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Normalized Least Mean Square with Fuzzy Variable Step Size for Time Series Prediction with Chaotic Dynamic Based on Autoregressive Moving Average Model

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Abstract: In this paper, the Autoregressive Moving Average (ARMA) model is used for the time series prediction with chaotic dynamic. The estimation of the weights vector of ARMA model can be obtained through an adaptive algorithm based on stochastic gradient descent, such that the prediction performance of a chaotic time series performed through ARMA model is influenced by the value of the step size. For this, in this paper, a new version of Normalized Least Mean Square (NLMS) algorithm is proposed with the step size adapted by a Mamdani Fuzzy Inference System (MFIS) used for estimation of the weights vector of ARMA model, for the time series prediction with chaotic dynamic.

Keywords: ARMA Model, Chaotic Systems, Fuzzy Systems, NLMS, Step Size, Time Series.

1. INTRODUCTION

A time series can be defined as a samples set ordered and obtained at given time instants (Dong and Pedrycz, 2008). One application that is very popular and of great interest to academic community and in general is the time series prediction, which is applied in various areas of knowledge as in biological systems (Gharehbaghi and Lindén, 2017), econometrics (Nonejad, 2021), control systems theory (Lee et al., 2018), meteorology (Lorenz and Brunke, 2021), and others. Due to possibility of outliers and interdependence in the observed samples, to obtain a satisfactory prediction based on a training data set is not a simple task. Furthermore, one complexity that a time series may present is the presence of chaotic dynamic (Farmer and Sidorowich, 1987). The main characteristics that define a time series as chaotic are that its behavior must have a stationary nature and it must be sensitive to variations in initial conditions, such that a small variation in its initial conditions makes that the observed values at each time instant evolve dramatically.

The Autoregressive Moving Average (ARMA) model has been applied in the time series prediction of a stationary nature (Ansari et al., 2019). To perform of time series prediction based on an ARMA model, initially it is necessary to obtain a model for a previously collected data set and then predict t steps ahead the next time observations based on the obtained model. Since a chaotic time series exhibits stationary behavior, then it is justified the use of an ARMA model for prediction. To estimate the weights vector of an ARMA model, several adaptive algorithms based on stochastic gradient descent have been used, where it is possible to cite the most traditional such as the Least Mean Square (LMS) (Rahman et al., 2019) and the Normalized Least Mean Square (NLMS) (Garroppo and Callegari, 2020). This work is limited to study only the NLMS adaptive algorithm for the estimation of the weights vector of an ARMA model, since it presents a lower sensitivity to variations of the input signal power and has a good performance in correlated signals. In Golshan and Samet (2009); Samet et al. (2013), an ARMA model with the weight vector estimated by the NLMS adaptive algorithm with the fixed step size was used for prediction of reactive power required for electric arc furnace operation.

For a good performance of NLMS adaptive algorithm during the update of the estimate of the weight vector of ARMA model, it is necessary to perform a good choice of step size (Strutz, 2019; Shin et al., 2004). The step size is so important for the performance of NLMS adaptive algorithm such that, a fast convergence speed of the weights vector will be obtained if the step size is large, but the steady-state Mean Square Error (MSE) will be large. On the other hand, a slow convergence speed of the weights vector will be obtained if the step size is small, but the steady-state MSE will be small Aslam et al. (2021). A possible solution to obtain a good performance of NLMS adaptive algorithm is to make the step size be variable at each time instant Strutz (2019); Aslam et al. (2021); Casco-Sanchez et al. (2019); Peng et al. (2020). A disadvantage to the methodologies cited above is that the adjustment of the step size at each time instant is performed dependent on high-order statistical measures. Furthermore, the performance of these methodologies is dependent on the choice of weighting parameters, damping factor, and others.

Since the Mamdani Fuzzy Inference System (MFIS) has been used to solve problems of difficult mathematical formulation in various areas of knowledge (Iancu, 2012; Ahmadi et al., 2020), its use is a possible alternative for adaptation of the step size of NLMS adaptive algorithm at each time instant. It is important to mention, according to the bibliographic studies performed in the specialized literature by the author of this paper, few works have proposed the use of a MFIS to adapt the step size of NLMS adaptive algorithm. In Orozco-Tupacyupanqui et al. (2015), a neuro-fuzzy system was used for tuning a non-adaptive optimal step size, according to the learning curve obtained for different step sizes. The methodology proposed in Ng et al. (2009) is closer to the proposal in this paper, where an MFIS is used to adapt the step size as a function of the error used to update the estimate of the weights vector of an adaptive equalizer of communication channels. A disadvantage of the methodology proposed in Ng et al. (2009), is that the step size is not adapted as a function of the time instant, such that the quantity of time instant is an important parameter for the convergence of the weights vector.

