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BOND GRAPH METHODOLOGY BASED ON THE POSITION OF THE CENTERS OF MASS APPLIED TO SMALL WIND TURBINES

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Abstract: A methodology is developed to obtain Bond Graph models from the position vectors of centers of mass and rotation matrices in mechanical systems. In this methodology, the equations of tangential velocity, the rotation matrix and the inertia associated with the centers of mass and potential energy of the physical system are used. This method is applied to the mechanical part of a small fixed blade wind turbine and a Rotational-Rotational (RR) robot. The objectives are to model the yaw dynamics and the rotational speed of the rotor in the wind turbine and the rotational angles of the RR robot. The implementation and comparison of the results is carried out, both in 20-sim and in Matlab-Simulink from the Lagrange equations and Simscape Toolbox, these results are compared to verify the proposed models. For the yaw angle model, the wind turbine steering vane is removed and a mechanism is proposed that allows the wind turbine to move to the desired angle, around the vertical axis.

Keywords: Bond Graph Methodology, Tangential Speed, Rotation Matrix, Inertia, Wind Turbine.

1. INTRODUCTION

In this work, a methodology is proposed that allows the construction of the Bond Graph model through the position of the centers of mass of physical systems, for which it is proposed to use the equations of tangential and angular velocities and the inertia associated with these centers of mass. The methodology can be applied to mechanical systems, in the specific case of this article it is applied as an example to the mechanical part of small fixed-blade wind turbines and a RR robot. This methodology is intended to validate through computational simulations and compare them with simulations obtained through the Euler-Lagrange equations, Goldstein et al. (2001) and Simscape Toolbox. In addition, there are publications on the modeling of Bond Graph from position vectors for two-dimensional systems, Mukherjee and Karmakar (2000) and three-dimensional systems where Newton-Euler equations are used for the construction of the Bond Graph model, Zeid and Chung (1992) . The proposed methodology does not require the transformations of the Euler angle between rigid bodies as in the work of Agarwal et al. (2012). The increase in the demand for electrical energy, the decrease in fossil fuels and the damage to the environment, have considerably increased the use of renewable energies, including the production of electricity through wind energy, this has given the need to achieve more efficient systems through new simplified models, to this need is added the obtaining of models through Bond Graph as one of the existing modeling techniques. In the literature there are different Bond Graph models for wind turbines, most of them are focused on obtaining the relationships that exist between the blades, the hub and the generator torque, Gonzalez and Lopez (2017), Khaouch et al. (2016), where the ratio of the yaw angle is obtained mathematically, Bakka and Karimi (2011), Agarwal et al. (2012).

This article is developed as follows, first in section 2 the proposed methodology is developed step by step, second in section 3 and 4 it is applied to the mechanical part of the wind turbine and the RR robot, respectively and as a finally the comparisons between the results are made, by computational simulations.

2. METHODOLOGY

The proposed methodology is built on a basic procedure to obtain the Bond Graph diagram of the desired system from the position vectors of the centers of mass and rotation matrices; tangential velocity equations, rotation matrices, and inertia associated with centers of mass, as well as the potential energy of the physical system, are used. For this methodology, a Force-Voltage analogy is selected (F-V) Karnopp et al. (2012), but it is applicable for the Force-Current analogy (F-i), it is only necessary to readapt each analogy to the generalized variables of our system as shown in Table 1

Table 1. Bond Graph Analogies

System	"e"	"f"	"p"	"q"
F-V				
Translational Mechanic	F	v	ρ	x
Rotational Mechanic	τ	ω	ρ_ω	θ
Electric	V	i	λ	q_e
F-i				
Traslational Mechanic	v	F	x	ρ
Rotational Mechanic	ω	τ	θ	ρ_ω
Electric	V	i	λ	q_e

e-Effort	τ-Torque	x-Position
f-Flow	V-Voltage	θ-Angle
ρ-Moment	v -Linear velocity	q_e-Electric charge
q-Displacement	ω- Angular Velocity	λ-Flow
F-Force	i-Current	ρ_ω-Angular Momentum

Other considerations are that in the F-V analogy, the 1-junctions are a summation of forces, the inertias are related to masses (I:m) and the capacitances are related to the stiffness constants (C:1/k), but in the F-i analogy the 1-junctions are summations of velocities, the inertias are related to the stiffness constants (I:1/k) and the capacitances are related to the masses (C:m).

2.1 Steps of the Methodology.

The steps of the proposed methodology are listed below:

Step 1: Identification of the centers of mass and obtaining the position vectors of each one.

Analyzing the desired physical system to identify their centers of mass to get their position vectors, which can be obtained in the following ways:

a) Denavit-Hartenberg Convention (Includes rotation matrices)

b) System geometry

The use of the two methodologies varies depending on the complexity of the system and user familiarity with the convention or system geometry. The position vectors are represented by:

$$r_i = \left[x_i\left(q\right)\ y_i\left(q\right)\ z_i\left(q\right)\right]^T \tag{1}$$

where: $q = \left[q_1, ..., q_n\right]^T$ being q the time-dependent generalized variable and i is the $i-$th center of mass, n is the number of generalized variables.

