

# Two-temperature Tracking Control of Batch Binary Batch Distillation Columns

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**Abstract:** In this paper the joint optimal operation and Lyapunov-based tracking control design for binary batch distillation columns with temperature measurements is addressed. The combination of standard batch distillation dynamics concepts with nonlinear constructive control theory allows to design a temperature driven control policy that enforces the distillate composition to reach its prescribed value in finite time, including two sequential operation mode. First, the system is operated at total reflux (i.e., with no production) for a period of time, to set a proper temperature (i.e., composition) profile in the column. Then, the production period is performed with a suitable temperature-to-temperature Lyapunov tracking controller that enforces the prescribed constant distillate purity policy, without significant overpurification. The methodology includes: (a) guides to select the proper sensor locations, (b) a temperature measurement-driven criteria to set the switching time from total reflux to production period, (c) the on-line generation of temperature policies that ensure constant distillate purity, and (d) a temperature-to-temperature Lyapunov tracking controller that enforces the prescribed constant distillate product purity policy. The methodology is tested with representative binary systems.

**Keywords:** Batch distillation, temperature control, Lyapunov function, operation design, tracking control

## 1. INTRODUCTION

Distillation is one of the most common operations in the chemical industry, as it is a required operation for the production of many intermediate and final products in the chemical and petrochemical industries. Distillation columns are energy intensive processes [Humphrey et al., 1991], in which the regulation of distillate and/or bottoms compositions is required. The related multivariable feedback control design for continuous distillation units has been extensively studied with a diversity of linear and nonlinear approaches [Castellanos-Sahagún et al., 2006a; Skogestad, 1997], including composition (C) measurements. Given the complexities associated with composition measurements (operation and installation costs, and dead times), temperature control has been a useful alternative [Skogestad, 1997] with the limitation that sensible control trays alone usually do not yield accurate product purity regulation. For that purpose, several cascade controllers with (secondary) temperature (T) measurement driven schemes have been proposed, and their related setpoints (i.e., primary loops) are given by: (a) steady-state material balance-based (MB) precomputed setpoint policies [Castellanos-Sahagún et al., 2005, 2006b-c], (b) delayed composition measurement driven setpoint generators (denoted cascade CT controller), and (c) additional temperature measurements are used to build the required setpoints [Castellanos-Sahagún et al., 2008] (denoted as cascade TT scheme). In [Castellanos-Sahagún et al. 2010], the rather simpler multitemperature scheme proposed in Castellanos-Sahagún et al., (2008) was redesigned successfully as a TT-Lyapunov controller that improved notably the previous performance. These schemes are capable of proper distillate and/or bottom purity regulation. However, the scheme [Castellanos-Sahagún 2005] requires the steady

state solution of a nonlinear model of the column, and the CT cascade scheme is affected by dead times related to C measurements.

Batch distillation columns (BDC) offer advantages over continuous columns, especially, their flexibility and the possibility of separating multicomponent mixtures using the same unit [Muhner et al., 1992; Diwekar, 1995]. The resulting operation and control problems are complex due to the resulting transient nonlinear and finite time dynamics, and they have been addressed in sequential manner. First, the operation is designed via open-loop optimization [Mujtaba et al., 1996], yielding a nominal output signal to be followed by means of a feedback control that is designed in a second stage, using either linear gain scheduled [Finefrock et al., 1994], nonlinear model predictive techniques [Bosley et al., 1992], or geometric techniques [Barolo et al., 1998].

Other works have studied the constant distillate composition problem, i.e., the design of a control scheme that computes a variable reflux rate policy that keeps constant distillate purity, by means of modeling error compensation techniques ([Monroy-Loperena et al., 2003, 2004] and references therein), driven by possibly uncertain and dead-time affected composition measurements. On the other hand, an observer-based optimal closed-loop operation technique for binary batch distillation columns has been proposed [Alvarez et al., 2005]. The main drawbacks of this technique are its high model dependency, the complexity of the required nonlinear observer, and noise propagation.

