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Optimal Distributed Filters for Robust Estate Spate Estimation. ⋆

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Abstract: The nature of Wireless sensor networks (WNS) allows the implementation of a distributed estimation process which has proven to be a more robust solution than individual estimation. Furthermore, filters of an unbiased finite impulse response nature have proven themselves as a robust alternative for WSNs applications, which are often deployed in harsh environments, where electromagnetic interference, damaged sensors, or the landscape itself cause the network to suffer from faulty links and missing data. In this paper, we present a distributed unbiased finite impulse response (dUFIR) algorithm for optimal consensus on estimates in WSNs. We compare the performance of dUFIR filter against a distributed Kalman filter (dKF) and prove with simulations that better robustness is achieved by the dUFIR filter against data loss, unknown noise statistics and faulty measurements.

Keywords: Robust estimation, Optimal estimation, WSN.

1. INTRODUCTION

Wireless sensor networks(WSNs) are a collection of interconnected smart sensors, also called nodes, that are able to measure, process, and communicate data among each other. Applications for this technology can be found in healthcare, object tracking, environmental monitoring, etc. (Cook et al., 2009; Feng Zhao, 2004). As technology furthers the miniaturization of smart sensors, it is possible to deploy a massive number of cheap sensors, which allows redundant measurements of a desired quantity $\mathcal{Q}$.

The smart sensors components are prone to failure due to cost reduction during the manufacturing process; therefore, the nodes may present very noisy measurements, intermittent behavior, loss of data, or even invalid measurements communication(Vazquez-Olguin et al., 2021). In addition, in real-life applications, an accurate dynamic model of the physical process and noise statistics may not be available; hence, optimal estimation is required along with adequate sensor fusion techniques (Akyildiz et al., 2002; Mahmoud and Xia, 2014; Chen et al., 2014; Rao and Durrant-Whyte, 1991), which must demonstrate a sufficient robustness against missing data, model errors (mismodeling), and incomplete information about noise statistics.

Distributed filtering has been introduced to take advantage of the large number of nodes in WSNs. Each node is tasked with the estimation of $\mathcal{Q}$ and a consensus protocol is implemented by averaging the estimates, measurements, or information matrices (Li et al., 2015) so that all the nodes agree in a common value called the group decision value (Olfati-Saber and Murray, 2004). By implementing distributed filtering, battery life may be prolonged as the nodes no longer need to communicate with a base station. The estimation is locally performed, and the group decision value may be accessed en any node of the network.

Algorithms for distributed estimation have been developed based on the Kalman filter(KF) mainly due to its low computational burden, optimal estimation, and easy implementation (Bai et al., 2018). In Olfati-Saber (2007), the author has proposed a KF structure that requires each node to locally aggregate its measurement and covariance matrix with those of its neighbors and, in a posterior step, compute the estimate using a KF with a consensus term. In Carli et al. (2008), the KF approach has been developed for local estimation and a consensus matrix as fusion technique. In Stanković et al. (2009), the authors have presented an algorithm based on the KF to address an issue with missing data.

An important issue with KF is that to ensure optimal estimates, noise processes must be strictly white Gaussian (Pomarico-Franquiz and Shmaliy, 2014; Shmaliy et al., 2016; Shmaliy, 2012; Contreras-Gonzalez et al., 2013). Also, KF requires an adequate model and complete knowledge of the noise statistics; otherwise, the estimates

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may present large errors or even diverge, making the Kalman filter lack the required robustness for real-life applications (Vazquez-Olguin et al., 2018).

The principal reason behind the lack of robustness for KF algorithms is its Infinite Impulse Response structure. It has been proven in Jazwinski (2007); Ramirez-Echeverria et al. (2014); Zhao et al. (2016); Shmaliy et al. (2017a) that better robustness can be achieved by using filters operating on finite data horizons. Under such an assumption, a moving average estimator has been designed in Farina et al. (2010) for weak observability. In Vazquez-Olguin et al. (2017b), an unbiased finite impulse response (UFIR) filter was developed for consensus on measurements and in Vazquez-Olguin et al. (2018), the consensus on estimates was developed for the UFIR structure; however, the effect of missing measurements on the filter performance was not considered.