This paper aims to propose the chaotic time series prediction based on ARMA model with updated of the estimate of the weights vector performed through NLMS adaptive algorithm, with variable step size adapted by a MFIS. This new version of NLMS adaptive algorithm is namely Fuzzy Variable Step Size - Normalized Least Mean Square (FVSS-NLMS) algorithm. Unlike the proposed methodology in Ng et al. (2009), this paper proposes the adaption of the step size as a function of the squared value of the error used to update the estimate of the weights vector of ARMA model and of the normalized time instant by the Min-Max method. Through a linguistic description implemented in a fuzzy rule base, it is possible to obtain a good performance of ARMA model in the training stage and, consequently, in the prediction stage, when compared to the use of a fixed step size. Thus, the contribution proposed in this work is clear and justified. In order to compare the performance of FVSS-NLMS algorithm, the results obtained in the training stage and in the prediction stage were compared with the results obtained by the methodology proposed in Ng et al. (2009) and by the traditional versions of LMS and NLMS adaptive algorithms. This paper is organized as follows: in Section 2, the structure of ARMA model is presented; in Section 3, the FVSS-NLMS algorithm is presented; in Section 4, the procedure for time series prediction based on ARMA model via FVSS-NLMS algorithm is presented; in Section 5, the computational results obtained are presented.

2. AUTOREGRESSIVE MOVING AVERAGE MODEL

Box and Jenkis (Makridakis and Hibon, 1997) described that an Autoregressive Moving Average (ARMA) model can represent a stationary stochastic process, where the sample $y(k)$ observed at time k not only depends on the samples observed at past instants $y(k-1), y(k-2), \ldots, y(k-n)$, but also has an interdependent relationship with the observed noises $\epsilon(k), \epsilon(k-1), \epsilon(k-2), \ldots, \epsilon(k-n)$. After the parameterization of ARMA model based on a training data set, then it is possible to obtain the future estimates from $y(k)$ by the prediction t steps ahead.

For a stationary time series represented by a samples set $\{y(k)\}$, the ARMA(p,q) model can be expressed as follows:

$$\begin{aligned}\hat{y}(k) = \psi_1\hat{y}(k-1) + \psi_2\hat{y}(k-2) + \ldots + \psi_p\hat{y}(k-p)\\ +\epsilon(k) + \vartheta_1\epsilon(k-1) + \vartheta_2\epsilon(k-2) + \ldots + \vartheta_q\epsilon(k-q),\end{aligned} \tag{1}$$

where p indicates that there are p delayed versions of the sample $\hat{y}(k)$ estimated by the ARMA model and q indicates that there are q delayed versions of $\epsilon(k)$. Furthermore, ψ_u for $u = 1, 2, \ldots, p$ and ϑ_v for $v = 1, 2, 3 \ldots, q$, are the coefficients or weights of ARMA(p,q) model that satisfy the stationarity and invertibility conditions, and $\epsilon(k)$ is a noise described by a stochastic process with zero mean and constant variance σ^2_ϵ. Another manner of rewriting (1) is given in vector form, as follows:

$$\hat{y}(k) = \mathbf{\Theta}^T(k)\mathbf{\Gamma}(k) = \mathbf{\Gamma}^T(k)\mathbf{\Theta}(k), \tag{2}$$

where $\mathbf{\Gamma}(k) = [y(k-1)\ y(k-2)\ \ldots\ y(k-p)\ \epsilon(k)\ \epsilon(k-1)\ \epsilon(k-2)\ \ldots\ \epsilon(k-q)] \in \mathbb{R}^{(p+q+1)\times 1}$ is the regressors vector and $\mathbf{\Theta}(k) = [\psi_1 \ldots \psi_p\ 1\ \vartheta_1\ \ldots\ \vartheta_q] \in \mathbb{R}^{(p+q+1)\times 1}$ is the weights vector of ARMA(p,q) model. According to Wiener criterion Prasad and Patil (2016), the update of the estimate of the weights vector of ARMA(p,q) model can be obtained as follows:

$$\mathbf{\Theta}(k+1) = \mathbf{\Theta}(k) - \frac{1}{2}\mu\nabla_{\mathbf{\Theta}(k)}(\mathrm{E}[e^2(k)]), \tag{3}$$

where $J = \nabla_{\mathbf{\Theta}(k)}(\mathrm{E}[e^2(k)])$ is the cost functional described by the stochastic gradient of the squared error $e^2(k) = (y(k) - \hat{y}(k))^2$ and μ is the step size used to update the estimate of the weights vector. For real-time application, it is quite expensive to accumulate samples to work with the expectation $\mathrm{E}[\bullet]$ of the squared error. Instead of working with the mathematical expectation, one can work with instantaneous values of the squared error, thus emerging the LMS adaptive algorithm. In this manner, the cost functional is rewritten as:

$$J = \nabla_{\mathbf{\Theta}(k)}(e^2(k)), \tag{4}$$

substituting (4) into (3) is obtained that:

$$\begin{aligned}\mathbf{\Theta}(k+1) &= \mathbf{\Theta}(k) - \frac{1}{2}\mu\nabla_{\mathbf{\Theta}(k)}(e^2(k))\\ &= \mathbf{\Theta}(k) - \frac{1}{2}\mu\nabla_{\mathbf{\Theta}(k)}[(d(k) - \mathbf{\Theta}^T(k)\mathbf{\Gamma}(k))^2]\\ &= \mathbf{\Theta}(k) + \mu e(k)\mathbf{\Gamma}(k),\end{aligned} \tag{5}$$

where $\nabla_{\mathbf{\Theta}(k)}(e^2(k)) = -2e(k)\mathbf{\Gamma}(k)$. To obtain the NLMS adaptive algorithm, it is necessary to normalize the LMS adaptive algorithm by the input signal power, as follows:

$$\mathbf{\Theta}(k+1) = \mathbf{\Theta}(k) + \mu\frac{e(k)\mathbf{\Gamma}(k)}{\mathbf{\Gamma}^T(k)\mathbf{\Gamma}(k)} \quad (6)$$

3. FVSS-NLMS ALGORITHM

So that the ARMA(p,q) model can track the dynamic of the training data set of a time series, it is necessary that the estimative of the weights vector $\mathbf{\Theta}(k)$ be updated at each time instant. In this work, the update of the estimate of the weights vector $\mathbf{\Theta}(k)$ is performed by the FVSS-NLMS algorithm, as follows:

FVSS-NLMS

$$\begin{aligned} &\mu(k) = \text{MFIS}(e^2(k), \mathcal{K}(k)) \\ &\mathbf{\Theta}(k+1) = \begin{cases} \mathbf{\Theta}(k) + \mu(k)\dfrac{e(k)\mathbf{\Gamma}(k)}{h(k)}, & \text{if } h(k) \neq 0 \\ \mathbf{\Theta}(k), & \text{if } h(k) = 0 \end{cases} \\ &k \in [1, K], \end{aligned} \quad (7)$$

where $\mu(k)$ is the step size adapted by the MFIS, $h(k) = \mathbf{\Gamma}^T(k)\mathbf{\Gamma}(k)$, $\mathbf{\Theta}(k)$ is the weights vector of ARMA (p,q) model, $e^2(k)$ is the squared error, $\mathbf{\Gamma}(k)$ is the regressors vector, $\mathcal{K}(k)$ is the time instant k normalized by the Min-Max method, and K is the total number of time instants. It is important to note that $e^2(k)$ and $\mathcal{K}(k)$ are the input variables of MFIS.

Definition 1 (**Membership Function**) (Wang, 1999). *Let be the variable x and a universe of discourse $U \subset \mathbb{R}$, such that $x \in U$. Let be also a set $F \subset U$ also known as fuzzy set, which is characterized by a Membership Function (MBF) that, through a mapping, associates x to the fuzzy set F with a membership degree belonging to the interval* $[0,1]$, *given by:*

$$m(x) : U \to [0,1], \quad (8)$$

in which $m(\bullet)$ is a Membership Function (MBF) of the fuzzy set F.