Step 2: Obtaining Tangential velocities from the position vectors, where the transformer modules (MTF) are identified.

$$\dot{r}_i = \frac{dr_i}{dt} = \left[\dot{x}_i\left(q,\dot{q}\right)\ \dot{y}_i\left(q,\dot{q}\right)\ \dot{z}_i\left(q,\dot{q}\right)\right]^T \tag{2}$$

$$\left[\dot{x}_i\left(q,\dot{q}\right)\ \dot{y}_i\left(q,\dot{q}\right)\ \dot{z}_i\left(q,\dot{q}\right)\right]^T = \sum_{j=1}^{n} MTF_{ij}\dot{q}_j \tag{3}$$

$$MTF_{ij} = \frac{\partial r_i}{\partial q_j},\ \text{where } MTF_{ij} \in \Re^{3\times 1}. \tag{4}$$

Step 3: Identification of the 1-junctions representing angular velocities associated with the motion variables.

First, we identify the angular velocities associated with the 1-junctions, are identified, and their representation in Bond Graph is shown in Fig. 1

$$1_{\dot{q}_j} \tag{5}$$

Step 4: Identification of 0-junction, if necessary (associated with "+" or "-", in the vector of tangential velocities).

Expanding (3) for $\dot{x}_i\left(q,\dot{q}\right)$ we have (6) where its representation in Bond Graph is shown in Fig. 1, being analogous for $\dot{y}_i$ and for $\dot{z}_i$.

$$\dot{x}_i\left(q,\dot{q}\right) = MTF_{i1k}\dot{q}_1 \pm \ldots \pm MTF_{ink}\dot{q}_n. \tag{6}$$

where: k is the k-th MTF_{ijk} being $MTF_{ijk} \in \Re^{1\times 1}$

Step 5: Representation of the 1-junction associated with each tangential velocity vector ($\dot{x}$, $\dot{y}$, $\dot{z}$), and the inertia related to the mass of that position vector ($I_i : m_i$).

$$1_{\dot{x}_i} \quad , \quad 1_{\dot{y}_i} \quad , \quad 1_{\dot{z}_i} \tag{7}$$

$$1_{\dot{x}_i} \rightharpoonup I_i : m_i\ ,\ 1_{\dot{y}_i} \rightharpoonup I_i : m_i\ ,\ 1_{\dot{z}_i} \rightharpoonup I_i : m_i \tag{8}$$

To these 1-junctions converge the previous steps, from (2) to (8) the representation in Bond Graph associated with each tangential velocity is obtained. The representation of $\dot{x}_i$ can be seen in Fig. 1, being analogous to the representations for $\dot{y}_i$ and $\dot{z}_i$.

Step 6: Determine if there are external or internal forces acting on the 1-junctions of the steps 3 or 5, representing themselves as sources of effort or flow ("Se" or "Sf").

The external forces acting on each axis (x, y, z) of the system are analyzed, which are represented as sources of effort or flow, according to the analogy used, among these external forces is the force of gravity, this representation is shown in Fig. 1, where only the force acting on the x-axis is shown, being analogous those of y and z.

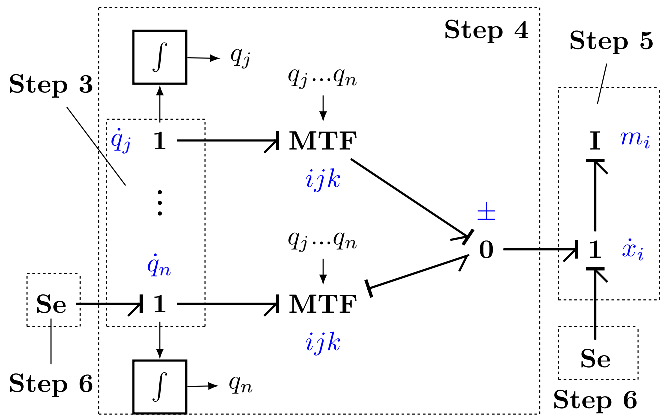

Fig. 1. Steps 3 to 6 representation in Bond Graph with a Se associated with the x-axis.

Step 7: Determine the absolute angular velocity through the Rotation Matrices.

The absolute angular velocities are determined as follows:

$$w_i = \left(R_0^i\right)^T w_0^i \tag{9}$$

where: R_0^i is the Rotation Matrix, w_0^i is the relative angular velocity concerning the inertial reference frame:

$$w_0^i = \rho_i w_0^1 + R_0^1 w_1^2 + \cdots + R_0^{i-1} w_{i-1}^i \tag{10}$$

where: $\rho_i = 1$ if there are rotations else $\rho_i = 0$.

$$w_i = [w_{ix}\ w_{iy}\ w_{iz}]^T = \sum_{j=1}^{n} \left(MTF_{w-ij}\dot{q}_j\right). \tag{11}$$

being $MTF_{w-ij} \in \Re^{3\times 1}$

Step 8: Representation of the 1-junction related to the absolute angular velocity.

$$1_{w_i}\ \text{, where this 1-junction is a } 3\times 1 \text{ vector} \tag{12}$$

Step 9: Identification of 0-junction, if necessary (associated with "+" or "-", in the vector of absolute angular velocities).

