Memorias del Congreso Nacional de Control Automático
ISSN: 2594-2492

Kalman-like Disturbance Observer-based Control. ⋆

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Abstract: State estimation using the Kalman Filter (KF) is one of the principal topics in areas such as guidance, navigation, and control. The disturbance observer (DOB) is a control scheme with a proven efficiency reported in the literature. In this article, we develop an alternative method, in which we estimated the compensation of unknown disturbances with the Kalman filter. We use the estimation to cancel the effects of the disturbance in the system. It gave a simulation example to show the effectiveness of the proposed method against classical algorithms.

Keywords: Disturbance Observer, Kalman Filter

1. INTRODUCTION

Disturbances broadly exist in control systems and consequently appear deviations in the steady-state errors. Therefore, disturbance rejection is one of the main issues in control systems. A disturbance usually appears from external unknown-interaction, a direct compensation in the control system is not possible. In control systems, *soft sensors* [González (2010)] are used to estimate non-measurable states in the system, they need a set of measurements and an inferential plant model. In the same way, a DOB used in a loop control can estimate and compensate a bounded unknown disturbances. Prof. Ohnishi has proposed for the first time in 1983 [Ohishi et al. (1983)] looking for an alternative control method to the integral action in the controllers. His efforts lead to the concept of equivalent disturbance used to compensate for the input and minimize the effects of uncertainties in the model. Disturbance Observer-based control gains a relevant role in further investigations.

The conventional design methods for DOB are based on frequency domain techniques[Sariyildiz and Ohnishi (2013)] but exists many implementations of disturbance observers [Radke and Gao (2006)]. In time-domain formulations, a multi-input multi-output system with disturbances can be controlled by computing an observer gain matrix (L)[] and a constant for a proportional control law (K). The closed-loop control is asymptotically stable with the fitting values for parameters K and L. Other solutions [Phuong et al. (2018),Hosseinnajad and Loueipour (2021)] use the Kalman Filter to smooth the output before doing the feedback to the Disturbance Observer-based Controller. Recent approaches include hybrid solutions[Zhu et al. (2019)] with two stages: controller and disturbance-estimator. Some other elegant control methods reject disturbance by feedback control and employ advanced algorithms like Adaptative Control [Cui (2019)], Robust Control[Hosseinnajad and Loueipour (2021)], and Sliding Mode Control [Wang et al. (2020)]. These control strategies achieve disturbance rejection by tracking the error between measured outputs and their input control signal.

The settling time of these kinds of controllers cannot react fast in presence of strong perturbations, and there suppresses the disturbance slowly. In this article, we propose a Disturbance Observer design with the structure of a Kalman Filter. An augmented model can estimate the disturbance with zero bias. For the control stage, we choose the Proportional Velocity Integral (PVI) control architecture with an additional Proportional-Derivative (PD) control loop as feedforward compensation. The performance for Kalman Disturbance Observer (KDOB)and stability in a closed-loop is explored via simulations.

2. PROBLEM FORMULATION

It is proven that the Kalman Filter is an optimal estimator in the mean square error(MSE) sense, and with only a measurement vector y_n and a set of initial estimation values $\hat{x}_0, P_n$ can estimate the internal states $\hat{x}_n$ for a discrete stochastic system. A linear system can be described in state-space representation as shown in equation (1)

⋆ This investigation was partly supported by the Mexican CONACyT-SEP Project A1-S-10287, Funding CB2017-2018.

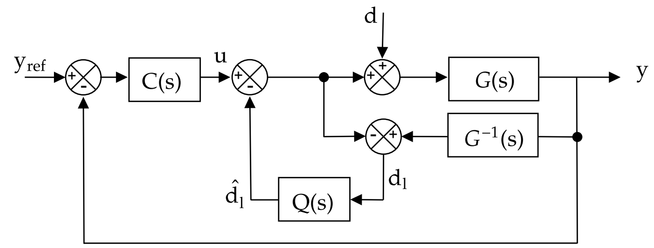

Fig. 1. General realization of continuous time DOBC

$$\begin{aligned} x_n &= A_n x_{n-1} + B_n u_n + w_n \,, \\ y_n &= C_n x_n + v_n \,, \end{aligned} \tag{1}$$