Starting with the results obtained with the temperature measurement driven (TT cascade) controller for continuous columns [6], and the developments in [Alvarez et al, 2005], a cascade TT scheme was applied successfully for a BDC system [Castellanos-Sahagún et al, 2013].

In this work, we propose an optimal control-operation design for batch distillation columns, considering the following objectives: the use of temperature measurements only, to design an operation policy that enforces the distillate composition to its target value in minimum time, and avoiding overpurification. The application on nonlinear constructive control techniques leads to a control structure selection methodology, implying that the column operation can be carried out in two periods: (a) a total reflux (no production) period, whose duration is determined with temperature measurements, and then, switching to (b) a closed-loop (CL) operation period, based on a Lyapunov controller driven by temperature measurements, similar to the one in Ref. [7]. The proposed scheme was applied successfully to binary BDC. The resulting closed loop behavior outperforms the ones obtained with previous cascade CT and TT controllers.

## 2. STATEMENT OF THE PROBLEM

Consider the  $N$ -tray binary BDC (depicted in Figure 1), with reboiler, condenser and accumulator vessel to collect the distillate product. From standard assumptions [Luyben, 1990; Castellanos-Sahagun et al, 2005, 2013; Alvarez et al, 2005] (constant pressure; equilibrium in all trays, equimolal overflows), the batch column dynamics are given by:

$$\begin{aligned}\dot{c}_i &= [L(m_{i+1})\Delta^+c_i - V\Delta^+E(c_i)]/m_i, & 0 \leq i \leq N-1 \\ \dot{c}_N &= [R\Delta^+c_N - V\Delta^+E(c_N)]/m_N, & \dot{c}_{N+1} = V[E(c_N) - c_{N+1}]/m_D \\ \dot{m}_i &= L(m_{i+1}) - L(m_i), & 1 \leq i \leq N-1; \\ \dot{m}_N &= R - L(m_N) & (1a-e)\end{aligned}$$

where:

$$\begin{aligned}\Delta^+c_i &:= c_{i+1} - c_i, & \Delta^+E(c_i) &:= E(c_i) - E(c_{i-1}) \\ E(c_{-1}) &:= c_0, & c_{N+1} &= c_D, & c_B &= c_0 \\ T_I &= \beta(c_I), & T_{II} &= \beta(c_{II})\end{aligned}$$

$$L(m_i) = \hat{R} + (m_i - \hat{m}_i)/\tau_i \quad (1a-d)$$

$c_i$  (or  $m_i$ ) is the light component mole fraction (or holdup) at the  $i$ -th tray,  $E$ ,  $\beta$  and  $L$  are the nonlinear liquid-vapor equilibrium, bubble point and hydraulic functions, respectively,  $T_I$ ,  $T_{II}$  are two temperature measurements (at locations to be determined) and  $\hat{(\cdot)}$  denotes the steady-state value of  $(\cdot)$  **at total reflux**. In compact notation, the column dynamics (1) are written as follows:

$$\dot{c} = F_c(c, m, R), \quad \dot{m} = F_m(m, R), \quad \psi = h(c) \quad (2a-b)$$

where:

$$c = (c_0, \dots, c_{N+1})', \quad m = (m_1, \dots, m_N)'$$

$$\psi = (T_I, T_{II})', \quad h(c) = [\beta(c_I), \beta(c_{II})]'$$

For BDC operations (i.e., during startup and production periods) the vapor flow rate  $V$  is kept constant, and the system is operated at total reflux, i.e.,  $R = V$ , until a switching time  $t_s$ . A temperature-based criterion to set  $t_s$  is required [Castellanos-Sahagun et al., 2013]. Then, the extraction period begins, with distillate product withdrawn at rate  $V-R$ , where the product composition must be maintained at the prescribed value  $\bar{c}_D$  by manipulating the reflux flow rate  $R$ , on the basis of two temperature measurements  $T_I$  and  $T_{II}$  (whose locations and possibly time-varying setpoints are to be determined). Usually, this constant distillate purity operation is maintained until a utility (i.e., profit) functional reaches its maximum.

