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Proposed of Fuzzy Parametric Adaptation of the Acceleration Coefficients and Inertial Weight in GA-PSO Hybridization

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Abstract: This paper aims to propose a new hybridization involving the Genetic Algorithm (GA) and the Particle Optimization Swarm (PSO). The objective of the proposed optimization algorithm is to perform the search process for optimal solutions in complex problems with a fast and non-premature convergence. Since the satisfactory convergence of the search process is a result of a good trade-off between global and local search, in order to achieve the objective of the proposed optimization algorithm, a Mamdani Fuzzy Inference System (MFIS) is used for fuzzy parametric adaptation of the acceleration coefficients and inertial weight of PSO. Through this parametric adaptation, which is performed using a linguistic description based on the expert's knowledge and implemented in a fuzzy rule base, it is possible to obtain a good trade-off between global and local search in complex optimization problems.

Keywords: Genetic Algorithm, Evolutionary Computation, Hybridization, Particle Swarm Optimization, Fuzzy System.

1. INTRODUCTION

Various real-world problems are solved by the fitness function optimization with various natures of complexity, such as multidimensionality, the presence of local and global optimal (multimodality), non-linearity, discontinuity in the search space, and others. Through the evolutionary computation theory, various stochastic optimization algorithms based on population and bio-inspired have been used to solve complex optimization problems that would be infeasible or quite costly through, for example, methods based on gradient. In evolutionary computation theory, the optimization methods have been used to solve problems in time series prediction (Panigrahi and Behera, 2020), and others. More specific to the topic addressed in this paper, the Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) have been widely used for solving problems in various fields of knowledge, which is due to its efficient performance in finding optimal solutions and easy computational implementation.

The GA is a bio-inspired optimization algorithm based on the genetic evolution theory, proposed by Holland et al. (1992). In the GA, the genetic evolution of individuals from a population is simulated through the use of genetic operators that insert a large diversity of positions in the search space to each generation or iteration, making the process multidirectional. Naturally, the search process performed by the GA, when compared to the PSO, has an great facility to avoid premature convergence in multimodal problems. However, the search process performed by the GA converges slowly due to large diversity of positions in the search space (Ghoshal et al., 2019). The PSO is an optimization algorithm belonging to family of swarm intelligence algorithms proposed by Kennedy and Eberhart (1995). The PSO is an optimization algorithm bio-inspired by the interaction between social and individual behavior developed by various species of flocking animals, such as the interaction of birds during flight in search of satisfying some goal, as an example, the search for food. Since its the search behavior is unidirectional, the search process quickly converges to optimal solution in unimodal problems. However, as described in Kennedy and Eberhart (1995), due to unidirectionality and the lack of a mechanism to insert diversity of positions, the premature convergence may occur in multimodal problems.

Through the above paragraph, it is possible to note that the undesirable characteristic of GA is the slow convergence and that the undesirable characteristic of PSO is the premature convergence, which occur due to the inefficient trade-off between global and local search (Kennedy and Eberhart, 1995; Shi and Eberhart, 1998). A possibile solution to solve these problems is through hybridization of these algorithms. The original purpose of GA-PSO hybridization is to perform a search process in complex optimization problems with a fast convergence and with a large diversity of positions inserted in the search space (Gandelli et al., 2007). In the literature some contributions to GA-PSO hybridization have been proposed. In

Kang et al. (2021), it was proposed the optimization for filament cylinders using GA-PSO hybridization coupled with local sensitivity analysis. In Roy and Das (2021), it was proposed a GA-PSO hybridization for demand management of electric power for cost optimization.

Although the GA-PSO hybridization has a better performance than GA and PSO, due to large diversity of positions inserted by the genetic operators, the search process performed by this hybridization may converge slowly. Although the search process performed by this hybridization has a great facility to avoid premature convergence in multimodal problems, just inserting diversity of positions in the search space may not be enough to avoid it. Furthermore, in complex optimization problems the premature convergence may occur due to excess or insufficiency of diversity in the search space. It is important to note that performing a trade-off between global and local search is nothing more than controlling the update of the particle positions between global and local search. Since the diversity of positions in the search space can be defined as a measure of distance between the particle positions, it is possible to note that through a good trade-off it is possible to insert diversity efficiently in the search space and, therefore perform a search process that converges quickly and non-prematurely.