Analyzing (11) each row of the MTF_{w-ij} vector can be rewritten as shown in (13); its representation in Bond Graph is shown in Fig. 2, being analogous to the representation of each of its components in y and z.

$$w_{ix} = MTF_{i1k}\dot{q}_1 \pm \ldots \pm MTF_{ijk}\dot{q}_n \tag{13}$$

where: k is consecutive of the MTFs of the previous steps for the combination ij being $MTF_{ijk} \in \Re^{1\times 1}$

Step 10: Associate the 1-junction of the previous step with the inertia of that center of mass. ("I:J")

As the absolute angular velocity in each axis is scalar and the inertia is a matrix, $J_i \in R^{3\times 3}$, it is necessary to built the vector of absolute angular velocity, using "PowerMux" as shown in Fig. 2. Although inertia is a

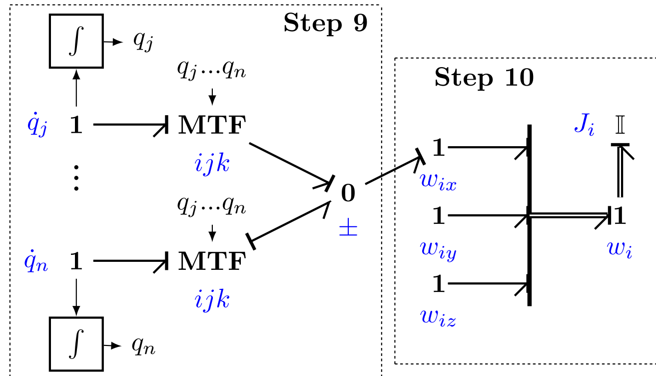

Fig. 2. Step 9 and 10 represented in Bond Graph with absolute angular velocity.

matrix, if the angular velocity has zero in two of its 3 components then it is represented in scalar form, that is, if $w_i = [0\ w_{iy}\ 0]^T$ then, ${w_i}^T J_i w_i = J_{iy} w_{iy}^2$, where the relationship with rotational kinetic energy is maintained.

Step 11: Calculate the Potential Energy.

The potential energy in each of the centers of mass is related to gravity or elasticities.

$$E_p = \sum_{i=1}^{m} m_i g z_i(q) + \frac{1}{2}\beta s^2 \tag{14}$$

where: m is the number of centers of mass, $z_i(q)$ is the position of the height of the i-th center of mass and β is the stiffness constant, s is the effective displacement.

The two terms of E_p are analyzed independently, the first term indicates on which center of mass the force of gravity acts, for this objective we must make use of (15)

$$F_g = \frac{\partial(\sum_{i=1}^{m=i} m_i g z_i(q))}{\partial q_j} \tag{15}$$

From this result we would obtain the centers of mass that are affected by the force of gravity, which are those that are affected by the generalized variables, its implementation in Bond Graph, is given by a source of effort, with the value of $-mg$, as shown later in the application section of the methodology.

The second term indicates the capacitive elements "C" that influence the generalized variables product of the elasticity of some element of the system as shown in Figures 3 (a) and 3 (b) when the elastic or capacitive element has velocity $\dot{q}_j$ and is between two centers of mass where its generalized variables have different speeds, respectively.

The following step is recommended to verify the Bond Graph model and to select the simplest equations for implementations in Bond Graph

Step 12: Determine the Euler-Lagrange equations of motion and compare them in simulations with

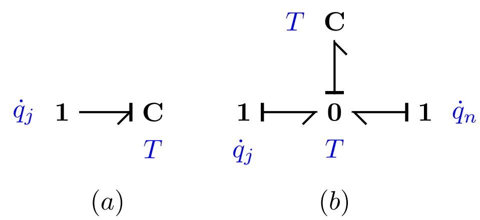

Fig. 3. Representation of capacitive elements related to potential energy a) one centers of mass b) Between two centers of mass.

the obtained Bond Graph model.

The Euler-Lagrange equations of motion are calculated, as shown below.

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = \tau - b\dot{q} \tag{16}$$

where: $L = (E_{k_t} + E_{k_r}) - E_p$, $E_{k_t} = \sum_{i=1}^{m} \frac{1}{2} m_i \dot{r}_i^T \dot{r}_i$ is the Translational Kinetic Energy, $E_{k_r} = \sum_{i=1}^{m} \frac{1}{2} w_i^T J_i w_i$ is the Rotational Kinetic Energy and b is the Viscous Coefficient of Friction.

3. BOND GRAPH MODEL OF THE WIND TURBINE

This model is based only on the mechanical part of the wind turbine, to obtain the model in Bond Graph we will use the methodology proposed in section 2, using a force-voltage analogy.

STEP 1: Analyzing the physical system in Fig. 4, obtained from modeling the wind turbine in SolidWorks, five centers of mass are localized, where D_c are the distances to reach each center of mass, Ω and γ are the angles of rotation around the horizontal or hub axis and vertical (Yaw) axis, respectively. In the specific case of our application we use the Denavit-Hartenberg convention as shown in Table 2. In this analysis the blades have a distribution of 120 degrees, so the angular velocity of each one presents this lag. where: $h = 1, 2, 3$ and