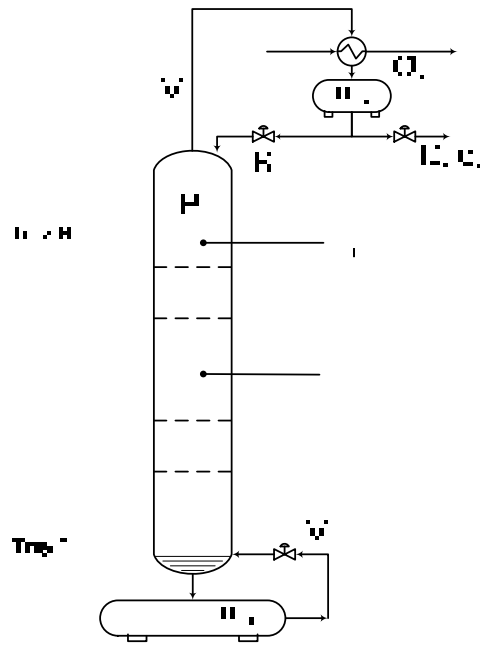


Fig. 1. Batch distillation column

Our BDC control problem consists in obtaining a temperature measurement-driven control scheme, consisting of:

- An *event controller* to determine the duration of the (non-productive) total reflux operation period,  $t_s$ , and
- A *Lyapunov-like control* scheme driven by two temperature measurements ( $T_I$ ,  $T_{II}$ ) at locations to be determined (see Figure 1) that manipulates the reflux flow rate  $R$ , in such a way that the distillate effluent composition ( $c_D$ ) is maintained at its prescribed value during the production period, minimizing/avoiding overpurification. The control scheme must be linear, with reduced model-dependency and simple construction and tuning guidelines. The behavior of the proposed temperature driven Lyapunov control scheme must be compared with the ones of its previously studied counterparts based on composition and/or temperature measurements.

### 3. CONTROL DESIGN

As explained previously, BDC operation requires two operation periods: (i) a total reflux period, whose duration must be set by an event controller, and (ii) an extraction period, with constant distillate purity and time varying reflux flow rate.

From the nonlinear control theory, we know that optimal nonlinear SF controllers [Freeman, et al., 1996; Sepulchre et al., 1997]: (i) are inherently robust and passive, (ii) cannot in general be constructed in analytic form via direct optimality, (iii) can be constructed in analytic form via inverse optimality by starting with a passive controller and verifying for which objective function the controller is optimal, and (iv) can be constructed, on the basis of non-passive models via backstepping procedure. From previous distillation column control studies [Castellanos-Sahagún et al., 2006; Monroy Loeppena 2004; Alvarez et al., 2005], we know that: (i) the behavior of a passive nonlinear SF controller can be recovered with a linear OF controller made of conventional PI components, given that the system relative-degree structure and observability property conditions are met.

Following these ideas, in Section 3.1 (i) a nonlinear passive (NLP) model is drawn for constructive control analysis, and (ii) the corresponding SF controller for constant distillate purity is recalled in Section 3.2, and it is reinterpreted as a material balance-like controller that can be used to generate a temperature tray policy. Then, In Section 3.3 the NLP model of Section 3.1 is realized in terms of a single input-two output (SI-20) model, that is used in Section 3.4 to obtain a linear model for the Lyapunov controller. In Section 3.5 we present the proposed OF Lyapunov control based on temperature measurements. The event controller is explained in Section 3.6.

#### 3.1 Nonlinear passive model

Here, a passive model for control design purposes is drawn. As it is known in distillation column control, the hydraulic dynamics are faster than composition dynamics [Levy et al., 1969; Skogestad 1997], so that they can be assumed in quasi-steady state in the design stage, and their effect must be accounted for in the tuning stage. Then, Eq. (2b) can be set as (3) with liquid flows given by (4):

$$\dot{m} = F_m(m, R) \approx 0; \quad L(m_i) \approx R, \quad 1 \leq i \leq N \quad (3-4)$$