Definition 2 (**Linguistic Variable**) (Wang, 1999). *Since numeric variable receives numeric values, a linguistic variable x defined in the universe of discourse U receives linguistic values representing fuzzy sets defined in U. For example, x is a linguistic variable defined as the "speed" that can be assigned the linguistic values "low", "medium" or "high".*

Definition 3 (**Fuzzy Rule**) (Wang, 1999). *The expert's knowledge about how to solve a given problem inserted in a linguistic context, is represented in a fuzzy rule base of the type **If** propositions of the antecedent **then** propositions of the consequent. In the fuzzy rule base, the antecedent and consequent are defined by fuzzy propositions of the type x is A, where A is a linguistic value.*

The input variables of MFIS are defined as linguistic variables of the antecedent, which receive linguistic values due to fuzzification, through the following mappings performed by the j-ths MBFs $m_j(\mathcal{K}(k)) : U \to [0,1]$ and $m_j(e^2(k)) : V \to [0,1]$, with the universes of discourse $U = [0,1]$ and $V = [0.1 \times 10^{-5}, 3 \times 10^{-5}]$. For each linguistic variable of the antecedent, three linguistic values were defined, which are small (S) for $j = 1$, medium (M) for $j = 2$ and large (L) for $j = 3$, whose parameters of MBFs are shown in Table 1.

Table 1. Interval of triangular MBFs.

$\mathcal{K}(k)$		$e^2(k)$		$\bar{\mu}(k)$	
Interval		Interval		Interval	
S	[0 0.2 0.3]	S	[0.001 0.01 0.3]	S	[0.1 0.5 1.0]
M	[0.2 0.3 0.5]	M	[0.01 0.3 0.9]	M	[0.5 1.0 1.5]
L	[0.3 0.5 1.0]	L	[0.3 0.9 1.3]	L	[1.0 1.5 2.0]

The linguistic variable of the consequent $\bar{\mu}$ receives linguistic values through mapping performed by the j-th. MBF $m_j(\bar{\mu}(k)) : Z \to [0,1]$, with the universe of discourse $Z = [0, 0.1]$. For the linguistic variable of the consequent, three linguistic values were defined, which are small (S) for $j = 1$, medium (M) for $j = 2$ and large (L) for $j = 3$, whose parameters das MBFS are shown in Table 1.

$$\begin{aligned} &\mathcal{R}^1: \textbf{ If } \mathcal{K}(k) \textit{ is } S \textbf{ and } e^2(k) \textit{ is } S \textbf{ then } \bar{\mu}(k) \textit{ is } M \\ &\mathcal{R}^2: \textbf{ If } \mathcal{K}(k) \textit{ is } S \textbf{ and } e^2(k) \textit{ is } M \textbf{ then } \bar{\mu}(k) \textit{ is } M \\ &\mathcal{R}^3: \textbf{ If } \mathcal{K}(k) \textit{ is } S \textbf{ and } e^2(k) \textit{ is } L \textbf{ then } \bar{\mu}(k) \textit{ is } M \\ &\mathcal{R}^4: \textbf{ If } \mathcal{K}(k) \textit{ is } M \textbf{ and } e^2(k) \textit{ is } S \textbf{ then } \bar{\mu}(k) \textit{ is } S \\ &\mathcal{R}^5: \textbf{ If } \mathcal{K}(k) \textit{ is } M \textbf{ and } e^2(k) \textit{ is } M \textbf{ then } \bar{\mu}(k) \textit{ is } S \\ &\mathcal{R}^6: \textbf{ If } \mathcal{K}(k) \textit{ is } M \textbf{ and } e^2(k) \textit{ is } L \textbf{ then } \bar{\mu}(k) \textit{ is } L \\ &\mathcal{R}^7: \textbf{ If } \mathcal{K}(k) \textit{ is } L \textbf{ and } e^2(k) \textit{ is } S \textbf{ then } \bar{\mu}(k) \textit{ is } S \\ &\mathcal{R}^8: \textbf{ If } \mathcal{K}(k) \textit{ is } L \textbf{ and } e^2(k) \textit{ is } M \textbf{ then } \bar{\mu}(k) \textit{ is } M \\ &\mathcal{R}^9: \textbf{ If } \mathcal{K}(k) \textit{ is } L \textbf{ and } e^2(k) \textit{ is } L \textbf{ then } \bar{\mu}(k) \textit{ is } L \end{aligned} \quad (9)$$