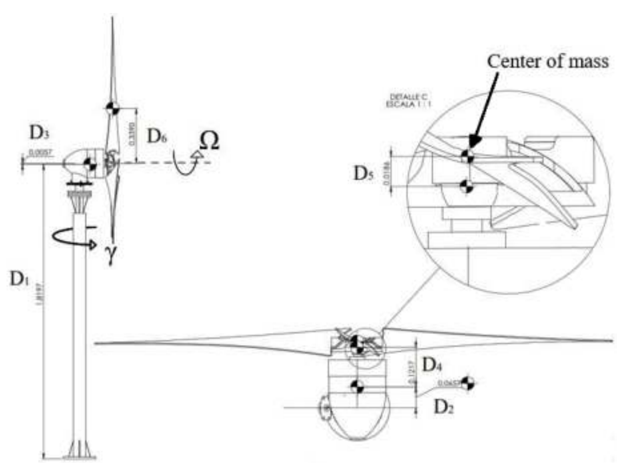

Fig. 4. Location of wind turbine centers of mass

Table 2. Denavit-Hartenberg parameters

	T_{ij}	θ_{ij}	D_{ij}	α_{ij}	d_{ij}
$Nacelle$	T_{01}	γ	D_1	0	D_2
Hub	T_{12}	0	D_3	0	D_4
	T_{23}	0	0	0	D_5
$Blade_h$	T_{34}	0	0	Ω_h	0
	T_{45}	0	D_6	0	0

$$\Omega_1 = \Omega \ ; \ \Omega_2 = \Omega - \frac{2\pi}{3} \ ; \ \Omega_3 = \Omega + \frac{2\pi}{3} \tag{17}$$

The position vectors are:
$Nacelle$

$$r_1 = \begin{bmatrix} D_2 \cos(\gamma) \\ D_2 \sin(\gamma) \\ D_1 \end{bmatrix} \tag{18}$$

Hub

$$r_2 = \begin{bmatrix} D_2 \cos(\gamma) + D_4 \cos(\gamma) \\ D_2 \sin(\gamma) + D_4 \sin(\gamma) \\ D_1 + D_3 \end{bmatrix} \tag{19}$$

$Blade_h$

$$r_{(h+2)} = \begin{bmatrix} (D_2 + D_4 + D_5)\cos(\gamma) + D_6 \sin(\Omega_h)\sin(\gamma) \\ (D_2 + D_4 + D_5)\sin(\gamma) - D_6 \sin(\Omega_h)\cos(\gamma) \\ D_1 + D_3 + D_6 \cos(\Omega_h) \end{bmatrix} \tag{20}$$

STEP 2: From the position vectors of each center of mass given (17) to (20) and applying (2), (3), and (17) we obtain:

$$Nacelle \qquad \dot{r_1} = \begin{bmatrix} (MTF_{111})\dot{\gamma} \\ (MTF_{112})\dot{\gamma} \\ 0 \end{bmatrix} \tag{21}$$

where: $MTF_{111} = -D_2 \sin(\gamma)$ and $MTF_{112} = D_2 \cos(\gamma)$

$$Hub \qquad \dot{r_2} = \begin{bmatrix} (MTF_{211})\dot{\gamma} \\ (MTF_{212})\dot{\gamma} \\ 0 \end{bmatrix} \tag{22}$$

where: $MTF_{211} = -D_2 \sin(\gamma) - D_4 \sin(\gamma)$ and $MTF_{211} = D_2 \cos(\gamma) + D_4 \cos(\gamma)$

$Blade_h$

$$\dot{r}_{(h+2)} = \begin{bmatrix} (MTF_{(h+2)21})\dot{\Omega} - (MTF_{(h+2)11})\dot{\gamma} \\ (MTF_{(h+2)22})\dot{\Omega} + (MTF_{(h+2)12})\dot{\gamma} \\ (MTF_{(h+2)23})\dot{\Omega} \end{bmatrix} \tag{23}$$

where: $h = 1, 2, 3$, $MTF_{(h+2)21} = D_6 \cos(\Omega_h)\sin(\gamma)$, $MTF_{(h+2)22} = -D_6 \cos(\Omega_h)\cos(\gamma)$, $MTF_{(h+2)11} = (D_2 + D_4 + D_5)\sin(\gamma) - D_6 \sin(\Omega_h)\cos(\gamma)$, $MTF_{(h+2)12} = (D_2 + D_4 + D_5)\cos(\gamma) + D_6 \sin(\Omega_h)\sin(\gamma)$, $MTF_{(h+2)23} = -D_6 \sin(\Omega_h)$

STEP 3:

$$1_{\dot{\Omega}} \quad and \quad 1_{\dot{\gamma}} \tag{24}$$

STEP 4: In this case, there are zeros in all equations, this will be represented in the following steps.

STEP 5:

$$1_{\dot{x}} \qquad 1_{\dot{y}} \qquad 1_{\dot{z}} \tag{25}$$

$$1_{\dot{x}} \rightharpoonup I_i : m_i \quad 1_{\dot{y}} \rightharpoonup I_i : m_i \quad 1_{\dot{z}} \rightharpoonup I_i : m_i \tag{26}$$

Joining from (21) to (26) the representation in Bond Graph associated with each tangential velocity is obtained, Blade 2 is represented in Fig. 5, while Blades 1 and 3 are represented analogously.

STEP 6: In this case, there are external forces associated with the force of the wind on the blades that generates a torque applied on the hub axis, an applied torque on the vertical axis due to the motor and mechanism added, and the force of gravity acting (see below Step 11) on the 1-junction related to the position in z (S_{emg}), being represented in Bond Graph as shown in Fig. 5.