However, it is not trivial to obtain a good trade-off between global and local search. According to Kennedy and Eberhart (1995), since the acceleration coefficients pondering respectively the individual and social knowledge sharing acquired by the particles in the search space, through the parametric adaptation of the acceleration coefficients of PSO it is possible to control the individual and social knowledge sharing about the individual and global best positions in the search space. According Shi and Eberhart (1998), since the velocity orientates the update of the position of each particle and that in the equation of update of the velocity the inertial weight has the function of to ponder the contribution of velocity of the previous iteration in the current iteration, through the parametric adaptation of the inertial weight of PSO, it is possible to control the update of the position of each particle. Thus, through the parametric adaptation of the acceleration coefficients and inertial weight, it is possible to obtain a good trade-off between the global and local search.

Aiming to propose a new optimization algorithm referring to GA-PSO hybridization, this work proposes the use of a Mamdani Fuzzy Inference System (MFIS) to perform the fuzzy parametric adpatation of the acceleration coefficients and inertial weight of PSO. The use of a fuzzy system to parametrically adapt the acceleration coefficients and the inertial weight is justified due to ability of an MFIS to represent the expert's subjective knowledge in a fuzzy rule base using linguistic descriptions. The inputs of MFIS are a diversity measure and the iteration normalizeds by the Min-Max method; the outputs are the acceleration coefficients and the inertial weight. The motivation for using the diversity as input of MFIS is due it provides information about the distance of the particles positions in the search space, making it possible to infer whether the particles are performing a global or local search. The motivation for using the iteration as another input of MFIS is due to the iterations number be an important parameter for convergence, since it is expected that the convergence occurs at the end of the total iterations number. This paper is organized as follows: in Section 2 the statements of the optimization problem are presented; in Section 3 the proposed optimization methodology is presented; in Section 4 the computational results obtained through benchmark functions optimization are presented.

2. OPTIMIZATION PROBLEM STATEMENTS

The proposed optimization methodology aims to perform the search for solutions in an n-dimensional search space that maximize or minimize a fitness function:

$$\begin{cases} J = J(x_{i,1}[k], x_{i,2}[k], \ldots, x_{i,n}[k]), \text{ subject to :} \\ \min(\mathbf{x}_i) \leq \mathbf{x}_i[k] \leq \max(\mathbf{x}_i) \\ \min(\mathbf{v}_i) \leq \mathbf{v}_i[k] \leq \max(\mathbf{v}_i), \end{cases} \tag{1}$$

where $J : \mathbb{R}^n \rightarrow \mathbb{R}$ is the fitness function, $\mathbf{v}_i[k] = [v_{i,1}[k], v_{i,2}[k], \ldots, v_{i,n}[k]] \in \mathbb{R}^{1\times n}$ is the velocity vector and $\mathbf{x}_i[k] = [x_{i,1}[k], x_{i,2}[k], \ldots, x_{i,n}[k]] \in \mathbb{R}^{1\times n}$ is the position vector of the i-th particle belonging to a population containing N particles. The population of N particles is evaluated at each iteration $k \in [1, K]$, where K is the total iterations number. For a minimization problem, the vector of best position $\mathbf{p}_i[k]$ of the i-th particle is updated as follows:

$$\mathbf{p}_i[k+1] = \begin{cases} \mathbf{p}_i[k] \text{ if } J(\mathbf{x}_i[k+1]) \geq J(\mathbf{p}_i[k]) \\ \mathbf{x}_i[k+1] \text{ if } J(\mathbf{x}_i[k+1]) < J(\mathbf{p}_i[k]), \end{cases} \tag{2}$$

for a maximization problem, the update of the vector of best position $\mathbf{p}_i[k]$ of the i-th particle is given by:

$$\mathbf{p}_i[k+1] = \begin{cases} \mathbf{p}_i[k] \text{ if } J(\mathbf{x}_i[k+1] \leq J(\mathbf{p}_i[k]) \\ \mathbf{x}_i[k+1] \text{ if } J(\mathbf{x}_i[k+1]) > J(\mathbf{p}_i[k]) \end{cases} \tag{3}$$

