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UFIR Filter for Networked Systems with Multiplicative Process noises, Uncertain Stochastic Parameters and One-step Random Delays Observations. ⋆

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Abstract: The problem of network systems with stochastic uncertain failures at the transmitted measurements and undetermined knowledge of the system parameters is addressed with the UFIR filter. The measurement output is multiple transmitted which random one-step packet delay and described by Bernoulli process according to the probability detected of this phenomena. the multiplicative noise is detected in the model and the observation, which are described by random variables. Linear filters such as the Kalman filter, the game theory H_∞, and the UFIR filter are developed based on the transformed model, which not depend on delays in the sense to obtain a minimum variance despite the errors at the system and achieve comparison the effectiveness and robustness obtained in real situations. A simulation example using the GPS coordinates of a vehicle illustrates the effectiveness o the proposed methodology.

Keywords: Delayed data, missing data, unbiased FIR filter

1. INTRODUCTION

In the network systems, the high influence of the disturbances, saturation channels or failures at the sensors has brought problems at the communicated data. To date, one of the most challenges in this system is the deterioration in the measurements taken by the sensors when these are sent to the central station via wireless communication, due to environment failures, saturate communication channels, or sensor gain degradation, which may cause constrains at the observation such as missing measurements or intermittent delays at different rates. To ensure appropriate performance of wireless sensors networks (WSN), new approaches have been attracted wide attention considering the real behavior of the packet data.

The state estimation has become an indispensable technology for WSN in view of the difficulty to obtain direct measurements of inaccessible system states and bounded signals. In the past decades, considerable research effort has demonstrated the importance of arrival measurement analysis to obtain veracious estimation stability. Consequently, new methodologies has been studied to according to the arrival observation at the processor, where delayed information, lost data, packet dropouts, or both problems can happen. The Bernoulli distributed random variables are commonly utilized to describe the phenomena when the estimator receives measurements successfully or with some sampling time delay Nikfetrat and Esfanjani (2018); Sun and Wang (2014). The stochastic uncertainties, the randomly delayed measurements, and measurement noises are treated by the orthogonal projection theorem using a robust recursive estimation in Feng et al. (2020), in the same way, more than one-step delay is analyzed in Qian et al. (2017); Wang et al. (2017) where two-step delay measurement and multiple-step delay observations are modeled with a combination of random variables and address with a robust Kalman filter. The mixed uncertainty, delays, packet dropouts, and lost data is also investigated in Zhang et al. (2012) a multi-rate distributed fusion estimation is proposed for a network system with multiple sensors, where the packet loses is compensate in two faces, when the measurement is taken by the sensor and when it arrives at the CS. The existence of random disturbances in the system and full knowledge of system parameters are uncommonly monitoring but it makes critical difficult the estimation behavior. The parameter uncertainties and multiplicative noise characterize these unreliable factors and there are not considered in the above literature. Qu et al. (2010); Wang and Sun (2019).To improve the reliability, few fil-

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tering methodologies have been presented, including the optimal filtering based on the Minimum MSE estimation principle and state augmentation approach to address the effect of the multiplicative and auto-correlated process noise Chen et al. (2016).

At the wide area of estimation methodologies, the Kalman filter has demonstrated the best accuracy when all information works healthily. However, faulty measurements and undetermined noise information can lead to unsuitably inaccurate estimation. The iterative unbiased finite impulse response (UFIR) filter Shmaliy (2011, 2006) can also be used under the uncertain observations with multiple delays and packet dropouts also with incomplete system knowledge Uribe-Murcia et al. (2019, 2021). This filter does not require any information about noise and initial conditions Shmaliy et al. (2017) and is thus more robust. In this paper, The UFIR filtering problem for a network system with one-step random delay and multiplicative noises is introduced. The possible one-step delay packet data at the filter is described by Bernoulli random variables with known probability. Finally, an example to compare the effectiveness of the filter proposed is provided based on the GPS navigation data of a moving vehicle.