The issue of missing measurements has been addressed by the KF and UFIR approaches. In Hu et al. (2012) an extended KF was modified to address the issue of missing measurement while in Vazquez-Olguin et al. (2017a) a UFIR filter was developed as a robust estimator that neglects noise statistic. Regarding WSNs, in Sinopoli et al. (2004) a KF was modeled for intermittent observations and in Uribe-Murcia et al. (2018) a UFIR alternative for missing and delayed data was developed. In Vazquez-Olguin et al. (2021), an implementation of a UFIR filter with distributed consensus on estimates shown good performance against vary unstable links.

In this work, we compare the robustness of a distributed UFIR filter with consensus on estimates and a Kalman filter against a large number of missing and faulty measurements with unknown statistics. The rest of the paper is organized as follows. In Section 2, the problem is formulated and the model presented. Section 3 presents a predictive version of KF algorithm while in Section 4 the predictive version of a UFIR with consensus of estimates is given. Simulations are provided in Section 5 and conclusions are given in Section 6.

2. PROBLEM FORMULATION AND PRELIMINARIES

The dynamic K-state space model for a physical quantity $\mathcal{Q}$, considering a distributed WSN, is given by the following equations:

$$x_k = F_k x_{k-1} + B_k w_k \,, \quad (1)$$

$$\bar{y}_k^{(i)} = H_k^{(i)}(F_k x_{k-1}) \,, \quad (2)$$

$$y_k^{(i)} = \gamma_k (H_k^{(i)} x_k + v_k^{(i)}) + (1-\gamma_k)\bar{y}_k^{(i)} \,, \quad (3)$$

$$y_k = H_k x_x + v_k \,, \quad (4)$$

where $x_k \in \mathbb{R}^K$, $F_k \in \mathbb{R}^{K\times K}$, and $B_k \in \mathbb{R}^{K\times M}$. The wireless sensor network is regarded as an undirected graph with n being the total amount of nodes. The ith node, with j neighbors and $J = j \cup i$ inclusive neighbors, is able to measure the x_k state vector by $y_k^{(i)} \in \mathbb{R}^p$, $p \leqslant K$, with $H_k^{(i)} \in \mathbb{R}^{p\times K}$. Local data $y_k^{(i)}$ are united in the observation vector $y_k = [\, y_k^{(i)^T} \, \dots \, y_k^{(j)^T} \,]^T$ with $H_k = [\, H_k^{(i)^T} \, \dots \, H_k^{(j)^T} \,]^T$. Noise vectors $w_k \in \mathbb{R}^M$ and $v_k = [\, v_k^{(i)^T} \, \dots \, v_k^{(j)^T} \,]^T$ are zero mean, white Gaussian, uncorrelated, and with the covariances $Q_k = E\{w_k w_k^T\} \in \mathbb{R}^{M\times M}$, $R_k = \mathrm{diag}[\, R_k^{(i)^T} \, \dots \, R_k^{(j)^T} \,]^T \in \mathbb{R}^{Jp\times Jp}$, and $R_k^{(i)} = E\{v_k^{(i)} v_k^{(i)^T}\}$. A binary variable γ_k serves as an indicator of whether a measurement exist ($\gamma_k = 1$) or not ($\gamma_k = 0$), in which case the measurement prediction $\bar{y}_k^{(i)}$ (2) is used by substituting x_{k-1} with the estimate. The problem can be stated as follows. Given model (1)-(4), prove that better robustness is achieved by a predictive dUFIR filter against missing of faulty measurements, with unknown noise statistics.

3. PREDICTIVE DISTRIBUTED KALMAN FILTER

The distributed Kalman filter (dKF) with consensus on estimates was first introduced in Olfati-Saber (2007). In order to address the issue of missing data, we modify the original algorithm by introducing a prediction feature (lines 3–5). For dKF, we consider that every node locally performs the operations in line 6–7 and shares the message $\mathrm{msg}_{\mathrm{dKF}}^{(i)}(z_k^{(i)}, Z_k^{(i)}, \bar{x}_k^{(i)})$ with its first order neighbors. When the node receives all the information from its neighbors, it then preforms the rest of the steps.