The unique root of (4) for a given  $R$  is given by:

$$m_i^* = G_i(R) = (R - \hat{R})\tau_i + \hat{m}_i, \quad 1 \leq i \leq N \quad (5)$$

where  $\tau_i$  is the tray hydraulic time constant. Substituting Eq. (4) and (5) in Eq. (1a) yields the *reduced-order passive model*:

$$\dot{c}_0 = [R(c_1 - c_0) - V \Delta^+ E(c_0)] / \hat{m}_B := f_0(c, R) \quad (6a)$$

$$\dot{c}_i = [R \Delta^+ c_i - V \Delta^+ E(c_i)] / [(R - \hat{R})\tau_i + \hat{m}_i] := f_i(c, R) \quad (6b)$$

$$\dot{c}_D = V[E(c_N) - c_D] / \hat{m}_D := f_{N+1}(c, R) \quad (6c)$$

$$\psi_I = \beta(c_I), \quad \psi_{II} = \beta(c_{II}) \quad (6d)$$

#### 3.2 Nonlinear SF passive controller

For the sake of analysis, consider the dynamics of the distillate composition, Eq (1b), and observe that, for constant distillate composition  $c_D$  at the desired value  $\bar{c}_D$ , it is required that  $E(c_N) = \bar{c}_D$ , which yields the constant value of the  $N$ -th tray composition

$$\bar{c}_N = E^{-1}(\bar{c}_D) \quad (7)$$

Consequently, the regulation of the  $N$ -th tray composition (or temperature) around the desired value  $\bar{c}_N$  (or  $\bar{T}_N$ ) implies constant distillate purity. Then, the SF control problem for constant distillate purity can be solved in this way, i.e., by using backstepping [Sepulchre et al., 1997]. The resulting SF control problem resumes to keeping constant the  $N$ -th tray composition by regulating the reflux flow rate, and the corresponding control law has been drawn previously [Castellanos Sahagún and Alvarez 2006a], and is given by:

$$R = \{[-k_I(c_N - \bar{c}_N)(\hat{m}_N - \hat{R}_{th})] + [V(E(c_N) - E(c_{N-1}))]\} / (c_D - c_N) \quad (8a-b)$$

that results from the enforcement of the stable first order dynamics for the composition error regulation (8b), where  $k_I$  is a positive control gain. The corresponding zero dynamics (ZD) controller is given by:

$$R_\zeta = [V(E(\bar{c}_N) - E(c_{N-1}))] / (c_D - \bar{c}_N) \quad (9)$$

that can be reinterpreted as a material balance controller [11, 16], i.e., the controller input  $R_\zeta$  exactly compensates the material balance so that  $c_N = \bar{c}_N$ . Additionally, the column (6) under the ZD controller (9) and the restriction (7) yields the system ZD that are assumed stable [16].

The preceding nonlinear passive controller is robust, but requires the states of the reduced system (6). This controller is to be reinterpreted in the following sections as a linear decentralized cascade controller driven by two temperature measurements.

#### 3.3 Linear Passive model

To simplify the control design, first consider the following coordinate change:

$$x_I = \beta(c_I), \quad x_{II} = \beta(c_{II}) \quad (10)$$

and rewrite the reduced passive model (6) as follows:

$$\dot{x}_I = g_I(x_I, x_{II}, x_z, R), \quad y_I = x_I \quad (11a-b)$$

$$\dot{x}_{II} = g_{II}(x_I, x_{II}, x_z, R), \quad y_{II} = x_{II} \quad (11c-d)$$

$$\dot{x}_z = f_z(x_I, x_{II}, x_z, R) \quad (11e)$$

$\dot{m}_B = -V + R$ , where:

$$g_I(x_I, x_{II}, x_z, R) = \beta'(c_I)f_i(c, R)$$

$$g_{II}(x_I, x_{II}, x_z, R) = \beta'(c_{II})f_{II}(c, R)$$

$$\mathbf{x}_z = (c_1, c_2, \dots, c_{II-1}, c_{II+1}, \dots, c_{N-1}, c_D)$$

$$\mathbf{f}_z = (f_1, f_2, \dots, f_{II-1}, f_{II+1}, \dots, f_{N-1}, f_D)'$$

$x_I$  and  $x_{II}$  are the temperatures at the measurement trays after a bubble point function-based coordinate change,  $x_z$  are the remaining compositions, and  $R$  is the reflux flow rate input.

