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Static Output Feedback Controller Design for Nonlinear Descriptor Systems Via Relaxed LMI Conditions ⋆

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Abstract: This paper presents an alternative to design a static output feedback controllers for nonlinear descriptor systems. The methodology is based on the direct Lyapunov method, from which, after a convex rewriting of the original nonlinear systems, conditions in the form of linear matrix inequalities are obtained. The proposal is shown to be more relaxed than previous ones in two ways; first, unmesaurable nonlinearities can be directly considered and, second, more flexibility can be obtained with the selection of different slack variables; such advantages are illustrated via numerical examples.

Keywords: Static Output Feedback, Convex Descriptor Model, Lyapunov Method, Linear Matrix Inequality

1. INTRODUCTION

Obtaining convex conditions for the design of static output feedback controllers (SOFCs) is a hard task in control theory; in fact, Syrmos et al. (1997) established that the problem itself is non-convex, even in the case of linear time-invariant systems (Geromel et al., 1998). Since then, many researchers have proposed different ways to reduce conservativeness when achieving conditions in the form of linear matrix inequalities (LMIs) (Boyd et al., 1994). LMI conditions are preferred because they are solved in polynomial time via convex optimization techniques (Scherer and Weiland, 2000). First attempts occurred for linear systems, for instance in (Crusius and Trofino, 1999) the so-called W and P problem have been proposed, they consist on solving simoultaneaously a set of LMIs and a linear matrix equality (LME). In (Cao et al., 1998), an Iterative LMI (ILMI) procedure has been presented.

In the case of nonlinear systems, particularly, in the context of convex models [1], the authors (Kau et al., 2007) proposed a set of LMIs combined with a set of LMEs, the problem becomes impossible to solve when convex outputs are considered. In (Huang and Nguang, 2007), following the idea of Cao et al. (1998), it is established a set of ILMIs; this is not optimal in any sense. In (Chadli et al., 2008), a novel way to cast bilinear matrix inequeality (BMI) conditions as LMI ones is proposed, it needs slack variables, but once again, these variables must be fixed before hand; an extension of this work is given by Chadli and Guerra (2012). All these approaches have been developed for standard convex models. More recently, few works (Estrada-Manzo et al., 2019) treat a larger family of nonlinear systems: descriptor models (Luenberger, 1977); which naturally appear in electromechanical, mechatronic, biomechanical systems modeled by the Euler-Lagrange formalism (Lewis et al., 2003).

Contribution: This work is intended to develop less conservative LMI conditions for the SOFC design of descriptor systems. The proposal is based on Finsler's Lemma and follows the idea of (Chadli and Guerra, 2012; Estrada-Manzo et al., 2019), that is, it employs slack variables but in a different manner so relaxed results are obtained. Moreover, previous works only consider measurable premise variables, hence the family systems under consideration is limited. Based on the aforementioned drawback, our proposal also covers systems with unmeasurable premise variables.

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[1] Convex models are a collection of linear submodels blended together by convex scalar functions; if the sector nonlinearity approach (Ohtake et al., 2001) is employed, thus, the convex model is an exact representation of the nonlinear one (Bernal et al., 2022).

The rest of the paper is organized as follows: Section 2 places this research by mentioning previous works on the subject and providing the reader with some technical tools; Section 3 establishes the main result with some remarks about it; Section 4 illustrates how former approaches are overcome by the proposal via numerical examples; finally, Section 5 gathers some concluding remarks and future work.

2. PROBLEM STATEMENT

Consider the following nonsingular descriptor system:

$$\begin{aligned} E(x)\dot{x}(t) &= f(x) + g(x)u(t), \\ y(t) &= h(x), \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the input vector, $y \in \mathbb{R}^o$ is the output vector. The vector fields $f(x) : \mathbb{R}^n \mapsto \mathbb{R}^n$, $g(x) : \mathbb{R}^n \mapsto \mathbb{R}^{n\times m}$, $h(x) : \mathbb{R}^n \mapsto \mathbb{R}^o$, and $E(x) : \mathbb{R}^n \mapsto \mathbb{R}^{n\times n}$ are assumed to be bounded and smooth in a region around the origin $\Omega_x \subset \mathbb{R}^n$.

The task is to design a nonlinear control law that feeds back only available signals, this is to say, a static output feedback of the form

$$u(t) = k(y), \quad k(y) : \mathbb{R}^o \mapsto \mathbb{R}^m. \tag{2}$$

Seeking LMI conditions such that the origin of the closed-loop system

$$E(x)\dot{x}(t) = f(x) + g(x)k(y)$$

is asymptotically stable is a hard task. In fact in (Syrmos et al., 1997) it has been established that the problem is not convex. Many approaches have proposed different ways to tackle this issue: for example, Crusius and Trofino (1999) fixed one of the decision variables variables, Huang and Nguang (2007) developed an algorithm based on iterative LMI (ILMI), authors in (Chadli and Guerra, 2012; Estrada-Manzo et al., 2019) proposed adding slack variables. Nonetheless, all these previous works only consider linear systems $\dot{x} = Ax + Bu$ or nonlinear ones of the form $\dot{x} = A(y)x + B(y)u$, where the nonlinear terms in $A(\cdot)$ and $B(\cdot)$) depending exclusively on the output y. This work is devoted to develop an alternative set of LMI conditions for a larger family of nonlinear systems.