Algorithm 1: Predictive dKF Algorithm

Data: $P_0^{(i)}, \bar{x}_0^{(i)} = x_0, Q_k, R_k^{(j)}, z_k^{(j)}, Z_k^{(j)}$
Result: $\hat{x}_k^{(i)}$
1 **begin**
2   **for** $k = 0 : \infty$ **do**
3   **if** $\gamma_k = 0$ **then**
4       $y_k^{(i)} = H_k^{(i)} F_k \hat{x}_{k-1}^{(i)}$;
5   **end if**
6     $z_k^{(i)} = H_k^{(i)^T} R_k^{(i)^{-1}} y_k^{(i)}$;
7     $Z_k^{(i)} = H_k^{(i)^T} R_k^{(i)^{-1}} H_k^{(i)}$;
8     $s_k^{(i)} = \sum_{l\in J} z_k^{(l)}$;
9     $S_k^{(i)} = \sum_{l\in J} Z_k^{(l)}$;
10     $M_k^{(i)} = (P_k^{(i)^{-1}} + S_k^{(i)})^{-1}$;
11     $\hat{x}_k^{(i)} = \bar{x}_k^{(i)} + M_k^{(i)}(s_k^{(i)} - S_k^{(i)}\bar{x}_k^{(i)}) + \epsilon M_k^{(i)} \sum_{l\in J}(\bar{x}_k^{(l)} - \bar{x}_k^{(i)})$;
12     $P_k^{(i)} \leftarrow F_k M_k^{(i)} F_k^T + B_k Q_k B_k^T$;
13     $\bar{x}_k^{(i)} \leftarrow F_k \hat{x}_k^{(i)}$;
14   **end for**
15 **end**

4. PREDICTED ITERATIVE DISTRIBUTED UFIR FILTER

The distributed UFIR filter was developed in Vazquez-Olguin et al. (2020) and modified to address the issue of missing data in Vazquez-Olguin et al. (2019). The batch structure of the filter over a horizon $[m, k]$ of N points is

$$\hat{x}^c_k = K_{m,k}Y_{m,k} + J\lambda^{\text{opt}}_k K_{m,k}Y_{m,k} - J\lambda^{\text{opt}}_k K^{(i)}_{m,k}Y^{(i)}_{m,k}\,, \quad (5)$$

where

$$Y^{(i)}_{m,k} = \left[\, y^{(i)^T}_m \;\; y^{(i)^T}_{m+1} \;\ldots\; y^{(i)^T}_k \,\right]^T , \quad (6)$$

$$Y_{m,k} = \left[\, y^T_m \; y^T_{m+1} \;\ldots\; y^T_k \,\right]^T , \quad (7)$$

and λ^{opt}_k is a consensus factor chosen to minimize the mean squared error as

$$\lambda^{\text{opt}}_k = \arg\min_{\lambda_k} \{\operatorname{tr} P(\lambda_k)\}\,, \quad (8)$$

with $P(\lambda_k) = E\{(x-\hat{x}^{ic})(x-\hat{x}^{ic})^T\}$. The centralized and individual filter gains $K_{m,k}$,$K^{(i)}_{m,k}$ are found by following the unbiasedness condition

$$E\{\hat{x}^c_k\} = E\{\hat{x}_k\} = E\{\hat{x}^{(i)}_k\} = E\{x_k\}\,. \quad (9)$$

The batch structure of the filter may be unsuitable for WSNs, specially if the number of inclusive neighbors is large. For this reason, an iterative alternative can be derived by describing (5) as a linear combination of unbiased centralized $\hat{x}_k$, and individual $\hat{x}^{(i)}_k$ estimates

$$\hat{x}^c_k = \hat{x}_k + J\lambda^{\text{opt}}_k(\hat{x}_k - \hat{x}^{(i)}_k)\,. \quad (10)$$

Following the procedure described in Shmaliy et al. (2017b), including a variable l that starts at $l = k - N + K + 1$ and ending in $l = k$. The recursions for a centralized estimation are given by

$$G_l = [H^T_l H_l + (F_l G_{l-1} F^T_l)^{-1}]^{-1}\,, \quad (11)$$

$$\hat{x}^-_l = F_l \hat{x}_{l-1}\,, \quad (12)$$

$$\hat{x}_l = \hat{x}^-_l + G_l H^T_l (y_l - H_l \hat{x}^-_l)\,. \quad (13)$$

The initial values G_{l-1} and $\hat{x}_{l-1}$ are computed at $s = k - N + K$ in batch forms as