Observe that the model (6) or (11) has relative degrees (RD's) equal to one for both inputs, and any choice of measured temperatures, excepting the distillate. Assuming that the CL column forces a unique material balance [Alvarez et al., 2005], then the resulting system's zero dynamics are stable. Thus, system (6) with the ZD control (9) is *passive*, implying that related robust nonlinear SF control problem is solvable.

### 3.4 Linear Model

Next, a linear-decentralized model with reconstructible load inputs is set for OF control design purposes. Following previous developments in two-point temperature and composition-temperature cascade control designs [5-7], on the basis of the preceding RD structure and the linearity-decentralization feature specifications for the OF control design, rewrite the passive model (11) as follows:

$$\dot{x}_I = a_I R + b_I, \quad b_I = \varphi_I(x_I, x_{II}, x_z, R), \quad y_I = x_I \quad (12a-c)$$

$$\dot{x}_{II} = a_{II} R + b_{II}, \quad b_{II} = \varphi_{II}(x_I, x_{II}, x_z, R), \quad y_{II} = x_{II} \quad (12d-f)$$

$$\dot{x}_z = f_z(x_I, x_{II}, x_z, R) \quad (12g)$$

where  $\Delta^+ \tilde{T}_k$  is an average temperature gradient at the  $k$ -th stage during the extraction period, and

$$\varphi_I(x_I, x_{II}, x_z, R) = g_I(x_I, x_{II}, u, d) - a_I R, \quad a_I = (\Delta^+ \tilde{T}_I / \tilde{m}_I)$$

$$\varphi_{II}(x_I, x_{II}, x_z, R) = g_{II}(x_I, x_{II}, u, d) - a_{II} R, \quad a_{II} = (\Delta^+ \tilde{T}_2 / \tilde{m}_{II})$$

The inputs ( $b_I$ ,  $b_{II}$ ) satisfy the matching condition [Herman & Krener, 1977], as they enter in the same channels as the control inputs. Since the temperature states  $x_I$  and  $x_{II}$  are measured, the load disturbances ( $b_I$ ,  $b_{II}$ ) are instantaneously observable, as they can be reconstructed from the inputs and the measured output derivatives, according to the expressions:

$$b_I = \dot{y}_I - a_I R, \quad b_{II} = \dot{y}_{II} - a_{II} R, \quad (13)$$

These load inputs can be quickly reconstructed with linear-decentralized reduced-order filters. Therefore, the dynamics (12g) is not necessary. Accordingly, our *model for OF control design* is given by:

$$\dot{x}_I = a_I R + b_I, \quad \dot{b}_I \approx 0, \quad y_I = x_I \quad (14a)$$

$$\dot{x}_{II} = a_{II} R + b_{II}, \quad \dot{b}_{II} \approx 0, \quad y_{II} = x_{II} \quad (14b)$$

where ( $b_I$ ,  $b_{II}$ ) are unknown-reconstructible load inputs.

### 3.5 Linear OF Lyapunov Control

To build the controller, consider the dynamics of the primary temperature  $T_I$  (i.e., the one associated to the  $N$ -th tray). For this aim, impose the closed-loop first order regulation dynamics (15) ( $k_I > 0$  is the corresponding control gain) in (14a), and solve for  $R$  to obtain the "virtual" controller  $R^*$  [eq. (16)], which in turn is applied to the dynamics (14b) to obtain the "sensitive" tray temperature SP generator, Eq. (17):

$$\dot{T}_I = -k_I (T_I - \tilde{T}_I), \quad k_I > 0 \quad (15)$$

$$R^* = [-k_I (T_I - \tilde{T}_I) - b_I] / a_I \quad (16)$$

$$\dot{T}_{II}^* = -(a_{II}/a_I) [k_I (T_I - \tilde{T}_I) + b_I] + b_{II}, \quad T_{II}^*(0) = T_{IIo}^* \quad (17)$$