For a minimization problem, the vector of best global position $\mathbf{p}_g[k]$ of the N particles is updated as follows:

$$\begin{aligned} &\mathbf{p}_g[k+1] \in \{\mathbf{p}_1[k+1], \mathbf{p}_2[k+1], \ldots, \mathbf{p}_N[k+1]\} \\ &\mathbf{p}_g[k+1] = \underset{1\leq i\leq N}{\arg\min}\ J(\mathbf{p}_i[k+1]), \end{aligned} \tag{4}$$

for a maximization problem, the update of the vector of best global position $\mathbf{p}_g[k]$ of the N particles is given by:

$$\begin{aligned} &\mathbf{p}_g[k+1] \in \{\mathbf{p}_1[k+1], \mathbf{p}_2[k+1], \ldots, \mathbf{p}_N[k+1]\} \\ &\mathbf{p}_g[k+1] = \underset{1\leq i\leq N}{\arg\max}\ J(\mathbf{p}_i[k+1]) \end{aligned} \tag{5}$$

3. PROPOSED OPTIMIZATION METHODOLOGY

In this section, the proposed optimization methodology referring to GA-PSO hybridization with fuzzy parametric adpatation of the acceleration coefficients and inertial weight is presented. The position vector $\mathbf{x}_i[k]$ of the i-th particle, at each iteration k, is updated by the genetic operators of parent selection, crossover and mutation. Right after, the new position vectors are evaluated by the fitness function J. After this, the vector of best position $\mathbf{p}_i[k]$ of the i-th particle and the vector of best global position $\mathbf{p}_g[k]$ of the N particles are updated. After this, the diversity and iteration normalizeds by the Min-Max method are calculated, which are the inputs of MFIS. After performing the fuzzy parametric adpatation of the acceleration coefficients and inertial weight, the velocity vector $\mathbf{v}_i[k]$ and the position vector $\mathbf{x}_i[k]$ of the i-th particle are updated. After this, the vector of best position $\mathbf{p}_i[k]$ of the i-th particle and vector of best global position $\mathbf{p}_g[k]$ of the N particles are updated. If the stopping condition is satisfied, then the solution to the problem is obtained; otherwise, the search for the optimal solution is continued until the stopping condition be satisfied.

3.1 Genetic Operator of Parent Selection

According to the crossover rate r_c, the genetic operator of parent selection is used with the objective of selecting the particles with best fitness J to be reproduced by the genetic operator of crossover. The selected particles will be the parent particles of the next generation of particles. For selecting the particles, in this methodology it is used the operator genetic of parents selection of tournament type, given by following pseudocode:

Algorithm 1 Tournament Method

Set the crossover rate r_c;
for $g = 1$ to Nr_c **do**
  for $l = 1$ to 2 **do**
    Choose randomly m particles;
    $parent_l$ = the particle with fitness best J;
  end for
end for

3.2 Genetic Operator of Crossover

After selecting the particles with best fitness, according to the crossover rate $r_c \in [0.5,\ 1]$, the particles selected (parent particles) will be reproduced by the genetic operator of crossover. Thus, the new generation of particles will have a greater quantity of characteristics of the parent particles, which will facilitate the search by the optimal solution of the problem. For reproduction of particles, in this methodology it is used the operator genetic of uniform type, given by the pseudocode **Algorithm 2**.

Algorithm 2 Uniform Crossover Method

for $g = 1$ to $Nr_c/2$ **do**
  Choose two particles randomly and obtain randomly $\alpha \in [0, 1]$;
  Obtain randomly $\alpha \in [0, 1]$;
  $son_1 = \alpha parent_1 + (1 - \alpha) parent_2$;
  $son_2 = \alpha parent_2 + (1 - \alpha) parent_1$;
end for

3.3 Genetic Operator of Mutation

The genetic mutation of particles of a population is performed by the genetic operator of mutation. The genetic operator of mutation is used during the search process for optimal solutions, with the objective of exploring new regions in the search space. According to the mutation rate $r_m \in [0.005,\ 0.05]$, some characteristic of the particle are modified by the genetic mutation. For genetic mutation of particles, in this methodology it is used the operator genetic of mutation of random type, given by the following pseudocode:

Algorithm 3 Random Mutation Method

for $i = 1$ to N **do**
  Obtain randomly $\beta \in [0, 1]$;
  if $\beta < r_m$ **then**
    Obtain randomly $\gamma \in \{1, 2, \ldots, n\}$;
    Perform the gene mutation:
    $x_{i,\gamma}[k] = \max(x_{i,\gamma}) + \beta(\max(x_{i,\gamma}) - \min(x_{i,\gamma}))$;
  end if
end for

3.4 Fuzzy Parametric Adaptation of the Acceleration Coefficients and Inertial Weight

According to the Section 3, after performing the genetic operators, the next step of the proposed optimization methodology is to perform the fuzzy parametric adaptation of the acceleration coefficients and inertial weight, through an MFIS. The fuzzy parametric adpatation of the acceleration coefficients and inertial weight is described as follows:

$$[\omega[k], C_1[k], C_2[k]] = \mathrm{MFIS}(\mathcal{D}[k], \mathcal{K}[k])$$
$$\mathcal{D}[k] = \frac{D[k] - d_{\min}[k]}{d_{\max}[k] - d_{\min}[k]} \text{ and } \mathcal{K}[k] = \frac{k-1}{K-1}, \qquad (6)$$

where $\omega[k]$, $C_1[k]$ and $C_2[k]$ are the output variables of MFIS. The variable $D[k]$ is the diversity measure obtained through the mean Euclidean distance between the position vector $x_i[k]$ of the i-th particle and the vector of best global position $p_g[k]$ of the N particles. The variables $d_{\min}[k]$ and $d_{\max}[k]$ are, respectively, the smallest and largest value obtained by the diversity until the iteration k. The variables $\mathcal{D}[k]$ and $\mathcal{K}[k]$, which are the inputs of MFIS, are, respectively, the diversity and the iteration normalizeds by the Min-Max method. It is important to note that the inputs variables of MFIS are the linguistic variables of the antecedent of the r-th fuzzy rule.

A fuzzy rule is defined as **If** *fuzzy propositions of the antecedent* **then** *propositions of the consequent*, where the fuzzy propositions of the antecedent and consequent are formed by linguistic values, which are associated to linguistic variables of the antecedent and consequent (Wang,

Table 1. Fuzzy rule for parametric adaptation of the acceleration coefficients and inertial weight.

$\mathcal{R}^1$: **If** ($\mathcal{K}[k]$ *is* S) **and** ($\mathcal{D}[k]$ *is* S) **then** ($\bar{\omega}[k]$ *is* L)($\bar{C}_1[k]$ *is* L)($\bar{C}_2[k]$ *is* S)
$\mathcal{R}^2$: **If** ($\mathcal{K}[k]$ *is* S) **and** ($\mathcal{D}[k]$ *is* M) **then**($\bar{\omega}[k]$ *is* M)($\bar{C}_1[k]$ *is* ML)($\bar{C}_2[k]$ *is* M)
$\mathcal{R}^3$: **If** ($\mathcal{K}[k]$ *is* S) **and** ($\mathcal{D}[k]$ *is* L) **then** ($\bar{\omega}[k]$ *is* S)($\bar{C}_1[k]$ *is* ML)($\bar{C}_2[k]$ *is* MS)
$\mathcal{R}^4$: **If** ($\mathcal{K}[k]$ *is* M) **and** ($\mathcal{D}[k]$ *is* S) **then** ($\bar{\omega}[k]$ *is* M)($\bar{C}_1[k]$ *is* ML)($\bar{C}_2[k]$ *is* MS)
$\mathcal{R}^5$: **If** ($\mathcal{K}[k]$ *is* M) **and** ($\mathcal{D}[k]$ *is* M) **then** ($\bar{\omega}[k]$ *is* M)($\bar{C}_1[k]$ *is* M)($\bar{C}_2[k]$ *is* M)
$\mathcal{R}^6$: **If** ($\mathcal{K}[k]$ *is* M) **and** ($\mathcal{D}[k]$ *is* L) **then** ($\bar{\omega}[k]$ *is* M)($\bar{C}_1[k]$ *is* MS)($\bar{C}_2[k]$ *is* ML)
$\mathcal{R}^7$: **If** ($\mathcal{K}[k]$ *is* L) **and** ($\mathcal{D}[k]$ *is* S) **then** ($\bar{\omega}[k]$ *is* M)($\bar{C}_1[k]$ *is* M)($\bar{C}_2[k]$ *is* L)
$\mathcal{R}^8$: **If** ($\mathcal{K}[k]$ *is* L) **and** ($\mathcal{D}[k]$ *is* M) **then** ($\bar{\omega}[k]$ *is* S)($\bar{C}_1[k]$ *is* MS)($\bar{C}_2[k]$ *is* L)
$\mathcal{R}^9$: **If** ($\mathcal{K}[k]$ *is* L) **and** ($\mathcal{D}[k]$ *is* L) **then** ($\bar{\omega}[k]$ *is* S)($\bar{C}_1[k]$ *is* S)($\bar{C}_2[k]$ *is* L)