$$G_s = (C^T_{m,s} C_{m,s})^{-1}\,, \quad (14)$$

$$\hat{x}^c_s = G_s C^T_{m,s} Y_{m,s} \quad (15)$$

The same procedure is followed for the individual estimates $\hat{x}^{(i)}_k$, obtaining the recursions

$$G^{(i)}_l = [H^{(i)^T}_l H^{(i)}_l + (F_l G^{(i)}_{l-1} F^T_l)^{-1}]^{-1}\,, \quad (16)$$

$$\hat{x}^{(i)^-}_l = F_l \hat{x}^{(i)}_{l-1}\,, \quad (17)$$

$$\hat{x}^{(i)}_l = \hat{x}^{(i)^-}_l + G^{(i)}_l H^{(i)^T}_l (y^{(i)}_l - H^{(i)}_l \hat{x}^{(i)^-}_l)\,, \quad (18)$$

with the initial values

$$G^{(i)}_s = (C^{(i)^T}_{m,s} C^{(i)}_{m,s})^{-1}\,, \quad (19)$$

$$\hat{x}^{(i)}_s = G^{(i)}_s C^{(i)^T}_{m,s} Y^{(i)}_{m,s}\,. \quad (20)$$

Matrices $C_{m,s} = \bar{C}_{m,s} F_{m,s}$ and $C^{(i)}_{m,s} = \bar{C}^{(i)}_{m,s} F_{m,s}$ are found with the following equations:

$$\bar{C}_{m,s} = \operatorname{diag}(\, H_m \; H_{m+1} \;\ldots\; H_s \,)\,, \quad (21)$$

$$\bar{C}^{(i)}_{m,s} = \operatorname{diag}(\, H^{(i)}_m \; H^{(i)}_{m+1} \;\ldots\; H^{(i)}_s \,)\,, \quad (22)$$

$$F_{m,s} = [\, I \; F^T_{m+1} \ldots (\mathcal{F}^{m+1}_s)^T]^T\,, \quad (23)$$

$$\mathcal{F}^g_r = \begin{cases} F_r F_{r-1} \ldots F_g\,, & g < r+1 \\ I\,, & g = r+1 \\ 0\,, & g > r+1 \end{cases}\,. \quad (24)$$

The message that the ith node transmits to ist neighbors is $\text{msg}^{(i)}_{dUFIR}(H^{(i)}_k, R^{(i)}_k, y^{(i)}_k)$. If all the nodes observe the exact same states, then it is possible to avoid the transmission of $H^{(i)}_k$.

A pseudo code for the predictive iterative dUFIR algorithm with consensus on estimates is listed as Algorithm 2.

Algorithm 2: Iterative dUFIR Filtering Algorithm

Data: y_k, $R^{(i)}_k$, R_k, N,λ^{opt}_k
Result: $\hat{x}_k$

```
begin
    for k = N − 1 : ∞ do
        m = k − N + 1,   s = m + K − 1;
        G_s = (C^T_{m,s} C_{m,s})^{-1};
        G^{(i)}_s = (C^{(i)T}_{m,s} C^{(i)}_{m,s})^{-1};
        if γ_k = 0 then
            y^{(j)}_k = H^{(j)}_k F_k x̂^{(j)}_{k-1}; ∀i ∈ J
        end if
        x̃_s = G_s C^T_{m,s} Y_{m,s};
        x̃^{(i)}_s = G^{(i)}_s C^{(i)T}_{m,s} Y^{(i)}_{m,s};
        for l = s + 1 : k do
            x̂^-_l = F_l x̂_{l-1};
            x̂^{(i)-}_l = F_l x̂^{(i)}_{l-1};
            G_l = [H^T_l H_l + (F_l G_{l-1} F^T_l)^{-1}]^{-1};
            G^{(i)}_l = [H^{(i)T}_l H^{(i)}_l + (F_l G^{(i)}_{l-1} F^T_l)^{-1}]^{-1};
            x̂_l = x̂^-_l + G_l H^T_l (y_l − H_l x̂^-_l);
            x̂^{(i)}_l = x̂^{(i)-}_l + G^{(i)}_l H^{(i)T}_l (y^{(i)}_l − H^{(i)}_l x̂^{(i)-}_l);
        end for
        x̂^c_k = (I + J λ^{opt}_k) x̃_k − J λ^{opt}_k x̃^{(i)}_k;
    end for
end
```

† First data y_0, y_1,..., y_{N-1} must be available.