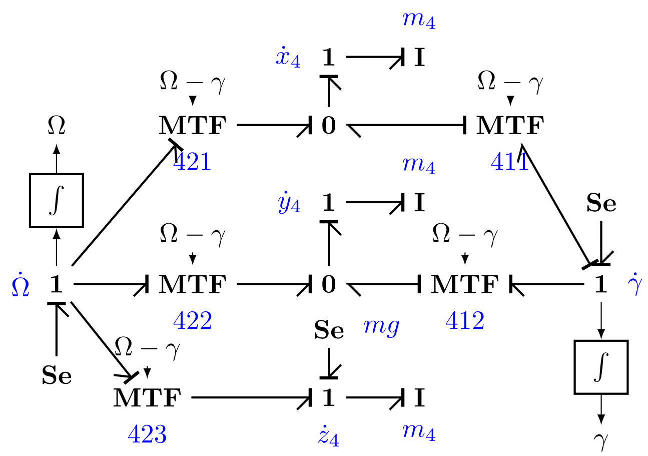

Fig. 5. Example of Bond Graph of Blade 2.

STEP 7: Applying (9) to (11) the absolute angular velocities of each center of mass are obtained:

$$Nacelle \qquad w_1 = [0 \;\; 0 \;\; \dot{\gamma}]^T \tag{27}$$

$$Hub \qquad w_2 = [0 \;\; 0 \;\; \dot{\gamma}]^T \tag{28}$$

$Blade_h$

$$w_{(h+2)} = \begin{bmatrix} \dot{\Omega} \\ \left(MTF_{(h+2)13}\right)\dot{\gamma} \\ \left(MTF_{(h+2)14}\right)\dot{\gamma} \end{bmatrix}, \text{ being } h = 1,2,3 \tag{29}$$

where: $MTF_{(h+2)13} = \sin(\Omega_h)$ and $MTF_{(h+2)14} = \cos(\Omega_h)$

STEP 8:

$$1_{w_1}, 1_{w_2}, 1_{w_3}, 1_{w_4}, 1_{w_5} \tag{30}$$

STEP 9: This step does not apply to this system.

STEP 10: As the inertia of the blades are matrices, a "PowerMux" is used, as shown in Fig. 6.

Although inertia is a matrix for all centers of mass, and analyzing (27) or (28) it is evident that the representation in Bond Graph of the angular velocity of the centers of mass of the nacelle and the hub can be represented as scalar.

$$w_1{}^T J_1 w_1 = J_{1z}\dot{\gamma}^2, \quad w_2{}^T J_2 w_2 = J_{2z}\dot{\gamma}^2 \tag{31}$$

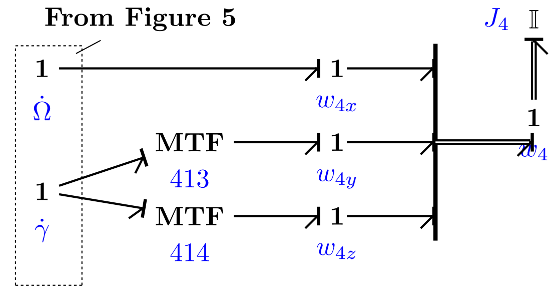

Fig. 6. Bond Graph with the absolute angular velocity for blade 2 (center of mass 4).

STEP 11: Obtaning $z_i(q)$ from (18) to (20) and applying (14), the potential energy in each of the centers of mass is related to gravity only (in this case elasticity is disregarded).

$$\begin{aligned} E_p &= m_1 g D_1 + m_2 g\,(D_1 + D_3) \\ &+ 3m_3 g\left(D_1 + D_3 + \sum \frac{D_6 \cos(\Omega_h)}{3}\right) \end{aligned} \tag{32}$$

where: $m_3 = m_4 = m_5$, $h = 1,2,3$, analyzing (15) and (32) it is evident that the force of gravity is present in the centers of mass 3 to 5, or what is the same is present in the z-axis of the blades, as shown in Fig. 5 (S_{emg}).

Figure 7 shows the extended model in Bond Graph of the wind turbine (Nacelle, Hub and Blade), this representation covers the steps of the methodology (from steps 2 to 11).

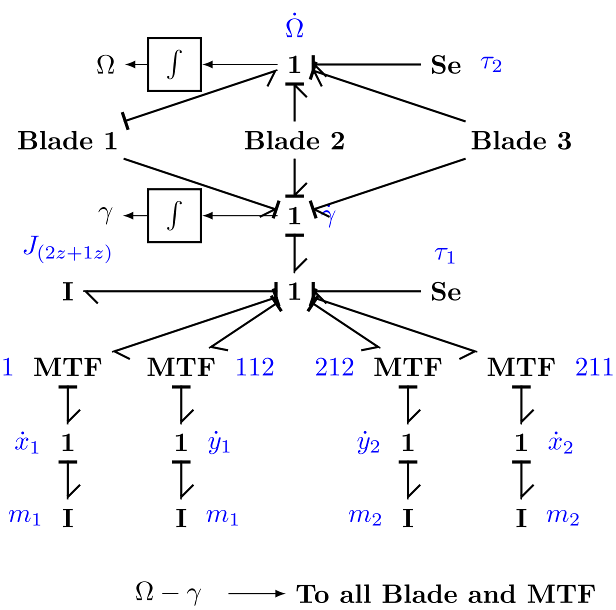

Fig. 7. Bond Graph (Extended) representation of the Wind Turbine

STEP 12: Making use of (16) we obtain the Euler-Lagrange equations of motion, for each generalized variable.