1999). It is important to note that a linguistic value represents a fuzzy set, which is characterized by a Membership Function (MBF). Due to the use of fuzzy propositions, each linguistic variable of the antecedent and consequent receives each linguistic value of the antecedent and consequent with a certain membership degree, through the following mappings performed by the MBFs $\mu_u(\mathcal{K}[k]) : U \to [0,1]$, $\mu_v(\mathcal{D}[k]) : V \to [0,1]$, $\mu_z(\bar{C}_1[k]) : Z \to [0,1]$, $\mu_q(\bar{C}_2[k]) : Q \to [0,1]$ and $\mu_w(\bar{\omega}[k]) : W \to [0,1]$. The universes of discourse, in which each MBF is defined, were set equal to $U,V = [0,1]$, $Z,Q = [0,3]$ and $W = [0,0.9]$.

For the fuzzy propositions of the antecedent, three MBFs of the triangular type were defined with the linguistic values Small (S) for $u,v = 1$, Medium (M) for $u,v = 2$ and Large (L) for $u,v = 3$. The parametric intervals of MBFs of the antecedent can be seen in Table 2. For the fuzzy propositions of the consequent, three MBFs of the triangular type were defined for the linguistic variable $\bar{\omega}$ of the consequent, with the linguistic values Small (S) for $w = 1$, Medium (M) for $w = 2$ and Large (L) for $w = 3$. For the linguistic variables $\bar{C}_1[k]$ and $\bar{C}_2[k]$, five MBFs of the triangular type were defined with the linguistic values Small (S) for $z,q = 1$, Medium Small (MS) for $z,q = 2$, Medium (M) for $z,q = 3$, Medium Large (ML) for $z,q = 4$ and Large (L) for $z,q = 5$. The parametric intervals of MBFs of the consequent can be seen in Table 2.

$$
\begin{aligned}
\mu_{\mathcal{R}^r_{\bar{C}'_1}} &= \min[\mu_u(\mathcal{K}[k]), \mu_v(\mathcal{D}[k]), \mu_z(\bar{C}_1[k])] \\
\mu_{\mathcal{R}^r_{\bar{C}'_2}} &= \min[\mu_u(\mathcal{K}[k]), \mu_v(\mathcal{D}[k]), \mu_q(\bar{C}_2[k])] \\
\mu_{\mathcal{R}^r_{\bar{\omega}'}} &= \min[\mu_u(\mathcal{K}[k]), \mu_v(\mathcal{D}[k]), \mu_w(\bar{\omega}[k])]
\end{aligned}
\tag{7}
$$

According to expert's knowledge about how the trade-off between global and local search should be performed, the fuzzy rule base and the MBFs were defined. The expert's knowledge can be obtained through past experiences during simulations and analysis of the problem. The fuzzy rule base developed for fuzzy parametric adaptation of the acceleration coefficients and inertial weight can be seen in Table 1. The fuzzy propositions of the antecedente and consequent are related through conditional fuzzy propositions, which are modeled by the fuzzy relations between the universes of discourse $U \times V$ of the antecedent and Z,Q,W of the consequent, given by the fuzzy implication (7).