Regard the candidate Lyapunov function (18)

$$V = [(e_I)^2 + (e_{II})^2] / 2 > 0, \quad (18)$$

where  $e_I = T_I - \tilde{T}_I$ ;  $e_{II} = T_{II} - T_{II}^*$ , and write its dissipation (19) along the column motion:

$$\dot{V} = e_I [a_I (R^* + \tilde{R}) + b_I] + a_{II} e_{II} \tilde{R} \quad (19)$$

where the actual controller has been rewritten as the sum of the virtual controller  $R^*$  and a deviation  $\tilde{R}$

$$R = R^* + \tilde{R}$$

Now, perform *backstepping* by transferring the term  $e_I a_I$   $\tilde{R}$  from the first to the second term of Eq. (19) [21]:

$$\dot{V} = e_I [a_I R^* + b_I] + [a_I e_I + a_{II} e_{II}] \tilde{R} \quad (20)$$

and enforce the (implicit) control expressions

$$a_I R^* + b_I = -k_I x_I, \quad \tilde{R} = -k_{II} [a_I e_I + a_{II} e_{II}] \quad (21)$$

to enforce the negative dissipation ( $k_I$ ,  $k_{II}$  are control gains):

$$\dot{V} = -k_I (x_I)^2 - k_{II} [a_I e_I + a_{II} e_{II}]^2 < 0 \quad (22)$$

Now, let us recall (14b-c), and invoke (21) to obtain the Lyapunov controller:

$$R^* = [-k_I e_I - b_I] / a_I \quad (23a)$$

$$\dot{T}_{II}^* = a_{II} R^* + b_{II}, \quad T_{II}^*(0) = T_{IIo}^* \quad (23b)$$

$$R = R^* - k_{II} [a_I e_I + a_{II} e_{II}] \quad (23c)$$

The combination of these components with two first order filters (24a-d) to estimate the loads ( $b_I$ ,  $b_{II}$ ) yields the temperature driven BDC Lyapunov control in internal model control form:

$$\dot{w}_I = -\omega_o (w_I + \omega_o T_I + a_I R), \quad \hat{b}_I = w_I + \omega_o T_I \quad (24a-b)$$

$$\dot{w}_{II} = -\omega_o (w_{II} + \omega_o T_{II} + a_{II} R), \quad \hat{b}_{II} = w_{II} + \omega_o T_{II} \quad (24c-d)$$

$$R^* = [-k_I e_I - \hat{b}_I] / a_I \quad (24e)$$

$$\dot{T}_{II}^* = a_{II} R^* + \hat{b}_{II}, \quad T_{II}^*(0) = T_{IIo}^* \quad (24f)$$

$$R = R^* - k_{II} [a_I e_I + a_{II} e_{II}] \quad (24g)$$

where  $\omega_o$  is an observer gain.

Observe that the generated temperature setpoint  $T_{II}^*$  given by Eq (24f) is time-varying, while the setpoint  $\bar{T}_I$  is constant. In this way, the distillate purity can be kept constant, because the computation of the time-varying temperature policy  $T_{II}^*$  and the resulting reflux flow rate can be performed on-line, implying that the proposed controller has reduced model dependency when compared to the previous observer based SF controllers.

### 3.6 Event Controller and Switching Time

Regarding the switching from total reflux operation to the application of the Lyapunov controller (24), we must ensure that the distillate is not overpurified. After a total reflux period, proper, monotonic composition (and temperature) column profiles are obtained. Then, after a sufficiently large time at total reflux, the composition and the temperature in the top tray will reach the desired values  $\bar{c}_N = E^{-1}(\bar{c}_D)$ ,  $\bar{T}_I = \bar{T}_N = \beta(\bar{c}_N)$ . If we maintain total reflux operation after this moment, the composition (or temperature) in the N-th tray will surpass the desired target value  $\bar{c}_N$  (or  $\bar{T}_N$ ), implying an overpurification of the distillate product. Consequently, we define the switching time  $t_s$ , as the time required to operate the column at total reflux, until  $T_I = \bar{T}_N$ . In this way, the proposed event controller

$$R = V, \quad \text{for } 0 \leq t < t_s, \quad (24g)$$

where  $t$  is the required time for  $T_I \approx \bar{T}_N = \beta(\bar{c}_N)$ .