Where $x_n = [x_1, \cdots, x_K]^T \in \mathbb{R}^K$ is a state vector, $y_n = [y_1, \cdots, y_M]^T \in \mathbb{R}^M$ is an observation vector, $u_n = [u_1, \cdots, u_L]^T \in \mathbb{R}^L$ is an exogenous input (control signal) vector, and the vectors $w_n \sim \mathcal{N}(0, Q_n)$ and $v_n \sim \mathcal{N}(0, R_n)$ are the process and measurement noises, repectivly. The matrices $A_n \in \mathbb{R}^{K\times K}$ (state-transition matrix), $B_n \in \mathbb{R}^{K\times L}$ (input matrix) and $C_n \in \mathbb{R}^{M\times K}$ (Observation matrix) are know. The matrices Q_n and R_n are the covariances of the white Gaussian uncorrelated noise vectors w_n, v_n.

We assume that a control signal u_n actuates the system but with an additive perturbation vector, so the signal control that feeds the plant is $\bar{u}_n = u_n + d_n$. The classical DOB control techniques [Li et al. (2017)] shown in Fig(1) uses an inverse model of the plant $G(s)$ and a transfer function $Q(s)$ which is a low pass filter to estimate the perturbation $\hat{d}_l$. More complex designs of observers are MIMO systems represented with a state equation [Chen et al. (2016)].

Thus, to determine the perturbation we represent $\hat{d}_n$ as a state variable, the augmented state vector is $\hat{x}_n = [x_1 \cdots x_K, d_1 \cdots d_L]^\intercal$, the corresponding augmentative model must add the behavior of the augmentative state, assuming that the perturbation is slow compared to the sampling time we get $\dot{d(t)} \approx 0$, and by the Euler forward method and incorporating the additive process noise $d(n+1) = d(n) + \bar{w}_n$ where $\bar{w}_n$ represents the augmented noise components. We give the augmented model defined as:

$$\begin{aligned} X_n &= \mathscr{A}_n X_{n-1} + \mathscr{B}_n U_n + \mathscr{W}_n \,, \\ Z_n &= \mathscr{C}_n X_n + \mathscr{V}_n \,, \end{aligned} \tag{2}$$

Where the matrices are:

$$\begin{aligned} X_n &= \begin{bmatrix} x_{n+1} \\ d_{n+1} \end{bmatrix} \qquad Z_n = \begin{bmatrix} y_n \\ d_n \end{bmatrix} \qquad U_n = \begin{bmatrix} u_n \\ \mathbf{0} \end{bmatrix} \\ \mathscr{A}_n &= \begin{bmatrix} A_n & B_n \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \qquad \mathscr{B}_n = \begin{bmatrix} B_n & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \qquad \mathscr{C}_n = \begin{bmatrix} C_n & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \\ \mathscr{W}_n &= \begin{bmatrix} w_n \\ \bar{w}_n \end{bmatrix} \qquad \mathscr{V}_n = \begin{bmatrix} v_n \\ \bar{v}_n \end{bmatrix} \end{aligned} \tag{3}$$

And the covariance of the augmented noise matrices $W_n, \mathscr{V}_n$ are $\mathscr{Q}_n = \mathsf{diag}(Q_n, \bar{Q}_n)$ and $\mathscr{R}_n = \mathsf{diag}(R_n, \bar{R}_n)$.

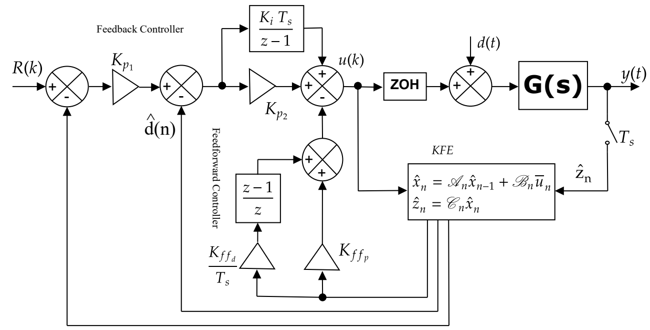

Fig. 2. Diagram block of Kalman Disturbance Observer-based Control

Figure 2 shown our proposal for a Kalman-like disturbance observer-based control (KDOBC). We used a classical PIV topology in the feedback controller. The major difference with a PID controller relies on the necessity to have measurements for position and velocity, each one for two different control loops.