For each fuzzy rule, due to performing of the fuzzy implications, are obtained the MBFs described in (7). All the MBFs obtained through the fuzzy implication are combined through the fuzzy aggregation in order to obtain a single MBF, which represents a total response, for each linguistic variable of the consequent, given by:

$$
\begin{aligned}
\mu_{Total_{\bar{C}'_1}} &= \max[\mu_{\mathcal{R}^1_{\bar{C}'_1}}, \mu_{\mathcal{R}^2_{\bar{C}'_1}}, \ldots, \mu_{\mathcal{R}^9_{\bar{C}'_1}}] \\
\mu_{Total_{\bar{C}'_2}} &= \max[\mu_{\mathcal{R}^1_{\bar{C}'_2}}, \mu_{\mathcal{R}^2_{\bar{C}'_2}}, \ldots, \mu_{\mathcal{R}^9_{\bar{C}'_2}}] \\
\mu_{Total_{\bar{\omega}'}} &= \max[\mu_{\mathcal{R}^1_{\bar{\omega}'}}, \mu_{\mathcal{R}^2_{\bar{\omega}'}}, \ldots, \mu_{\mathcal{R}^9_{\bar{\omega}'}}]
\end{aligned}
\tag{8}
$$

After obtained (8), it is necessary to obtain a numerical value for the acceleration coefficients and for the inertial weight, through the defuzzification of each MBF described in (8). In this methodology, the defuzzification method used is centroid type, such that the output variables of MFIS are given by:

$$
\begin{aligned}
C_1[k] &= \frac{\sum_{r=1}^{9} \bar{C}_1[k]\mu_{Total_{\bar{C}'_1}}(\bar{C}_1[k])}{\sum_{r=1}^{9} \mu_{Total_{\bar{C}'_1}}(\bar{C}_1[k])} \\
C_2[k] &= \frac{\sum_{r=1}^{9} \bar{C}_2[k]\mu_{Total_{\bar{C}'_2}}(\bar{C}_2[k])}{\sum_{r=1}^{9} \mu_{Total_{\bar{C}'_2}}(\bar{C}_2[k])} \\
\omega[k] &= \frac{\sum_{r=1}^{9} \bar{\omega}[k]\mu_{Total_{\bar{\omega}'}}(\bar{\omega}[k])}{\sum_{r=1}^{9} \mu_{Total_{\bar{\omega}'}}(\bar{\omega}[k])}
\end{aligned}
\tag{9}
$$

3.5 Velocity Vector and Position Vector Update Equations

In (10), it is presented the velocity vector update equation of the i-th particle. It is important to note that the velocity vector update equation is composed by $r_1, r_2 \in [0,1]$, which introduce the stochastic nature to PSO. The update of the j-th dimension of velocity vector of the i-th particle is given by:

$$
\begin{aligned}
&v_{i,j}[k+1] = \omega[k]v_{i,j}[k] + C_1[k]r_1(p_{i,j}[k] - x_{i,j}[k]) \\
&+C_2[k]r_2(p_{g,j}[k] - x_{i,j}[k]),
\end{aligned}
\tag{10}
$$

where $C_1[k], C_2[k] \in [0,3]$ and $\omega[k] \in [0,1]$. According to Kennedy and Eberhart (1995), the acceleration coefficients $C_1[k]$ and $C_2[k]$ control the individual and global knowledge sharing about the best positions obtained until the current iteration, respectively, by the pondering with the terms $(p_{i,j}[k] - x_{i,j}[k])$ and $(p_{g,j}[k] - x_{i,j}[k])$. According to Shi and Eberhart (1998), in (11) the inertial

Table 2. Parametric intervals of triangular MBFs.

	$\mathcal{K}[k]$	$\mathcal{D}[k]$	$C_1[k]$	$C_2[k]$	$\omega[k]$
Linguistic Value	Interval	Interval	Interval	Interval	Interval
Small	[0 0 0.1]	[0 0 0.5]	[0 0.5 1.0]	[0 0.5 1.0]	[0.001 0.09 0.15]
Medium Small	———	———	[0.5 1.0 1.5]	[0.5 1.0 1.5]	———
Medium	[0 0.1 0.3]	[0 0.5 1]	[1.0 1.5 2.0]	[1.0 1.5 2.0]	[0.09 0.5 0.35]
Medium Large	———	———	[1.5 2.0 2.5]	[1.5 2.0 2.5]	———
Large	[0.1 0.3 1.0]	[0.5 1.0 1.0]	[2.0 2.5 3.0]	[2.0 2.5 3.0]	[0.15 0.35 0.9]