5. SIMULATIONS

Simulations are performed considering the ground truth trajectory of a robot available for free from the MagPIE project dataset Hanley et al. (2017). A simulated WSN is placed over the trajectory as shown in Fig. 1. We consider that each sensor is able to measure the x and y coordinates along the entire trajectory. The process dynamics of the

Table 1. Standard deviation for each sensors measurements.

Node	σ_1
1	0.0967
2	0.1088
3	0.08
4	0.0921
5	0.0859
6	0.0837
7	0.0875
8	0.0938

moving object is described in state space by the following matrices:

$$F = \begin{bmatrix} 1 & \tau & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \tau \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad H^{(i)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad Q = \begin{bmatrix} \tau^3/3 & \tau^2/2 & 0 & 0 \\ \tau^2/2 & \tau & 0 & 0 \\ 0 & 0 & \tau^3/3 & \tau^2/2 \\ 0 & 0 & \tau^2/2 & \tau \end{bmatrix} \sigma_w^2,$$

where $\sigma_w = 0.76$ m/s. For dUFIR, the optimal horizon N_{opt} was found at a test stage to be 41 in average. Measurements are simulated considering that each sensor adds zero mean White Gaussian noise with different variances according to table 1.

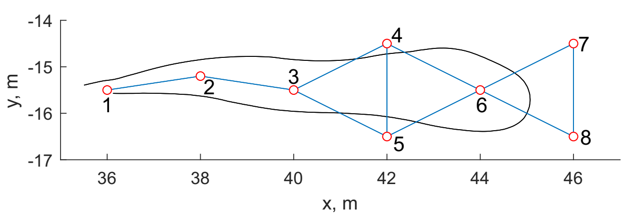

Fig. 1. Simulated WSN over the ground-truth trajectory.

We also consider random data loss after $k = 100$ for every node.The missing data follows the binomial distribution with probability of success of 0.02. We also simulate an unpredicted event, from $300 \leq k \leq 550$, that produced a very large amount of missing measurements for node 3. The measurements for this node are sketched in Fig. 2.

In order to test robustness of Algorithm 1 and Algorithm 2, we first assume complete knowledge of noise statistics and observe that, as expected, dKF presents slightly less error than dUFIR, this can be observed in Fig. 3.

By letting $Q_k \leftarrow (0.5)^2 Q_k$ and $R_k^{(i)} \leftarrow (p^{(i)})^2 R_k^{(i)}$, where $p^{(i)}$ is different for every node, larger errors are observed for dKF during the interval $300 \leq k \leq 550$ as it is sketched in Fig. 4.

To further the analysis of robustness, we consider an scenario where, for unknown reasons, node 3 repeats the

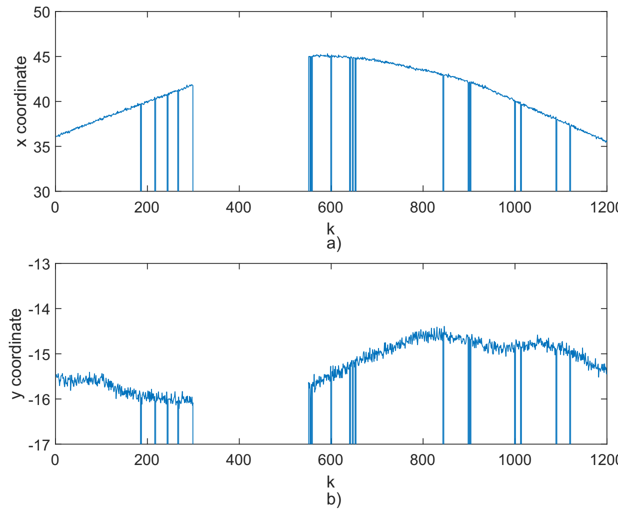

Fig. 2. Measurments of node 3. a) x coordinate and b) y coordinate.