2. PROBLEM FORMULATION

Consider the discrete time state system

$$x_n = (A + \bar{A}\alpha_n)x_{n-1} + w_n\,, \tag{1}$$

$$y_n = Cx_n + v_n\,, \tag{2}$$

where $x_n \in \mathbb{R}^K$ is the state vector, $y_n \in \mathbb{R}^M$ is the measurement vector by the sensor , and $A \in \mathbb{R}^{K\times K}$, $F \in \mathbb{R}^{K\times K}$, $C \in \mathbb{R}^{K\times M}$ and $G \in \mathbb{R}^{K\times M}$ are known matrices. $w_n \in \mathbb{R}^K$ and $v_n \in \mathbb{R}^K$ are white Gaussian uncorrelated noise vectors with zero mean and the covariances $Q = E\left\{w_n w_n^T\right\}$ and $R = E\left\{v_n v_n^T\right\}$. The multiplicative noise α_n and β_n are mutually independent and uncorrelated with w_n and v_n.

The measurement taken by the sensor are assumed to be transmitted for wireless communication to a central station multiply times with the intention to obtain always information at the estimator and not only noise. Assume that the system parameters as well as in the dynamic state and observation model are subject to a stochastic uncertain and there exist the probability to receive random delay measurement of one-step at the processor, this phenomenon is described by a binary sequence with a known probability as Bernoulli distribution. Combining the noise which is described as multiplicative noise, and delayed measurement the observed signal is eventually convert on:

$$z_n = \theta_{0,n} y_n + (1 - \theta_{0,n})\, y_{n-1}\,, \tag{3}$$

where z_n is the received measurement at the processor at current time n, $\theta_{0,n}$ is a uncorrelated random Bernoulli variable with probability $P\left\{\theta_{0,n} = 1\right\} = \tilde{\theta}_{0,n}$, $P\left\{\theta_{0,n} = 0\right\} = (1 - \tilde{\theta}_{0,n})$, and take values between $0 \leq \theta_{i,n} \leq 1$.

Model (3) indicate us that, the on time packet data is receives $z_n = y_n$ with the probability $\tilde{\theta}_{0,n}$ if $\theta_{0,n} = 1$ otherwise if $\theta_{0,n} = 0$ one-step delay data is obtained at the estimator $z_n = y_{n-1}$ with the probability $1 - \tilde{\theta}_{0,n}$.

To a better comprehension of the model (3) the following table 1 depicts a typical data transmission with only one-step delayed information. Table 1 shows that $y(1)$, $y(2)$, $y(5)$, $y(7)$ and $y(8)$ arrive on time, while $y(3)$ and $y(9)$ are delayed and $y(4)$ and $y(6)$ are lost.

Table 1. Data Transmission in the Network.

n	1	2	3	4	5	6	7	8	9
θ_0	1	1	0	0	1	0	1	1	0
Z_n	$y(1)$	$y(2)$	$y(2)$	$y(3)$	$y(5)$	$y(5)$	$y(7)$	$y(8)$	$y(8)$

the model (1)-(2) may not be yet applied due to the dependence of the delayed information and the noise, consequently a transformation of (1)-(2) are needed. The UFIR filter is derived considering the following transformed model.

3. UFIR FILTERING FOR TWO-STEP RANDOM MEASUREMENT DELAYS AND LOST DATA

Before preceding further, to apply the standard estimation techniques is necessary that the observation equation should not depend on previous data and multiplicative noise. Then, we introduce a following model transformation to accommodate the impact from the multiplicative random delays and noises.

3.1 Model transformation

The model transformation consist in rewrite the system (1)-(3) based on the random delayed measurement and the multiplicative noise in a equivalent to accommodate the impact from this failures as follows:

$$x_n = Ax_{n-1} + \bar{w}_n\,, \tag{4}$$

$$z_n = \bar{H}_n x_n + \bar{V}_n\,, \tag{5}$$

where the new process noise is

$$\bar{w}_n = \bar{A}\alpha_n x_{n-1} + w_n\,. \tag{6}$$

Introducing the definition of the state back on time as

$$x_{n-k_n} = A^{-k_n}\left[x_n - \sum_{i=0}^{k_n-1} A^i w_{n-i}\right]. \tag{7}$$