weight $\omega[k]$ has the function to ponder the j-th dimension of velocity vector obtained in previous iteration in the current iteration. So, by the correctly selection of values for the acceleration coefficients and inertial weight, it is possible to obtain a good trade-off between the global and local search. After this, the position vector is updated, given by:

$$\mathbf{x}_i[k+1] = \mathbf{x}_i[k] + \mathbf{v}_i[k+1] \tag{11}$$

4. COMPUTATIONAL RESULTS

In this section, it is performed the minimization of four benchmark functions described in Table 4 and given below:

$$\begin{aligned}
f_1(\mathbf{x}[k]) &= \sum_{k=1}^{K}(\mathbf{x}[k]^2) \\
f_2(\mathbf{x}[k]) &= \sum_{k=1}^{K-1}\left[100(\mathbf{x}[k+1]-\mathbf{x}[k]^2)^2 + (\mathbf{x}[k]-1)^2\right] \\
f_3(\mathbf{x}[k]) &= 10K + \sum_{k=1}^{K}\left(\mathbf{x}[k]^2 - 10cos2\pi\mathbf{x}[k]\right) \\
f_4(\mathbf{x}[k]) &= 1 + \frac{1}{4000}\sum_{K=1}^{k}\mathbf{x}[k]^2 - \prod_{k=1}^{K} cos\frac{\mathbf{x}[k]}{\sqrt{k}}
\end{aligned} \tag{12}$$

The results obtained by the optimization of benchmark functions through the proposed optimization methodology were compared with the results obtained by the GA, PSO and GA-PSO algorithms. In Table 3, are shown the results obtained by the optimization of benchmark functions through the algorithms mentioned above, with the crossover rate set equal to $r_c = 0.9$, mutation rate set equal to $r_m = 0.1$, acceleration coefficient set equal to $C_1 = C_2 = 2$ and inertial weight set equal to $\omega = 1$ for the GA, PSO and GA-PSO algorithms. The parameter m, used to select the parent particles by the tournament method, was set equal to $m = 3$. The initial position vectors $\mathbf{x}_i[1]$ were initialized randomly for all algorithms. The velocity vectors and the vectors of best position were initialized with $\mathbf{p}_i[1] = \mathbf{x}_i[1]$ and $\mathbf{v}_i[1] = \mathbf{0}$. The particle population size was set equal to $N = 100$ and, for each benchmark function, its positions were evaluated during $K = 300$ iterations.

Since all the algorithms evaluated in this section are stochastic, then the number of Monte Carlo simulations referring to optimization of each benchmark function was set equal to 100. In addition to multimodal problems, the proposed optimization methodology was evaluated on multidimensional problems, where the dimension of the search space was set equal to $n = 20$. Due to the optimization algorithms have developed the worst results in the optimization of benchmark $f_3(\mathbf{x}[k])$ as presented in the Table 3, thus the performance analysis was performed only for the worst case, i.e., for the results obtained for the benchmark $f_3(\mathbf{x}[k])$.

In Fig. 1 (a), it is possible to observe that the search process performed by the PSO converged prematurely, which is due to the benchmark $f_3(\mathbf{x}[k])$ be multimodal, since the its search process is performed in a unidirectional manner. Through the statistical metrics shown in Table 3, it is possible to confirm the unsatisfactory performance of PSO. In Fig. 1 (a), when compared to GA-PSO hybridization, it can be seen that the GA converged slower, which is due to its search process be performed in a multidirectional manner through the use of genetic operators. When compared to the GA, the GA-PSO hybridization obtained a more fast convergence than the GA, which is due to its search process be performed in both multidirectional and unidirectional manner, since it is a hybridization and, consequently, develops optimization features belonging to the GA and PSO. With respect to performance of the proposed optimization algorithm, it is noted that, besides to avoid premature convergence, a fastest convergence and the best results in the optimization of the benchmark $f_3(\mathbf{x}[k])$ were obtained.