Equation of motion for γ

$$\left(\frac{3J_{3y}}{2} + J_{2z} + \frac{3J_{3z}}{2} + J_{1z} + D_2^2 m_1 + m_2(D_2^2 + D_4^2 + 2D_2D_4) + 3m_3(D_2^2 + D_4^2 + D_5^2 + \frac{D_6^2}{2} + 2D_2D_4 + 2D_2D_5 + 2D_4D_5)\right)\ddot{\gamma} = J_{eq1}\ddot{\gamma} = \tau_1 - b_1\dot{\gamma} \tag{33}$$

Equation of motion for Ω

$$3\left(m_3D_6^2 + J_{3x}\right)\ddot{\Omega} = J_{eq2}\ddot{\Omega} = \tau_2 - b_2\dot{\Omega} \tag{34}$$

where b_1 and b_2 are coefficients of viscous friction. From the equations of motion the accelerations are multiplied by inertias, *i.e.*, the equations of motion are expressed analogously to Newton's laws where for rotation the sum of all torques is proportional to the acceleration, being the constant of proportionality is the inertia associated with the system:

$$\sum \tau = J\ddot{\theta} \tag{35}$$

So it is possible to implement it directly in Bond Graph Methodology, which is an advantage and particularity of this system, where the Coriolis $C(q,\dot{q})$ and Gravity $G(q)$ matrices are zero. In the specific case of this system, $G(q)$ in the Lagrange model is zero due to the balanced distribution of 120 degrees between each blade, counteracting the effect of gravity (mg) between each blade. This particularity allows an easier implementation and shows why it is important to arrive at Euler Lagrange's equations Karnopp (1977) Karnopp (2012) in this methodology to verify if it is possible to obtain a reduced system as shown in Fig. 8.

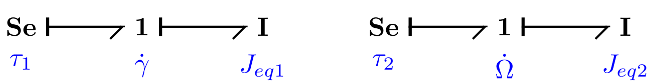

Fig. 8. Bond Graph (Simplified) representation of equations of motion

4. BOND GRAPH MODEL OF THE RR ROBOT

Step 1: Analyzing the physical system in Fig. 9, obtained from the modeling of the Robot RR in SolidWorks, two centers of mass are located, where l_1 and l_2 are the distances to reach each center of mass, θ_1 and θ_2 are the rotation angles around the vertical axis and horizontal axis respectively.

Table 3. Denavit-Hartenberg parameters

T_{ij}	θ_{ij}	D_{ij}	α_{ij}	d_{ij}
T_{01}	θ_1	l_1	90	0
T_{12}	θ_2	l_2	0	0

Analyzing the Fig. 9, the Denavit-Hartenberg parameters are obtained. Thus with this obtaining **Step 2**, represented by the tangential velocities and **Step 7** for absolute angular velocities.

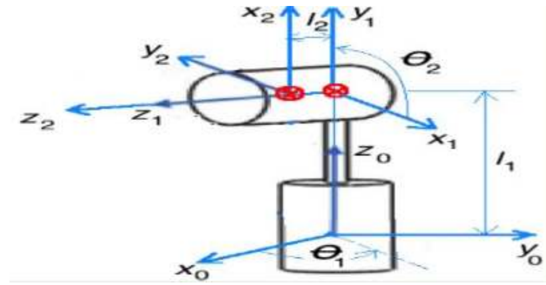

Fig. 9. Location of wind turbine centers of mass

$$\dot{r_1} = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix} \tag{36}$$

$$\dot{r_2} = \begin{bmatrix} (MTF_{211})\,\dot{\gamma} \\ (MTF_{212})\,\dot{\gamma} \\ 0 \end{bmatrix} \tag{37}$$

where: $MTF_{211} = l_2\cos(\theta_1)$ and $MTF_{212} = l_2\sin(\theta_1)$

$$w_1 = \begin{bmatrix} 0 \\ \dot{\theta}_1 \\ 0 \end{bmatrix} \tag{38}$$

$$w_2 = \begin{bmatrix} (MTF_{w-21})\dot{\theta}_1 \\ (MTF_{w-22})\dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \tag{39}$$

where: $MTF_{w-21} = \sin(\theta_2)$ and $MTF_{w-22} = \cos\theta_2$

Analyzing **Step 2 to 11** the representation is obtained of Fig. 10 in Bond Graph:

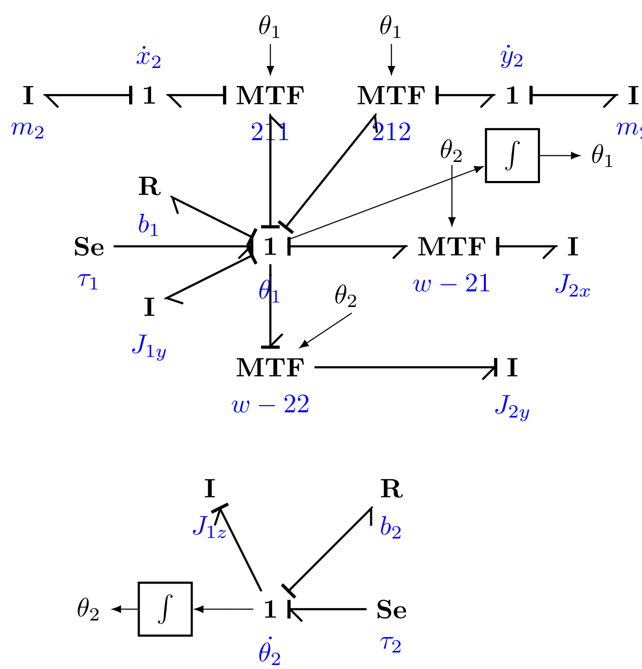

Fig. 10. Bond Graph RR robot

Developing **STEP 12:** we obtain the Euler-Lagrange equations of motion, for each generalized variable.