The parameters of MBFs presented in Table 1 and the fuzzy rule base presented in (9), were defined according to the expert's knowledge about how the adaptation of the step size of NLMS adaptive algorithm should be performed. The expert's knowledge is obtained, for example, during simulations of the analyzed problem. In (9), the fuzzy propositions of the antecedent and consequent are related through the fuzzy implication, such that its input is the degree of activation of i-th fuzzy rule, characterized by the following MBF:

$$\alpha^i = t[m_j(\mathcal{K}(k)), m_j(e^2(k))] = \min[m_j(\mathcal{K}(k)), m_j(e^2(k))], \quad (10)$$

where $t[m_j(\mathcal{K}(k)), m_j(e^2(k))]$ is the t-norm, which is used due to fuzzy propositions of the antecedent forming compound fuzzy propositions through the logical connective "**and**". The t-norm is defined as the minimal value between the j-ths MBFs $m_j(\mathcal{K}(k))$ and $m_j(e^2(k))$, such that $t : [0,1] \times [0,1] \to [0,1]$. After obtained α^i, it is obtained the output of the fuzzy implication, which is an MBF, given by:

$$m_{\mathcal{R}^i} = \min[\alpha^i, m_j(\bar{\mu}(k))] \quad (11)$$

As can be seen in (10) and (11), the fuzzy implication is performed for each fuzzy rule. To combine all the MBFs $m_{\mathcal{R}}$ with the objective of obtaining a single MBF that represents the total response of the fuzzy implication, it is performed the fuzzy aggregation, as follows:

$$m_{Total} = \max[m_{\mathcal{R}^1}, m_{\mathcal{R}^2}, \ldots, m_{\mathcal{R}^9}] \tag{12}$$

After to obtain m_{Total}, in order for the step size $\mu(k)$ receive a numerical value, it is necessary to defuzzify m_{Total}. To perform the defuzzification, in this paper the centroid method is used, given by:

$$\mu(k) = \frac{\sum_{i=1}^{9} \bar{\mu}(k) m_{Total}(\bar{\mu}(k))}{\sum_{i=1}^{9} m_{Total}(\bar{\mu}(k))} \tag{13}$$

4. PREDICTION PROCEDURE BASED ON ARMA MODEL VIA FVSS-NLMS ALGORITHM

The procedures for identification of ARMA model based on training data set via FVSS-NLMS algorithm and the performing the prediction are presented below:

- **Step 1**: Define the model order: it is necessary to define the parameters p and q of ARMA(p, q) model. The definition can be performed by trial and error or based on some optimality criterion.
- **Step 2**: Parameterize the ARMA(p, q) model: at each sample of a training data set it is performed the update of the estimate of the weights vector $\mathbf{\Theta}(k)$, according to (7). It is important to note that at each sample of the training data set the step size is adapted through the FVSS-NLMS algorithm.
- **Step 3**: Evaluate the obtained model: after obtained the ARMA(p, q) model for the training data set, it is necessary to evaluate it through statistical metrics. If the model performance is unsatisfactory, it is necessary to go back to **Step 1** and parameterize the ARMA(p, q) model again.
- **Step 4**: Perform the prediction: it is used the model obtained in **Step 2** to perform the prediction t steps ahead based on the samples observed in the previous instants.

5. COMPUTATIONAL RESULTS

In this section, are presented the computational results obtained through the implementation of FASS-NLMS algorithm for chaotic time series prediction based on ARMA model. The results obtained are divided into two stages, the first stage is referring to the training stage and the second stage is referring to the prediction stage based on the ARMA(p, q) model obtained in the training stage. The performance of ARMA(p, q) model obtained for the two stages was evaluated using the following metrics:

- Variance Accounted For (VAF):

$$\text{VAF}(\%) = \left[1 - \frac{var(\mathbf{y} - \hat{\mathbf{y}})}{var(\mathbf{y})}\right] \times 100, \tag{14}$$

where $var(\bullet)$ is the variance, $\mathbf{y}$ is the data vector from a time series, $\hat{\mathbf{y}}$ is the data vector estimated by the ARMA(p, q) model.

