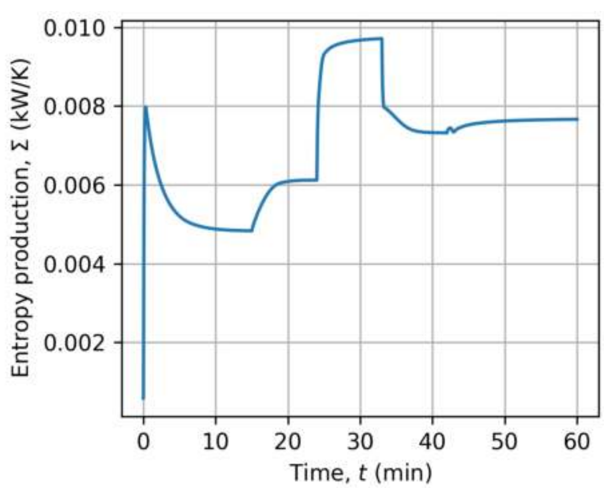

Fig. 4. Total heat exchanger entropy production closed loop dynamic response.

set-point change and disturbances applied to the system produced a variation in the total heat exchanger entropy production (see Fig. 4), however the controller was able to asymptotically stabilize the error ε associated to this total entropy production.

4. CONCLUSION

In the present work, a thermodynamics based regulation problem is proposed to control a counter flow shell-and-tube heat exchanger. The resulting controller, given by (11), 15, and (16), has a dynamic behavior that depends on the output variable ($T_{1,out}$), its reference ($T_{1,r}$), the control variable (v_2), and the measured disturbances ($v_1, T_{1,in}, T_{2,in}$), but it is independent of the spatial profile of temperatures inside the heat exchanger. This controller behaves as a PID with three adjustable parameters, K_p, τ_i, and τ_d. The controller's dynamic (15) and (16) was integrated directly with the PDEs (4) with boundary and initial conditions (2) and (3), where the time derivative $dT_{1,out}/dt$ was directly computed with the ODE of T_1 at $z = L$ from the discretization, however this is a preliminary work and as a future work it is proposed to robustify the controller by using a filter to estimate the time derivative $dT_{1,out}/dt$ from the measured temperature $T_{1,out}$ and to assume uncertainty in the model parameters. Finally, further analysis is required to evaluate if the proposed error (11), correlated with the total entropy production inside the heat exchanger, improves the efficiency of the heat exchanger operation, associated to the cost of irreversibility in the system, in comparison with a typical error of the form $\varepsilon = T_{1,out} - T_{1,r}$.

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REFERENCES

Ahn, J.K., So, G.B., Lee, J.Y., Lee, Y.H., So, M.O., and Jin, G.G. (2014). PID control of a shell and tube heat exchanger system incorporating feedforward control and anti-windup techniques. *Journal of Institute of Control, Robotics and Systems*, 20(5), 543–550. doi: 10.5302/j.icros.2014.14.0009.

Alonso, A.A. and Ydstie, B.E. (2001). Stabilization of distributed systems using irreversible thermodynamics. *Automatica*, 37(11), 1739–1755. doi:10.1016/S0005-1098(01)00140-6.

Alonso, A.A., Ydstie, B.E., and Banga, J.R. (2002). From irreversible thermodynamics to a robust control theory for distributed process systems. *Journal of Process Control*, 12(4), 507 – 517. doi:10.1016/S0959-1524(01)00017-8.

Alonso, A.A. and Ydstie, E.B. (1996). Process systems, passivity and the second law of thermodynamics. *Computers & Chemical Engineering*, 20, S1119–S1124. doi: 10.1016/0098-1354(96)00194-9.

Dammers, W. and Tels, M. (1974). Thermodynamic stability and entropy production in adiabatic stirred flow reactors. *Chemical Engineering Science*, 29(1), 83–90. doi:10.1016/0009-2509(74)85033-5.

de Groot, S. and Manzur, P. (1984). *Non-equilibrium thermodynamics*. Dover publications, New York.

García-Sandoval, J., González-Álvarez, V., and Calderón, C. (2015). Stability analysis and passivity properties for a class of chemical reactors: internal entropy production approach. *Computers & Chemical Engineering*, 75, 184–195. doi:10.1016/j.compchemeng.2015.01.021.

García-Sandoval, J., Hudon, N., Dochain, D., and González-Álvarez, V. (2016). Stability analysis and passivity properties for a class of thermodynamical systems: internal entropy production approach. *Chemical Engineering Science*, 139, 261–272. doi:10.1016/j.ces.2015.07.039.

Hindmarsh, A.C. (1983). Odepack, a systematized collection of ode solvers. *IMACS Transactions on Scientific Computation*, 1, 44–64.

Jin, Y., Gao, N., and Zhu, T. (2017). Controlled variable analysis of counter flow heat exchangers based on thermodynamic derivation. *Applied Thermal Engineering*, 129. doi:10.1016/j.applthermaleng.2017.10.025.

Kjelstrup, S., Bedeaux, D., Johannessen, E., and Gross, J. (2010). *Non-equilibrium thermodynamics for engineers*. World Scientific, Singapore.

Kumar, V.B., Sampath, D., Praneeth, V.N.S., and Kumar, Y.V.P. (2021). Error performance index based PID tuning methods for temperature control of heat exchanger system. In *2021 IEEE International IOT, Electronics and Mechatronics Conference (IEMTRONICS)*, 1–6. doi:10.1109/iemtronics52119.2021.9422613.

Lienhard, J.H. and Lienhard, J.H. (2019). *A heat transfer textbook*. Dover books on engineering. Dover Publ., Mineola, NY, 4 edition. Literaturangaben.

Maschke, B.M.J., Ortega, R., and Van Der Schaft, A.J. (1998). Energy-based lyapunov functions for forced hamiltonian systems with dissipation. 4, 3599–3604 vol.4. doi:10.1109/CDC.1998.761738.

Ortega, R., van der Schaft, A., Maschke, B., and Escobar, G. (2002). Interconnection and damping assignment passivity-based control of port-controlled hamiltonian systems. *Automatica*, 38(4), 585 – 596. doi:10.1016/S0005-1098(01)00278-3.

Padhee, S., Khare, Y.B., and Singh, Y. (2011). Internal model based PID control of shell and tube heat exchanger system. In *IEEE Technology Students' Symposium*, 297–302. doi:10.1109/techsym.2011.5783833.

Petzold, L. (1983). Automatic selection of methods for solving stiff and nonstiff systems of ordinary differential equations. *SIAM Journal on Scientific and Statistical Computing*, 4(1), 136–148. doi:10.1137/0904010.

Ydstie, B.E. (2002). Passivity based control via the second law. *Computers & Chemical Engineering*, 26(7-8), 1037–1048. doi:10.1016/S0098-1354(02)00041-8.