the one-step delayed can be defined as $x_n - 1 = A^{-1}x_n + A^{-1}\bar{w}_n$. Thus, from the above specification and the measurement equation (2), we can conclude that (5) was obtained as:

$$
\begin{aligned}
z_n &= \theta_0(Cx_n + v_n) + (1-\theta_0)(Cx_{n-1} + v_n - 1) \qquad (8)\\
&= \left[\theta_0 C + (1-\theta_0)CA^{-1}\right] x_n + [\theta_0 v_n + (1-\theta_0)v_{n-1}\\
&\quad (1-\theta_0)CA^{-1}\bar{w}_n] \,.
\end{aligned}
$$

where $\bar{H}_n$ and $\bar{v}_n$ are the new observation and noise matrices, and the random variable θ_0 satisfies the stochastic properties $E\{\theta_{0,n}\} = \bar{\theta}_0$, $E\{(1-\theta_{0,n})\} = (1-\bar{\theta}_0)$, $E\{(\delta_{j,n})^2\} = \bar{\delta}_j$ and $E\{\alpha_{j,n}\alpha_{i,n}\} = 0$ for $i \neq$.

$$
\begin{aligned}
\bar{H}_n &= \theta_0 C + (1-\theta_0)CA^{-1} \,, \qquad (9)\\
\bar{V}_n &= \theta_0 v_n + (1-\theta_0)v_{n-1} - (1-\theta_0)CA^{-1}\bar{w}_n \,, \qquad (10)
\end{aligned}
$$

The covariance matrices of noise $\bar{v}_n$ and $\bar{w}_n$ are given by

$$
\begin{aligned}
\bar{R} &= \theta_0 R_n + (1-\theta_0)R_{n-1} + (1-\theta_0)CA^{-1}\bar{A}Q_n\bar{A}^T A^{-T}C^T\\
&\quad + (1-\theta_0)CA^{-1}Q_n A^{-T}C^T \,, \qquad (11)\\
\bar{Q}_n &= \bar{A}E\left\{x_{n-1}x_{n-1}^T\right\}\bar{A}^T + Q_n \,, \qquad (12)
\end{aligned}
$$

where we have

$$
\bar{X}_n = E\left\{x_n x_n^T\right\} = AE\left\{x_{n-1}x_{n-1}^T\right\}A^T\bar{Q}_n \,, \qquad (13)
$$

based on (11), we note that $\bar{v}_n$ is correlated with $\bar{w}_n$, then we obtain

$$
E\left\{\bar{v}_n w_n T\right\} = -(1-\theta_0)CA^{-1}\bar{Q}_n \,, \qquad (14)
$$

To address the Kalman and H_∞ filter problem of the signal x_n from the observation z_n is necessary that the noise matrices don't be time correlated. A de-correlated noise transformation employing the Lagrange multiplier method is shown below to compensate the effect of the correlated noise at the estimation.

3.2 De-correlation of $\bar{v}_n$ and w_n

Following Shmaliy et al. (2019) we first rewrite the state equation (1) as

$$
\begin{aligned}
x_n &= A_n x_{n-1} + \bar{w}_n + \Lambda_n\left(z_n - \bar{H}_n x_n - \bar{v}_n\right)\\
&= A_n x_{n-1} + u_n + \zeta_n \,, \qquad (15)
\end{aligned}
$$

where the Lagrange multiplier Λ_n is to be determined, z_n is the vector of real data, and we obtain

$$
\begin{aligned}
\bar{A}_n &= A_n - \Lambda_n\bar{H}_n A_n \,,\\
u_n &= \Lambda_n z_n \,,\\
\zeta_n &= (I - \Lambda_n\bar{H}_n)\bar{w}_n - \Lambda_n\bar{v}_n \,, \qquad (16)
\end{aligned}
$$

from the definition of the new observed noise vector ζ_n, the covariance matrix $Q_\zeta = E\{\zeta_n\zeta_n^T\}$ is defined as,

$$
\begin{aligned}
Q_\zeta &= (I - \bar{\theta}_0\Lambda_n C)\bar{A}E\left\{x_{n-1}x_{n-1}^T\right\}\bar{A}^T(I - \bar{\theta}_0\Lambda_n C)^T\\
&\quad + (I - \bar{\theta}_0\Lambda_n C)Q_n(I - \bar{\theta}_0\Lambda_n C)^T\\
&\quad + \bar{\theta}_{0,n}\Lambda_n R_n\Lambda_n^T + (1-\bar{\theta}_{0,n})\Lambda_n R_{n-1}\Lambda_n^T \,. \qquad (17)
\end{aligned}
$$