- Mean Square Error (MSE):

$$\text{MSE} = \frac{1}{N}\sum_{k=1}^{N}(y(k) - \hat{y}(k)), \tag{15}$$

where N is the number of samples from the data vector.

- Normalized Root Mean Square Error (NRMSE):

$$\text{NRMSE} = \sqrt{\frac{1}{N}\sum_{k=1}^{N}\left[\frac{(y(k) - \hat{y}(k))^2}{max(\mathbf{y}) - min(\mathbf{y})}\right]} \tag{16}$$

- Non-Dimensional Error Index (NDEI):

$$\text{NDEI} = \frac{\text{RMSE}}{\text{std}(\mathbf{y})}, \tag{17}$$

where std$(\bullet)$ is the standard deviation.

- Best Fit Criterea (FIT):

$$\text{FIT}(\%) = \left(1 - \frac{||\mathbf{y} - \hat{\mathbf{y}}||}{||\mathbf{y} - \bar{\mathbf{y}}||}\right) \times 100, \tag{18}$$

where $\bar{\mathbf{y}}$ is the mean value of $\mathbf{y}$ and $|| \bullet ||$ is the Euclidean norm operator.

In this work, the Mackey-Glass chaotic time series (Mackey and Glass, 1977) was chosen to evaluate the predictive ability performed by the ARMA(p, q) model with the weights vector estimated by the FVSS-NLMS algorithm. The mathematical model of Mackey-Glass series is presented below:

$$y(k) = y(k-1) + \frac{\beta y_\tau}{1 + y_\tau^n} - \gamma x(k-1), \tag{19}$$

where the parameters $\beta, \gamma, n > 0$, $\beta/\gamma > 1$ and $y_\tau = y(k-\tau)$. For values starting at $y_\tau \geq 17$, the Mackey-Glass series exhibits chaotic dynamic. The parameters β and γ are defined, respectively, as the production and decay rates of variable y. The parameters that define the Mackey-Glass series were set equal to $\beta = 0,1$, $\gamma = 0,2$ and $\tau = 20$, and the initial conditions set equal to $y(k - \tau) = 0$. In order to compare results obtained through FVSS-NLMS algorithm, the step size for the LMS and NLMS adaptive algorithms was set equal to $\mu = 0.6$. For performing the training and prediction stages, 3000 samples were obtained in (19), where the first 2500 samples were used to obtain the ARMA(p, q) for the training stage and the 500 samples remaining were used to evaluate the prediction ability of ARMA(p, q) model referring to the prediction stage. The prediction stage was performed for $t = 2$ steps forward. The parameters (p, q) were set equal to $p = 4$ and $q = 2$, by trial and error. In Fig. 1, it is shown the outputs estimated by the ARMA$(4, 2)$ model for the training and prediction stages. Since the Mackey-Glass series has a rather chaotic dynamic behavior, to better visualize the tracking ability in the training stage and prediction stage of ARMA model, two preview zoom were performed which are seen in Figs. 2 and 3.

Table 2. Results of the statistical metrics obtained for the prediction stage.

Métricas	LMS	NLMS	**FVSS-NLMS**	Ng et al. (2009)
VAF(%)	78.1885	88.4505	**90.7802**	89.1223
MSE	0.0133	0.0104	**0.0056**	0.0076
NRMSE	0.0712	0.0698	**0.0545**	0.0613
NDEI	0.2717	0.2691	**0.1983**	0.2115
FIT(%)	53.2510	69.9854	**73.6132**	71.9912

Table 3. Results of the statistical metrics obtained for the prediction stage.

Métricas	LMS	NLMS	**FVSS-NLMS**	Ng et al. (2009)
VAF(%)	92.8175	95.6731	**97.3738**	96.4563
MSE	0.0060	0.0056	**0.0028**	0.0041
NRMSE	0.0639	0.0539	**0.0239**	0.0313
NDEI	0.2126	0.2028	**0.1829**	0.1945
FIT(%)	90.3929	91.4956	**97.9852**	96.2352

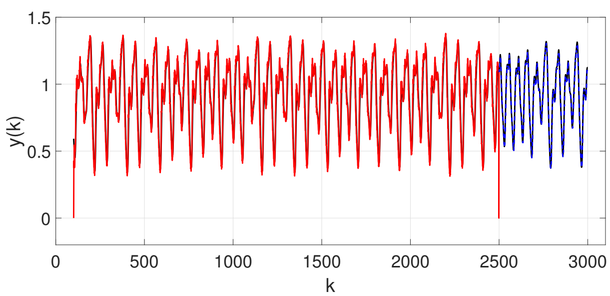

Fig. 1. Estimated outputs for the training stage (red color) and for the prediction stage (blue color).