In other words, the duration of the total reflux period can be established by measuring the Nth tray temperature. This is different from previous works, that regarded the distillate temperature as the variable for the switching time.

Summarizing, the control scheme for the BDC is given the combination of Eq. (25) and Eqs. (24), shown in Eq. (26):

Total Reflux Period: Event controller:

$$\text{For } 0 \leq t < t_s: \quad R = V \quad (26a)$$

Extraction Period: Lyapunov TT controller:

$$\text{For } t \geq t_s \quad R^* = [-k_I e_I - \hat{b}_I]/a_I \quad (26b)$$

$$\dot{T}_{II}^* = a_{II} R^* + \hat{b}_{II}, \quad T_{II}^*(0) = T_{IIo} \quad (26c)$$

$$R = R^* - k_{II} [a_I e_I + a_{II} e_{II}] \quad (26d)$$

$$\text{Where: } \dot{w}_I = -\omega_o(w_I + \omega_o T_I + a_I R), \quad \hat{b}_I = w_I + \omega_o T_I \\ \dot{w}_{II} = -\omega_o(w_{II} + \omega_o T_{II} + a_{II} R), \quad \hat{b}_{II} = w_{II} + \omega_o T_{II}$$

### 3.7 Controller implementation

1. Control Scheme (26) is based only on temperature measurements, i.e., it is not affected by measurement delays nor dead times. Consequently, the filter-based estimation of

the load disturbances ( $\hat{b}_I, \hat{b}_{II}$ ) will be limited only by the high frequency holdup dynamics, and the measurement noise.

2. The calculation of the virtual control  $R^*$  requires the inverse of the coefficient  $a_I$ . In high-purity columns, this can be a very large number, and consequently, this control structure can be prone to amplify the measurement noise. For this reason, the signal to noise ratio should be characterized, and the controller ( $k_I, k_{II}$ ) and observer ( $\omega_o$ ) gains must be detuned accordingly.

3. The controller (26) are expressed in the Internal Model Control form, and consequently, it is equipped naturally with an anti-windup control scheme, i.e., they can tolerate actuator saturations without significant performance degradation.

4. The proposed scheme requires the selection of an additional temperature sensor. From previous works [4-6,17], we recommend to perform some simulation work, and select the one with the largest tray-to-tray temperature gradients of the BDC during the total reflux period.

5. The controller and filter tuning can be executed in a systematic manner by following the tuning guidelines shown in [6-7]. Additionally, the filter gain  $\omega_o$  is chosen equal for all the filters. In addition, this assumption can be relaxed to assign different values to each filter's gain.

## 4. APPLICATION EXAMPLE

The studied BDC separates a methanol-water mixture. There are  $N = 8$  trays, the initial load is  $m_{Bo} = 12$  Kmole at composition  $c_F = 0.25$ , the vapor flow is  $V = 5400$  mol/h, the tray, hydraulic parameter set is  $(1/\tau_m, a, b) = (1000 \text{ 1/h}, 5400, 30)$ , the condenser holdup is  $m_D = 250$  mol, and the nominal product composition is  $\bar{c}_D = 0.985$ .

*Total reflux period.* At the beginning of the batch operation, total reflux policy is required. The switching time (i.e., the duration of the total reflux period) was chosen as follows:

when the top tray temperature  $T_I = \beta(\bar{c}_N)$ . In this case, the switching time was 17.7 min. After that, the Lyapunov controller Eq (26) was applied.

*Proposed Control structure.* As stated previously, the target temperature measurement corresponds to the one in the N-th tray. To place the secondary temperature measurement, some simulation work (not shown here) was required, in which the SF controller (8) was applied to the BDC after the total reflux period. It was found that the trays with the maximum gradients over the batch column operation are trays 2, 3 and 4. Here we chose tray 3 as the complimentary temperature measurement.