Now, the Lagrange multiplier Λ_n can be determined to satisfy the condition $E\{\zeta_n\bar{v}_n^T\} = 0$. This gives

$$
\begin{aligned}
\Lambda_n &= [-(1-\bar{\theta}_0)Q_n A^{-1T}C^T - (1-\bar{\theta}_0)\bar{A}E\left\{x_{n-1}x_{n-1}^T\right\}\\
&\quad \bar{A}^T A^{-T}C^T][\bar{\theta}_0 R_n + (1-\bar{\theta}_0)R_{n-1}]^{-1} \,. \qquad (18)
\end{aligned}
$$

Given the de-correlated covariance matrices $\bar{R}_n$ and Q_ζ, the UFIR, Kalman and game theory H_∞ filters can be developed as we will shown next.

3.3 UFIR filter design

Based on the observation vector $Y_{m,n} = \left[y_m^T\, y_{m+1}^T \,\ldots\, y_n^T\right]^T$ and the state vector $X_{m,n} = \left[x_m^T\, x_{m+1}^T \,\ldots\, x_n^T\right]^T$ in the horizon $[m,n]$, the UFIR Filter estimation of the state $\hat{x}_n$ will be derived in the sense of the unbiased condition $(E\{x_n\} = E\{\hat{x}_n\})$ to achieve minimize the estimation error as Shmaliy (2006):

$$
\begin{aligned}
X_{m,n} &= A_{m,n}x_m + S_{m,n}U_m + D_{m,n}W_{m,n} \,, \qquad (19)\\
Z_{m,n} &= C_{m,n}x_m + L_{m,n}U_m + G_{m,n}W_{m,n} + \bar{V}_{m,n} \,, \qquad (20)
\end{aligned}
$$

where the extended matrices are

$$
\bar{A}_{m,n} = \left[I \;\bar{A}^T \,\ldots\, (\bar{A}^{K-1})^T\right]^T \,, \qquad (21)
$$

$$
S_{m,n} = \begin{bmatrix}
\Lambda_m & 0 & \ldots & 0 & 0\\
A\Lambda_m & \Lambda_{m+1} & \ldots & 0 & 0\\
\vdots & \vdots & \ddots & \vdots & \vdots\\
A^{K-2}\Lambda_m & A^{K-3}\Lambda_{m+1} & \ldots & \Lambda_{n-1} & 0\\
A^{K-1}\Lambda_m & A^{K-2}\Lambda_{m+1} & \ldots & A\Lambda_{n-1} & \Lambda_n
\end{bmatrix}, \qquad (22)
$$

$$
C_{m,n} = \bar{C}_{m,n}A_N = \begin{bmatrix}
\bar{H}_m\\
\bar{H}_{m+1}A\\
\bar{H}_{m+1}A^2\\
\vdots\\
\bar{H}_n A^{K-1}
\end{bmatrix}, \qquad (23)
$$

$$
\begin{aligned}
L_{m,n} &= \bar{C}_{m,n}S_{m,n} \,, \qquad (24)\\
\bar{C}_{m,n} &= \mathrm{diag}\underbrace{\left[\bar{H}_m \;\bar{H}_{m+1} \;\cdots\; \bar{H}_n\right]}_{N} \,. \qquad (25)
\end{aligned}
$$

The UFIR filter is designed for the stochastic system with one-step random delay and multiplicative noise (15)-(5) discarding the initial state and the error covariance matrix P_n using the Batch form in the horizon $[m, m+K-1]$ where the K is the number of the state, and the iterative form to update the batch' estimation and reach the best estimation value $\hat{x}_n$ to the time index n