In practical applications it's common to use filters to estimate the velocity using the position measure, however, significant delays can cause loss of accuracy. The KFE algorithm will achieve the offline measurements of disturbance. We then incorporated the estimated perturbance into the control as a feed-forward compensation and the filtered output as a feedback compensation.

In the feed-forward control loop, we have implemented a discrete PD controller to predict the future value of the disturbance, but without minimizing the error in steady-state. To tune the PIV controller, we require the bandwidth (BW) and the damping ratio (ζ). The total inertia J and viscous friction B are also required. When the damping ratio is fixed, the bandwidth determines the rise time. The damping ratio is related to the overshoot for a fixed bandwidth. The best practices to syntonize controllers can be found in [Ziegler and Nichols (1942)].

Given an initial error covariance for the augmented model $\mathscr{P}_0 \in \mathbb{R}^{(K+L)\times(K+L)}$, the initial state vector $X_0^\intercal \in \mathbb{R}^{(K+L)}$ and the noise covariances matrices $\mathscr{Q}_n \in \mathbb{R}^{(K+L)\times(K+L)}$, $\mathscr{R}_n \in \mathbb{R}^{(M+L)\times(M+L)}$ and an observation vector $Z_n^\intercal \in \mathbb{R}^{(M+L)}$ the Kalman Filter Estimator (KFE) can recover the optimal estimated-value of disturbances $\hat{d}_n$ with zero-bias, the capabilities of the KFE to ensure controllability and observability in LTI systems regarding the process noise has been extensively estudied[Rhudy and Gu (2013); Su et al. (2015); Zhang (2017)]. The algorithm for the KF is given in algorithm 1.

3. APPLICATION: MECHANICAL SERVO SYSTEMS

Mechanical servo systems have been extensively used in CNC machining, industrial applications, cameras, tele-

Algorithm 1 KFE Algorithm

Input: $\hat{\mathbf{X}}_0$, $\mathbf{Z}_n$, $\mathbf{U}_n$, $\mathscr{P}_0$, $\mathscr{Q}_n$, $\mathscr{R}_n$
Output: $\hat{X}_n$, $\mathscr{P}_n$
for $n = 1, 2, \cdots$ **do**
$\mathbf{P}_n^- = \mathscr{A}_n \mathscr{P}_{n-1} \mathscr{A}_n^\intercal + \mathscr{Q}_n$
$\mathbf{S}_n = \mathscr{C}_n \mathbf{P}_n^- \mathscr{C}_n^\intercal + \mathscr{R}_n$
$\mathbf{K}_n = \mathbf{P}_n^- \mathscr{C}_n^\intercal \mathbf{S}_n^- 1$
$\hat{\mathbf{X}}_n^- = \mathscr{A}_n \hat{\mathbf{X}}_{n-1} + \mathscr{B}_n \mathbf{U}_n$
$\hat{\mathbf{X}}_n = \hat{\mathbf{X}}_n^- + \mathbf{K}_n(\mathbf{Z}_n - \mathscr{C}_n \hat{\mathbf{X}}_n^-)$
$\mathbf{P}_n = (\mathbf{I} - \mathbf{K}_n \mathscr{C}_n)\mathbf{P}_n^-$
end

scopes, elevators, and robotics[Preitl et al. (2007)]. In a servo application, a command signal is used to control the position of a load. As the signal actuates the servomotor, the load changes its position. At the same time, an encoder provides a negative feedback signal, sending to the controller the states of the load. The controller looks at this feedback and, based on a control law, calculates the appropriate value for the control signal to achieve the position given by the command signal. In the fig (3) a general control scheme for a servomotor is shown.