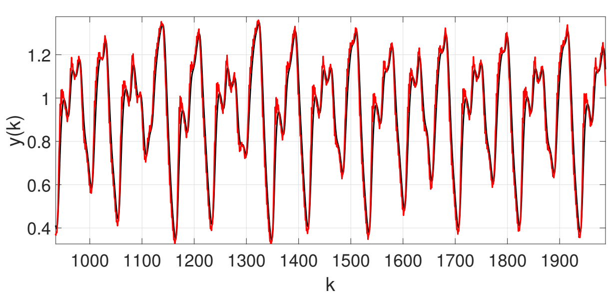

Fig. 2. Preview zoom performed at the estimated output for the training stage.

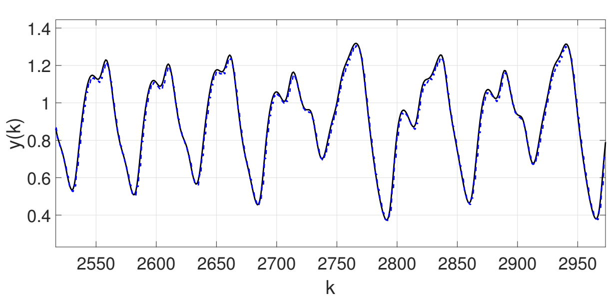

Fig. 3. Preview zoom performed at the estimated output for the prediction stage.

The temporal evolution of the variable step size adapted by the MFIS is shown in Fig. 4. The comparison of results obtained using the statistical metrics for the training and prediction stages can be seen in Tables 2 and 3, respectively. It can be seen that the ARMA$(4, 2)$ model with the weights vector estimated by the FVSS-NLMS algorithm obtained the best results with respect to the statistical metrics VAF(%), MSE, NRMSE, NDEI and FIT(%), for both training and prediction stages. When compared to methodology proposed in Ng et al. (2009) for adaptation of the step size of NLMS adaptive algorithm through an MFIS, it is possible to note the superior performance obtained by the FVSS-NLMS algorithm, for the training and prediction stages. The superior performance of FVSS-NLMS algorithm, when compared to proposed methodology in Ng et al. (2009), is due to the step size be adapted as a function of the squared error used to update the estimate of the weights vector of ARMA model and of the normalized time instant by the Min-Max method.

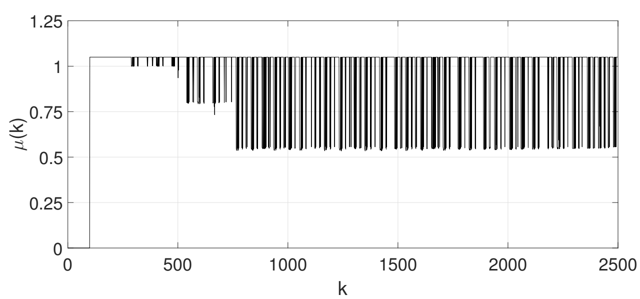

Fig. 4. Step size for FVSS-NLMS algorithm.

6. CONCLUSION

It was observed that the ARMA model with the weights vector estimated by the FVSS-NLMS algorithm, for both training and prediction stages, obtained the best results with respect to statistical metrics used, when compared to proposed methodology in Ng et al. (2009), LMS and NLMS adaptive algorithms with a fixed step size. Thus, it is possible to note that the linguistic description based on the expert's knowledge implemented in a rule base allowed a satisfactory ability to adapt the step size and, consequently, a satisfactory tracking of chaotic dynamic presented by the Mackey-Glass time series. Finally, it is noted that the step size adaptation by MFIS is independent of high-order statistical measures. On the other hand, a disadvantage of the proposed methodology is the tuning of the parameters of MBFs, which is dependent on the expert's knowledge.

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