Batch UFIR Filter The batch UFIR estimate can be written according to Shmaliy (2011) in the form

$$
\hat{x}_k = \mathcal{G}_k C_{m,k}^T\left(Z_{m,k} - L_{m,k}U_{m,k}\right) + S_{m,k}^{(K)}U_{m,k} \,, \qquad (26)
$$

where $Z_{m,k}$ is the observation vector with the arrival measurement, $U_{m,k}$ is the known input vector defined on (16) and the matrix $\mathcal{C}_{m,k}$ are represented on $[m,k]$ as

$$\mathcal{C}_{m,k} = \begin{bmatrix} \bar{H}_m F^{-K+1} \\ \vdots \\ \bar{H}_{k-1} F^{-1} \\ \bar{H}_k \end{bmatrix}, \tag{27}$$

The generalized noise power gain (GNPG) is $G_k = (\mathcal{C}_{m,k}^T \mathcal{C}_{m,k})^{-1}$.

Iterative UFIR Filtering Algorithm The iterative UFIR filtering algorithm likewise to the KF, computes the final estimation at the time n using previously estimated values in the batch form, it are calculated in a predictive and update wise. The iterations are organized using the following recursions,

$$\tilde{x}_n^- = (A_n - \Lambda_n \bar{H}_n A_n)\tilde{x}_{n-1} + u_n\,, \tag{28}$$
$$G_n = [\bar{H}_n^T \bar{H}_n + (\bar{A} G_{n-1} \bar{A}^T)^{-1}]^{-1}\,, \tag{29}$$
$$\tilde{x}_n = \tilde{x}_n^- + (G_n \bar{H}_n^T)(z_n - \bar{H}_n \tilde{x}_n^-)\,. \tag{30}$$

4. EXPERIMENTAL VERIFICATION

In this section, A numerical example is proposed using a GPS vehicle tracking signal of the Cook County of Illinois available from the University of Illinois at Chicago Databases and in University of Illinois at Chicago (2006) whit the aim to show the robustness and feasibility of the proposed algorithm. Two performance tests about the effect of the accuracy noise matrices and reliability transmission likelihood is given. The Fig. 1 illustrated the measured GPS trajectory in the north and east coordinate x, y.

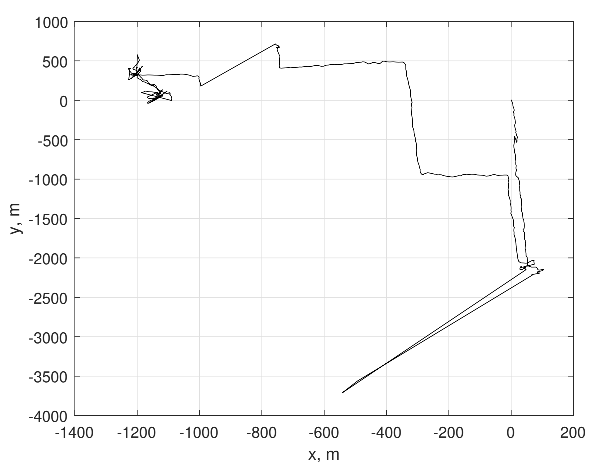

Fig. 1. GPS-measured vehicle trajectory in the local north (y) and east (x) coordinates.

The received information can be expressed via the model (5)–(15) assuming the following characteristics: $x_n = [x_{1n}\ x_{2n}\ x_{3n}\ x_{4n}]^T$, where x_{1n}, m is the x_n coordinate, x_{2n} is the velocity in m/s in the coordinate x_n, x_{3n}, m is the y_n coordinate, and x_{4n} is the velocity in m/s in y_n.

$$x_n = \left(\begin{bmatrix} 1 & \tau & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \tau \\ 0 & 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0.01 & 0 & 0 & 0 \\ 0 & 0.01 & 0 & 0 \\ 0 & 0 & 0.01 & 0 \\ 0 & 0 & 0 & 0.01 \end{bmatrix} \alpha_n \right) x_{n-1} + \begin{bmatrix} \tau/2 & 0 \\ \tau & 0 \\ 0 & \tau/2 \\ 0 & \tau \end{bmatrix} w_n\,.$$

$$y_n = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x_n + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} v_n\,.$$