*Control tuning.* Following the tuning guides given in [6-7, 17], the gains  $(\omega_o, k_{II}, k_I) = (120, 40, 20)$  1/hr were used:

*Control behavior.* The application of the total reflux period until  $t = 17.7$  min and the latter application of the proposed Lyapunov controller Eq (26) are depicted in Fig. 2. To

illustrate the advantages of the proposed Lyapunov controller over conventional cascade schemes, the BDC was also tested with a previous cascade CT [6] and TT [17] cascade schemes. For the CT cascade scheme [6], the N-th tray composition measurement, was affected by 1 min dead time. Because of the presence of dead time, in the case of the CT controller requires significant detuning to attenuate the oscillatory response. For the latter two cases, the switching time was 21.6 min. The temperature response is depicted in Fig. 2C, showing that: at the beginning of the production period .17.7 min for the proposed scheme, the temperature at tray 8 is driven smoothly to its desired value  $T_1 = 338.5$  °K, and consequently, the distillate composition reached in 40 minutes its desired value ( $c_D = 0.985$ ) (see Fig. 2A), with no overpurification. For the schemes proposed in [Alvarez et al., 2005] and [Castellanos-Sahagún and Alvarez al, 2013],  $t_s = 21.7$  min, yielding overshoots in  $T_1$  and in distillate purity. The CT controller shows some oscillations in product purity. These results show that the proposed scheme outperforms the CL behaviors obtained previously.

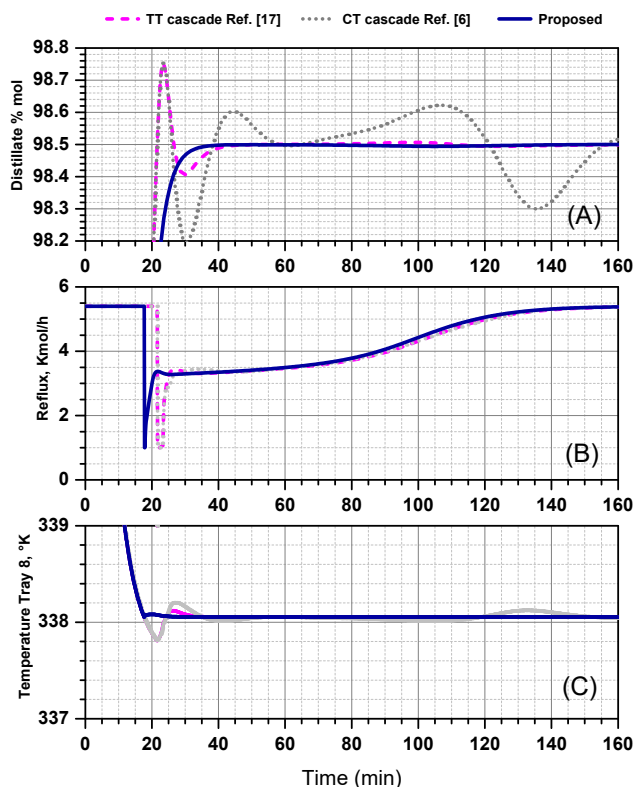


Fig. 2. CL behavior of the BDC: (A) Distillate (B) Control effort (C) Nth Tray temperature

## 5. CONCLUSIONS

The problem of designing a rather simple temperature measurement-driven Lyapunov controller for binary BDC has been addressed, by combining constructive control tools, conventional BDC notions and simple linear controls and

filters. The approach recalls the previously established solvability conditions for the nonlinear SF control problem for the regulation of the N-th tray composition, and improves the previous switching time criterion, to avoid product overpurification. The approach was successfully applied to an 8-tray methanol –water BDC, yielding the desired behavior, i.e., constant distillate purity and smooth control effort, matching the behavior of its more complex nonlinear counterparts, and outperforming existing CT and TT cascade control schemes. The formalization of the robust functioning (with stability proofs for nonautonomous nonlinear systems in the light of passivity and detectability properties) of the proposed design and its extension to multicomponent mixtures are matters of ongoing research.

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