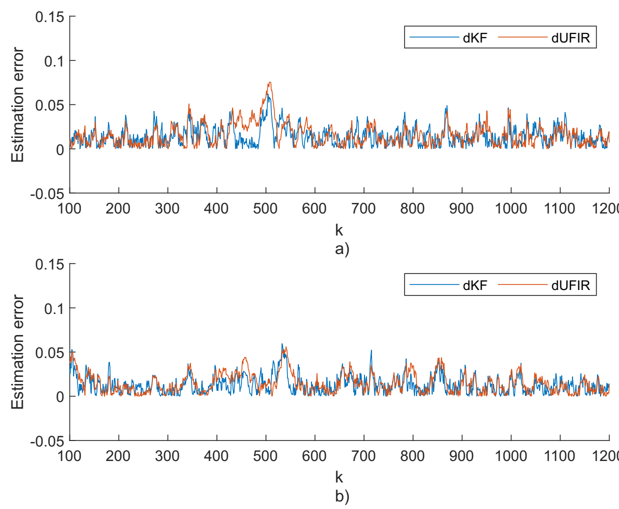

Fig. 3. Absolute estimation error for node 3, assuming perfect knowledge of noise statistics. a) x coordinate, b) y coordinate.

same measurement during the interval $300 \leq k \leq 550$ as shown in Fig. 5. Under this abnormal behavior, the measurements of node 3 cannot be regarded as missing data, therefore they are processed by both algorithms. In Fig. 6, we observe that both filters produce large errors, being the smallest error attainable to dKF; however, the dUFIR presents faster convergence when the node resumes normal operation. The filter dUFIR takes 41 samples in comparison to the almost 100 samples for dKF.

6. CONCLUSIONS

Real world applications of WSNs require robust estimators against unpredictable events. In this paper, we compare the performance of a predictive Kalman filter with a predictive distributed UFIR filter against missing

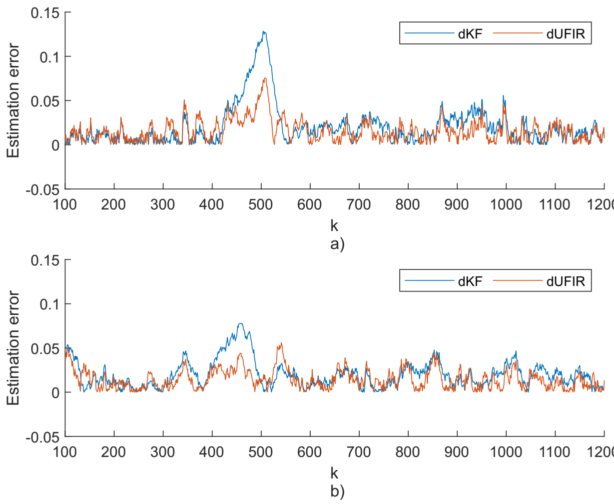

Fig. 4. Absolute estimation error for node 3, assuming imperfect knowledge of noise statistics. a) x coordinate, b) y coordinate.

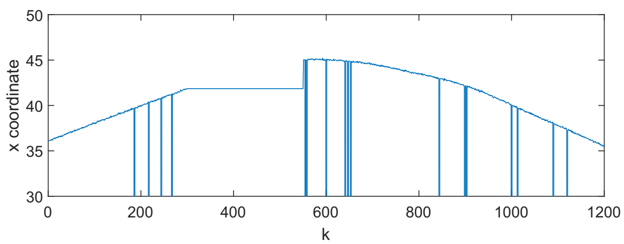

Fig. 5. Measurements of node 3 for x coordinate, assuming repeated incorrect data.

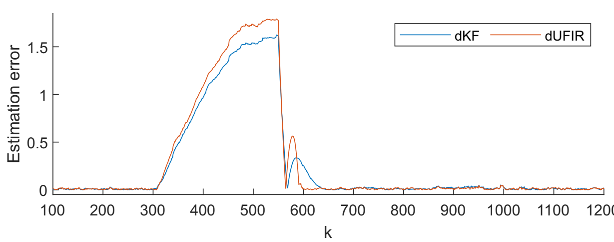

Fig. 6. Absolute estimation error for node 3, assuming repeated incorrect data.

measurements and incorrect observations that cannot be regarded as missing data. We prove with simulations that, when noise and model statistics are not perfectly known, better robustness in terms of absolute error is achieved by dUFIR against missing measurements. In the case of incorrect data, as presented in fig 6, larger errors are obtained by dUFIR; however, when the node resumes normal operation dUFIR takes less time than dKF to reduce the absolute error.

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