α_n is a white noise with unit covariance and uncorrelated with other noises. Q_n and R_n are specify with a general knowledge of the system. the vehicle moves with an average speed of 10 m/s, and a standard deviation of 15% and assign $\sigma_{2w} = 1.5$ m/s and $\sigma_{1w} = 0$ m/s. The GPS navigation produces errors of smaller than 15 meters with a probability of 95% in the 2σ sense and we thus assign $\sigma_v = 3.75$ m. Accordingly,

$$Q = \sigma_{w2}^2 \begin{bmatrix} \tau^2/4 & \tau^2/2 & 0 & 0 \\ \tau^2/2 & \tau^2 & 0 & 0 \\ 0 & 0 & \tau^2/4 & \tau^2/2 \\ 0 & 0 & \tau^2/2 & \tau^2 \end{bmatrix},\ R = \begin{bmatrix} \sigma_v^2 & 0 \\ 0 & \sigma_v^2 \end{bmatrix}.$$

To apply properly the UFIR and H_∞ filter, in the sense to minimize the estimated error the following tuning has to be provided:

- The UFIR filter require to define a horizon of point called N_{opt}, solving the minimizing problem
  $$N_{\text{opt}} = \arg\min_N [\text{tr} P_n(N)]\,,$$
  we obtain from the optimization $N_{\text{opt}} = 5$.
- Defining properly the tuning factor σ for H_∞ filter achieve minimize the estimated error obtained in the KF and hence is hard work. In our experiment we found $\sigma = 0.15 \times 10^{-2}$, nevertheless, it changes at this value produce that the filter diverge.

4.1 Tracking Errors

The estimation produced by the UFIR, KF, and $H\ _{infty}$ employing the on-time arrival probability $\ theta_0 = 0.7$ is shown in Figure ref Fig2. Consistent estimations of the vehicle position in the north direction by the three filters are displayed in this figure. The ability to return back to the actual trajectory when the vehicle rapidly maneuvers is highlighted by the UFIR filter despite the large excursion. A minimum estimation error is obtained by the $H\ _{infty}$ filter improving the KF estimation, but errors on $\ theta_0$ make it diverge. 3.

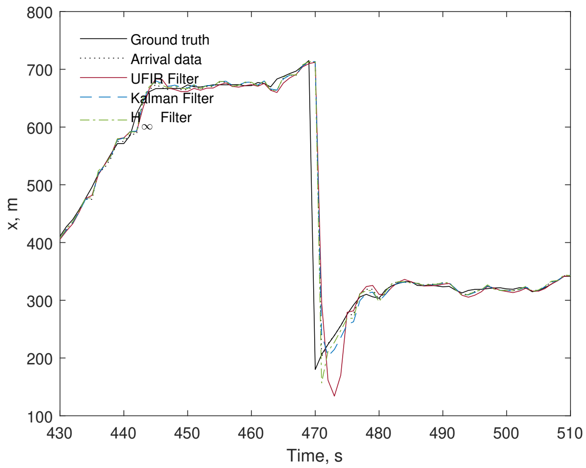

Fig. 2. Vehicle tracjectory estimation by the UFIR filter, KF, and H_∞ filter using model (5)–(15).

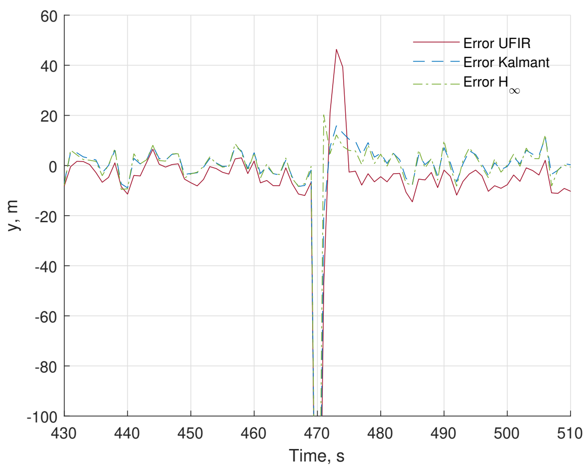

Fig. 3. Tracking error produced by the UFIR filter, KF, and H_∞ filter.