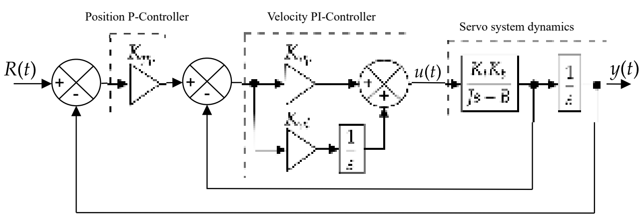

Fig. 3. Servo Control System

The equation 4 gives the transfer function for the servo system in Fig3.

$$\frac{C(s)}{U(s)} = \frac{K_t K_b}{s(Js + B)} \tag{4}$$

In a continuous-time state-space representation, we can write the equation above as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\dfrac{B}{J} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{K_t K_b}{J} \end{bmatrix} r(t) \,, \qquad y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \,, \tag{5}$$

Where $r(t)$ is the input voltage applied to the servo, x_1 is the output displacement of the servo, x_2 the velocity, J is the total motor's inertia, B is the viscous friction coefficient, K_t the torque voltage ratio and K_b the ball-screw constant. A controller for this application needs specific considerations to reduce or eliminate the effect of disturbance in the controller. Perturbance rejection is one of the principal objectives in controller design. In general, a disturbance is not only because of the environment but of uncertainties from the model. The traditional methods such as proportional-integral-derivative(PID) controller are not designed to consider the presence of perturbations, so they will be incapable to correct the output in the presence of severe perturbances.

However, the disturbance is not usually available for measurement, and their nature can't be represented in a parametrized form, therefore deterministic system identification methods[Ljung (2010)] can't be used.

The Kalman Filter needs a discrete-time state-space model, the matrix A_n is obtained using the Laplace Transform method and B_n by the integral approximation method, the system in Eqs (5) becomes:

$$\begin{aligned}
A_n &= \mathcal{L}^{-1}\{(s\mathbf{I} - \mathbf{A})^{-1}\}|_{t=T_s} \,, \\
&= \mathcal{L}^{-1}\left\{ \begin{bmatrix} s & -1 \\ 0 & s + \dfrac{B}{J} \end{bmatrix}^{-1} \right\}|_{t=T_s} \,, \\
&= \mathcal{L}^{-1}\left\{ \begin{bmatrix} 1/s & J/(Js^2 + Bs) \\ 0 & J/(Js + B) \end{bmatrix} \right\}|_{t=T_s} \,, \\
&= \begin{bmatrix} 1 & J(1 - e^{-BT_s/J})/B \\ 0 & e^{-BT_s/J} \end{bmatrix} \,, \\
B_n &= \int_0^{T_s} e^{\mathbf{A}\sigma} d\sigma \mathbf{B} \,, \\
&= \int_0^{Ts} \begin{bmatrix} 1 & J(1 - e^{-B\sigma/J})/B \\ 0 & e^{-B\sigma/J} \end{bmatrix} d\sigma \begin{bmatrix} 0 \\ \dfrac{K_t K_b}{J} \end{bmatrix} \,, \\
&= \frac{K_t K_b}{J} \int_0^{Ts} \begin{bmatrix} J(1 - e^{-B\sigma/J})/B \\ e^{-B\sigma/J} \end{bmatrix} d\sigma \,, \\
&= \frac{K_t K_b}{J} \begin{bmatrix} \dfrac{J}{B} T_s - \left(\dfrac{J}{B}\right)^2 (1 - e^{-BT_s/J}) \\ \dfrac{J}{B}\left(1 - e^{-BT_s/J}\right) \end{bmatrix} \,, \\
C_n &= \mathbf{C} \,, \\
&= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\end{aligned} \tag{6}$$

Finally, the augmented model for the KFE is given in Eqs. (3) can be calculated from Eqs. (6)

4. SIMULATIONS

To verify the effectiveness of the KDOBC, a simulation of a servomotor system for linear movement was conducted, the parameters for the model were taken from [Lin et al. (2008)], The tuning of the PD control loop is resumed in the next steps

- Set K_{ffd} and K_{ffp} gains to zero
- Increase K_{ffp} gain until the response to a disturbance produce damped oscillations
- Slightly increase K_{ffd} gain until his value does not affect the output

The simulation parameters are on table 1. To verify the robustness of the KDBOC, two experiments were conducted. First, the response to a short step perturbation.

The second, the effect of a periodic ramp as a disturbance signal.