As shown in Fig. 1 (c), through the fuzzy parametric adaptation of the acceleration coefficients by the MFIS, it is possible to note that during the first iterations, aiming a larger pondering in the sharing of individual knowledge, the acceleration coefficient $C_1[k] > C_2[k]$ so that particles can explore taking into considering the its individual learning about the regions in the search space with the best fitness. As the iterations advance, aiming at a larger pondering in the sharing of social knowledge for performing a local search near the particle with better fitness, it is observed that $C_2[k] > C_1[k]$. As shown in Fig. 1 (b), through the fuzzy parametric adaptation of the inertial weight by the MFIS, it is possible to note that during the first iterations, so that the particles can perform a global search, the inertial weight have a large value. As the iterations advance, so that the particles can perform a local search and, thus, convergence occurs, the value of the inertial weight was decreased. It is important to note that the fuzzy parametric adaptation of the acceleration coefficients and

Table 3. Results obtained through the Monte Carlo simulations of the search process.

Algorithm	Results	$f_1(\mathbf{x}[k])$	$f_2(\mathbf{x}[k])$	$f_3(\mathbf{x}[k])$	$f_4(\mathbf{x}[k])$
PSO	Mean Fitness	0.0213	1.0740×10^{-10}	163.0306	0.0017
	Standard Deviation	0.0328	3.3789×10^{-10}	162.1489	0.0028
GA	Mean Fitness	4.8759×10^{-4}	0.0725	8.0351	0.0069
	Standard Deviation	4.1236×10^{-4}	0.1640	8.1492	0.0032
GA-PSO	Mean Fitness	4.3499×10^{-12}	0	3.9898	0
	Standard Deviation	1.3724×10^{-11}	0	4.9871	0
Proposed	**Mean Fitness**	1.0936×10^{-21}	0	1.2919×10^{-15}	0
	Standard Deviation	3.4432×10^{-21}	0	4.2848×10^{-15}	0

Table 4. Description of benchmark functions.

Function	**Name**	**Search Space**	**Modes**
$f_1(\mathbf{x}[k])$	Sphere	[-5.12, 5.12]	Unimodal
$f_2(\mathbf{x}[k])$	Rosenbrock	[-5, 10]	Multimodal
$f_3(\mathbf{x}[k])$	Rastrigin	[-5.12, 5.12]	Multimodal
$f_4(\mathbf{x}[k])$	Griewank	[-600, 600]	Multimodal

inertial weight was not performed in a linear manner, since the behavior of the particles in the search space is non-linear. Through the fuzzy parametric adaptation of the acceleration coefficients and inertial weight, it was possible to obtain a good trade-off between the global and local search and, consequently, to perform a search process with a fast and not premature convergence.

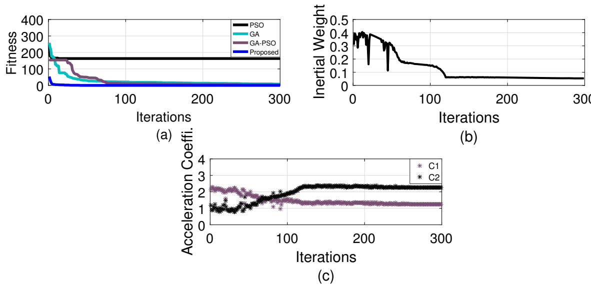

Fig. 1. Fitness curve obtained for the benchmark $f_3(\mathbf{x}[k])$ (a), Adaptive inertial weight for optimization of the benchmark $f_3(\mathbf{x}[k])$ (b) and Adaptive acceleration coefficients for optimization of the benchmark $f_3(\mathbf{x}[k])$ (c).

5. CONCLUSION

Through the fuzzy parametric adaptation of the acceleration coefficients and inertial weight through MFIS, a good trade-off between the global and local search was obtained. It was observed that the variations of values obtained during the parametric adaptation of the acceleration coefficients and inertial weight were performed with objective of providing better conditions so that the particles may balance the trade-off between global and local search. However, the variations of values obtained for the acceleration coefficients and inertial weight were not performed in a linear manner, which is a coherent result, since the search process, which is stochastic, does not allow the particles to develop a linear search behavior. The non-linear behavior of the values obtained for the acceleration coefficients and inertial weight allowed that, due to information provided about the distances between the particles and about the temporal advance of the search process, when stuck in local optimal, the particles would jump to better positions; thus, avoiding a premature convergence and developing a fast convergence.

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