4.2 Unsuitable transmissions

The wireless transmissions are usually affected by a wide kind of perturbations which are often be modeled by uncertain covariance matrices and multiplicative noise. To represent this situation where the matrices noise are inconsistent, the noise matrices Q and R are modified as $(1/\beta)^2 Q$ and $\beta^2 R$ where β take vales on $0.1 < \beta > 10$. From Figure 4 we can deduce that an optimal estimation is only obtained with accurate matrices. Nevertheless, when the stochastic noise parameters are unsure the UFIR filter estimation doesn't produce large variations at the estimation, nevertheless, its performance is affected by the dependence of the noise covariance in the new system matrices (16). The KF and H_{infty} produce big errors due to variations of the noise matrices.

4.3 Effect of Errors in the Transmission Probability

The importance of suitably knows the transmission probability is shown in Figure 5. A minimum estimation error

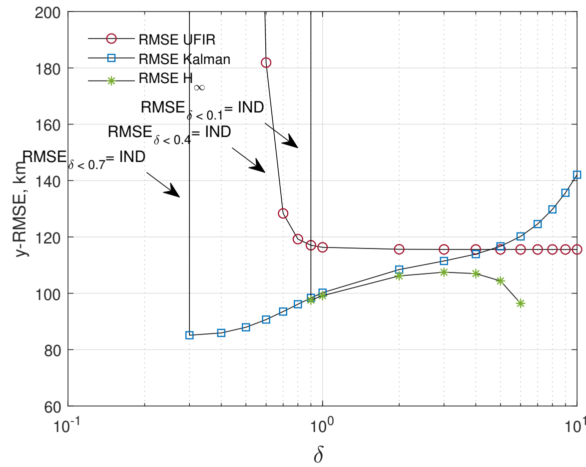

Fig. 4. Effect of unsuitable noise covariance matrices at the estimation produced by the UFIR filter, KF, and H_∞ filter.

is produced when the probability of transmission is equal to the model probability, thus increasing values of error are observed when we have variations on θ_0. An ideal observation vector is obtained when θ_0 tends to be 1, which means that estimations with minimum errors are obtained due to one-step delayed data is not received.

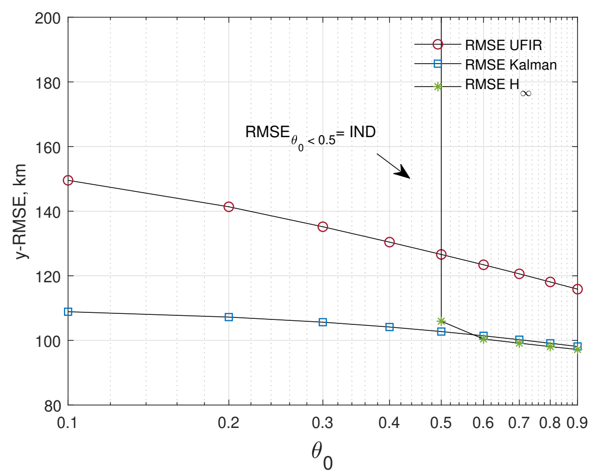

Fig. 5. Effect of the data transmission probability θ_0 on the RMSEs produced by the UFIR filter, KF, and H_∞ filter.

5. CONCLUSIONS

The problem of one-step delay measurements and multiplicative noise in network systems was solved using the UFIR, Kalman, and H_∞ filters. The random arrival packet data at the processor was presented using the Bernoulli distribution according to the approach probability detected. The algorithms designed have been developed based on a transformed model, which reduces the effect of uncertain failures at the system and brings the ideal model form to apply linear estimated methodologies.

Finally, an experimental example of GPS-based vehicle tracking has shown the advantages of the proposed UFIR algorithm to obtain a feasible estimation in practical applications with delays, and multiple uncertain parameters.

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