Equation of motion for θ_1

$$\begin{aligned}&(J_{1y} + J_{2y}\cos^2\theta_2 + l_2^2 m_2 + J_{2x}\sin^2\theta_2)\ddot{\theta}_1\\ &+ J_{2x}\sin 2\theta_2\dot{\theta}_2\dot{\theta}_1 - J_{2y}\sin 2\theta_2\dot{\theta}_2\dot{\theta}_1 = \tau_1 - b_1\dot{\theta}_1\end{aligned} \tag{40}$$

Equation of motion for θ_2

$$J_{2z}\ddot{\theta}_2 - \frac{\tan\theta_2(J_{2x} - J_{2y})\dot{\theta 2}_2^2}{\tan^2\theta_2 + 1} = \tau_2 - b_2\dot{\theta}_1 \tag{41}$$

Analyzing the equations of motion (40) and (41) , it can be seen by simple inspection that their implementation in Bond Graph is more complex than their implementation developed in the previous steps as show in Fig. 10.

5. VERIFICATION OF THE METHODOLOGY BY COMPUTATIONAL SIMULATIONS

Analyzing the developed examples and the proposed steps of the methodology, the implementation in 20-Sim (Bond Graph) of the simplified scheme of Fig. 8 for the wind turbine and the scheme of Fig. 10 for the RR robot is carried out. The parameters used for the simulation are as shown below: Wind Turbine parameters: $D_1 = 1.817$ m, $D_2 = 0.0657$ m, $D_3 = 0.0057$ m,$D_4 = 0.1217$ m, $D_5 = 0.0186$ m,$D_6 = 0.339$ m, mass (kg):$m_1 = 11.6760$, $m_2 = 0.5443$, $m_3 = 0.5987$, inertia $(kg * m^2)$: $J_{1x} = 0.0561910066$, $J_{1xy} = 0.0091358265$, $J_{1xz} = -0.004245226677227853$, $J_{1y} = 0.0732685013093770$, $J_{1yz} = 0.001842850879795179$, $J_{1z} = 0.070677543101644$, $J_{2x} = 0.0013146135416260$, $J_{2xy} = -2.41885235925522 \times 10^{-7}$, $J_{2xz} = -3.4916847045624 \times 10^{-9}$, $b_1 = b_2 = 0$, $J_{2y} = 0.00071655178635907$, $J_{2yz} = -1.67485386331549\times 10^{-7}$, $J_{2z} = 0.0006840017325117$, $J_{3x} = 0.0370823154229$, $J_{3xy} = -0.000546246474237$, $J_{3xz} = -0.00017857796137$, $J_{3y} = 0.000491678950127$, $J_{3yz} = -0.00011382205662520$, $J_{3z} = 0.0373784203097498$, $\tau_1 = 20$, $\tau_2 = 60$, RR Robot parameters: $l_2 = 0.271$, $m_1 = 11.5037$, $m_2 = 3.23121$, $b_1 = 1.5$, $b_2 = 1.5$, $J_{1y} = 0.137159$, $J_{2x} = 0.0323151$, $J_{2y} = 0.0338776$, $J_{2z} = 0.015552$, $\tau_1 = 60$, $\tau_2 = 60$, initial parameters equal zero:

Figure 11 and 12 show the Matlab representation of the wind turbine model and the RR robot respectively

Figures 13 and 14 show the comparison of the computational results of the implementation of Bond Graph and Matlab (simscape, Lagrange) for the mechanical part of the wind turbine.

The results show in Figures 13 and 14 are as expected due to Eq. (35) and Fig. (8) when constant inputs are applied.

Figures 15 and 16 show the comparison of the computational results of the implementation of Bond Graph and Matlab (simscape, Lagrange) for the RR robot