Symbol	Units
K_{p1}	$156.064\ 1/s$
K_{p2}	$3.67x10^{-2}\ Vs/mm$
K_i	$5.36\ V/mm$
K_{ffd}	8.25
K_{ffp}	1.02
F_s	$100\ KHz$
J	$3.655x10^{-5}\ Kgm^2$
B	$2.676x10^{-3}\ Kgm^2/s$
K_t	$0.423\ Nm/V$
K_b	$2/\pi\ mm/rad$

Table 1. Parameters of the servo system and controller

4.1 Case I: Simple Perturbation

In the figure 1 a disturbance $d(t)$ is additive with the control signal $u(k)$, as a result, the response of the servo system will be immediately affected, and the PIV control will compensate the perturbation over the time. A perturbation in step form with a magnitude of $0.8V$, phase-delay of $90ms$, and duration of $10ms$ is applied after using a unit step for the reference signal. Figures 4 and 5 show the simulation for a system without DOBC and with KDOBC, respectively.

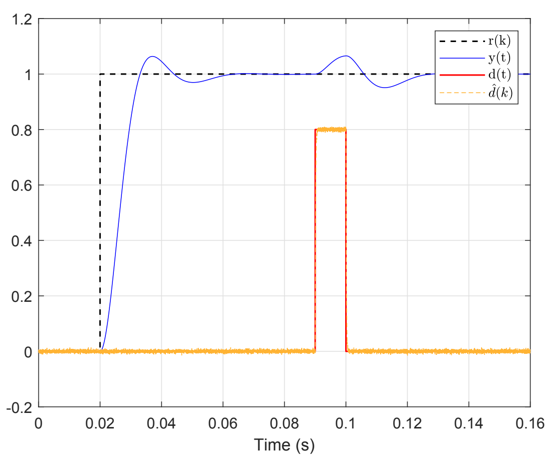

Fig. 4. Output of the servo system with PIV Controller

n the fig 4, we can observe an oscillation just after the perturbation was applied. Even without disturbance compensation, the feedback controller could regain a steady state. In the figure, we can also observe the disturbance $d(t)$ and the disturbance estimation $\hat{d}(k)$ the noisy components in the estimation is related to the covariance matrix R_n.

In the fig 5 the KDOBC uses the disturbance estimation to produce a feed-forward compensation, the oscillation caused by $d(t)$ is pretty attenuated. The effects of the perturbations are compared in the fig6. KDOBC weakens the amplitude of the perturbance.

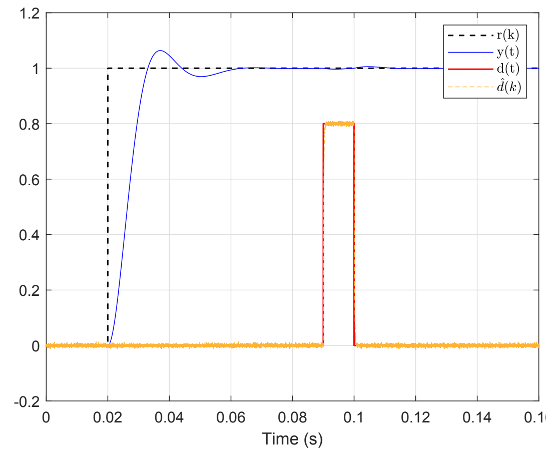

Fig. 5. Output of the servo system with disturbance compensation

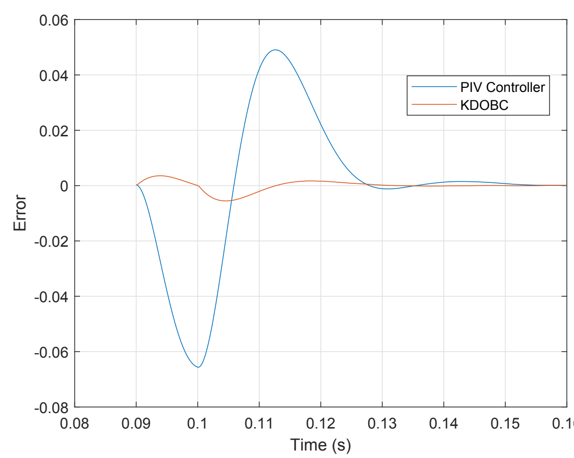

Fig. 6. Comparation of PVI controller and KDOBC

4.2 Case II: Periodic perturbation

A periodic ramp with a magnitude of $0.8V$, with phase delay of $20ms$, and a period of $10ms$ are applied as a disturbance signal. In the input, we have a unit step with a phase delay of $20ms$. Figures 7 and 8 show the simulation for a system with PIV and with KDOB controller, respectively.