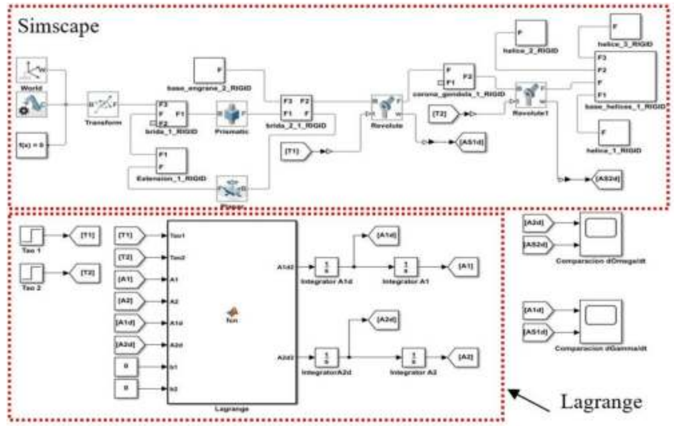

Fig. 11. Implementation in Matlab Wind Turbine

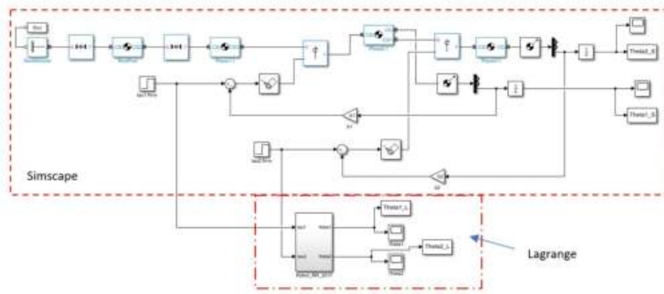

Fig. 12. Implementation in Matlab RR Robot

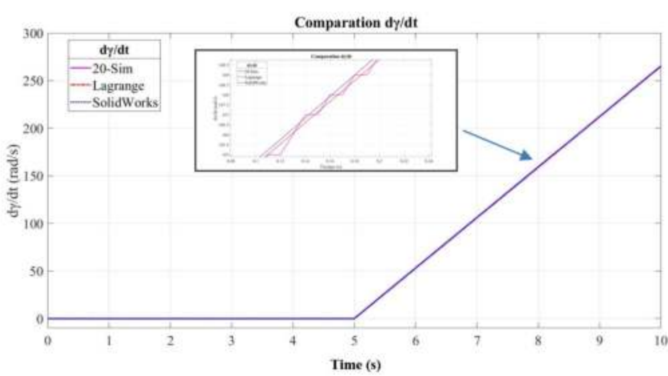

Fig. 13. Comparison of the results of the Yaw-angle-velocity ($\dot{\gamma}$)

Figure 14 to 16 show that all the system outputs are practically equals for the Simscape, Lagrange and Bond Graph models with small differences as show in the zoom boxes of these figure. These differences are due to the numeric algorithms used in the simulations.

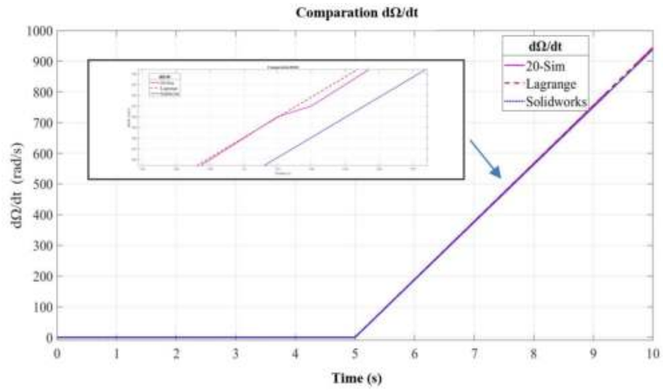

Fig. 14. Comparison of the results of the speed of the horizontal axis ($\dot{\Omega}$)

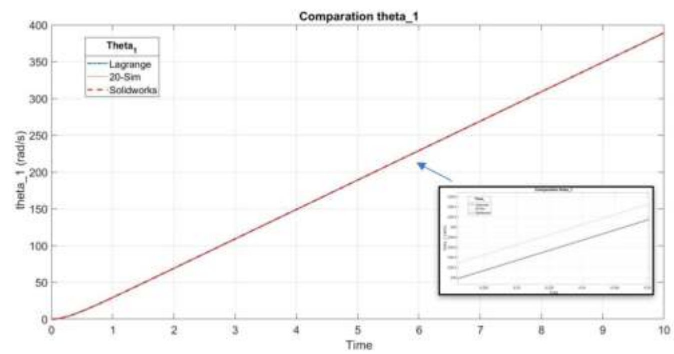

Fig. 15. Comparison of the results of the θ_1

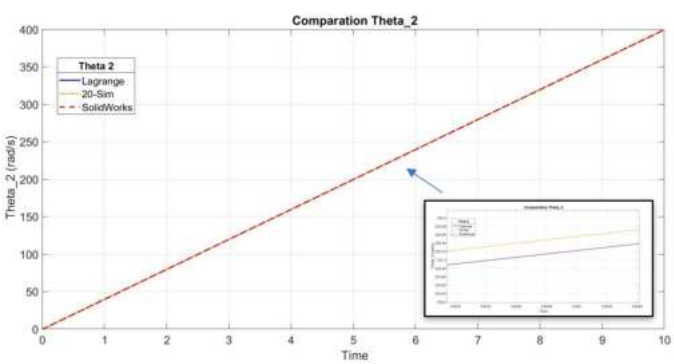

Fig. 16. Comparison of the results of the θ_2

CONCLUSIONS

The proposed methodology allows obtaining Bond Graph models based on the position vectors of the centers of mass and rotational matrices and Lagrange equations. As evidenced in the analyzes carried out, it can be concluded that it is always necessary to carry out the 12 steps of the methodology, since sometimes the implementation by Lagrange (wind turbine) or based on the position vectors (RR robot) is easier. A reduced Bond Graph model of the small wind turbine with fixed blades was obtained for the mechanical part, additionally, this reduced model is verified by comparing it with the models obtained in Lagrange and the Solidworks model exported to Matlab / Simulink / Simscape. An advantage of this methodology is that in some systems it is possible to obtain reduced models, which allows reducing the computational calculation methods, as well as their implementation. Also, the proposed methodology allows to obtain a simpler Bond Graph model compared to other methods such as Newton-Euler, which are more complex when obtaining Bond Graph models for three-dimensional systems.

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