On the fig 7 a unit step as a reference and a periodic ramp as disturbance is applied, the integral term in the PIV controller can compensate the ramp effect on the servo system, but small oscillations appear on the output. In the design of the KFE, the discrete-time LTI augmented model assumes that the perturbations remain almost constant between intervals, the ramp perturbation is not the case but, the difference between $T_s = 10\mu s$ and the ramp period $T = 10ms$ is enough to attenuate the errors on the output.

A lower error can be observed in the fig 8. One more time, the KDOBC can compensate for the disturbance effects. A comparative for the errors can be found in fig9.

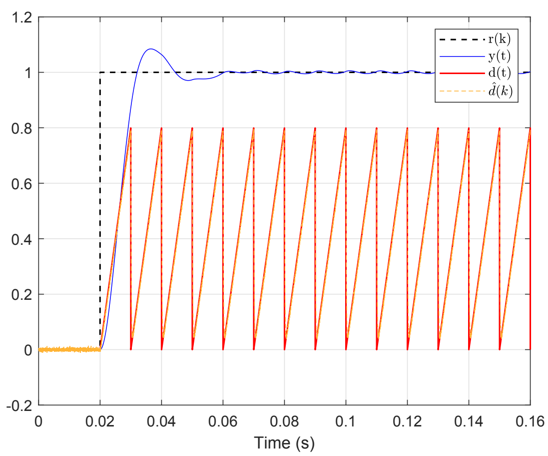

Fig. 7. Output of the servo system without DOBC

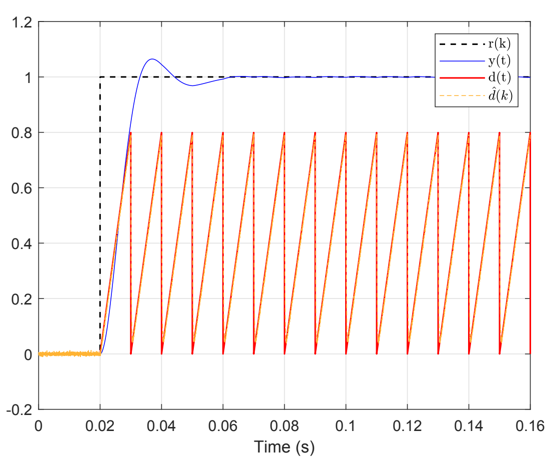

Fig. 8. Output of the servo system with disturbance compensation

5. CONCLUSIONS

Classical control techniques as PID or PIV can obtain disturbance rejection but with several losses of accuracy right after the perturbation appears. The KDOBC improves the steady-state tracking errors and the transient time response against a PIV controller. A Kalman Filter Estimator can estimate not only the states of the system but the magnitude of the disturbance, adding a feed-forward control loop the effects or perturbation can be included in the control signal. The uncommon Proportional-Derivative control was used to derive compensation for the disturbance. Even if this control has non-zero steady-state error the feed-forward control loop appears to balance it. Numerical simulations are also given to show the validity of this new modeling and control scheme.

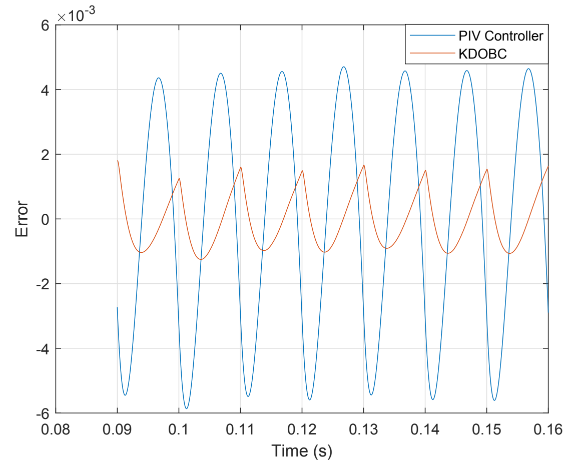

Fig. 9. Output of the servo system with disturbance compensation

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