2.1 Exact convex models

In order to obtain LMI conditions, system(1) needs to be expressed in a convex form. Among the exiting methodologies, the sector nonlinearity approach (Ohtake et al., 2003) provides an exact representation. This methodology is summarized as follows, it begins by expressing the vector fields in (1) as $f(x) = A(x)x$, $g(x) = B(x)$, and $h(x) = C(x)x$, where the entries of $A(x)$ and $C(x)$ should be well-defined in Ω_x, then:

(1) Separate the measurable and unmeasurable nonlinear terms in $A(x)$, $B(x)$, $C(x)$ and $E(x)$. Then, construct two premise vectors $z(y) \in \mathbb{R}^s$ (depending on only available signals) and $\zeta(x) \in \mathbb{R}^\sigma$ (gathering the rest of terms); their entries are assumed bounded and smooth in a region $\Omega_x \subset \mathbb{R}^n : 0 \in \Omega_x$, that is, $z_i \in [z_i^0, z_i^1], i \in \{1, 2, \dots, s\}$ and $\zeta_j \in [\zeta_j^0, \zeta_j^1], j \in \{1, 2, \dots, \sigma\}$. The region Ω_x could be seen as an operating region or the place where the controller is valid.

(2) Each entry of the premise vectors can be exactly written as a convex sum of its bounds, i.e.,

$$\begin{aligned} z_i(y) &= w_0^i(y)z_i^0 + w_1^i(y)z_i^1, \\ \zeta_j(x) &= \omega_0^j(x)\zeta_j^0 + \omega_1^j(x)\zeta_j^1, \end{aligned} \tag{3}$$

where the scalar functions are

$$\begin{aligned} w_0^i(y) &= \frac{z_i^1 - z_i(y)}{z_i^1 - z_i^0}, \quad w_1^i(y) = 1 - w_0^i(y), \\ \omega_0^j(x) &= \frac{\zeta_j^1 - \zeta_j(x)}{\zeta_j^1 - \zeta_j^0}, \qquad \omega_1^j(x) = 1 - \omega_0^j(x). \end{aligned} \tag{4}$$

These functions hold the convex sum property for $x \in \Omega_x$.

(3) Define the scheduling functions as

$$\begin{aligned} \mathbf{w}_i(y) &= w_{i_1}^1(y)w_{i_2}^2(y)\cdots w_{i_s}^s(y), \\ \boldsymbol{\omega}_j(x) &= \omega_{j_1}^1(x)\omega_{j_2}^2(x)\cdots \omega_{j_\sigma}^\sigma(x), \end{aligned} \tag{5}$$

with $i \in \{1, 2, \dots, r\}$, $r = 2^s$, $j \in \{1, 2, \dots, \rho\}$, $\rho = 2^\sigma$, $i_1, i_2, \dots, i_s \in \{0, 1\}$, and $j_1, j_2, \dots, j_\sigma \in \{0, 1\}$. Additionally, the set of indexes $[i_1 i_2 \cdots i_s]$ and $[j_1 j_2 \cdots j_\sigma]$ are a s-digit and σ-digit binary representation of $(i - 1)$ and $(j - 1)$; respectively.

(4) Calculate the vertex matrices $A_{ij} = A(x)|_{\mathbf{w}_i\boldsymbol{\omega}_j=1}$, $B_{ij} = B(x)|_{\mathbf{w}_i\boldsymbol{\omega}_j=1}$, $C_{ij} = C(x)|_{\mathbf{w}_i\boldsymbol{\omega}_j=1}$, and $E_{ij} = E(x)|_{\mathbf{w}_i\boldsymbol{\omega}_j=1}$.

Now, with the previous steps, one is ready to construct an exact convex model of (1), that is

$$E_{\mathbf{w}\boldsymbol{\omega}}\dot{x}(t) = A_{\mathbf{w}\boldsymbol{\omega}}x(t) + B_{\mathbf{w}\boldsymbol{\omega}}u(t), \quad y(t) = C_{\mathbf{w}\boldsymbol{\omega}}x(t), \tag{6}$$

with

$$E_{\mathbf{w}\boldsymbol{\omega}} = \sum_{i=1}^{r}\sum_{j=1}^{\rho}\mathbf{w}_i(y)\boldsymbol{\omega}_j(x)E_{ij},$$

$$A_{\mathbf{w}\boldsymbol{\omega}} = \sum_{i=1}^{r}\sum_{j=1}^{\rho}\mathbf{w}_i(y)\boldsymbol{\omega}_j(x)A_{ij},$$

$$B_{\mathbf{w}\boldsymbol{\omega}} = \sum_{i=1}^{r}\sum_{j=1}^{\rho}\mathbf{w}_i(y)\boldsymbol{\omega}_j(x)B_{ij},$$

$$C_{\mathbf{w}\boldsymbol{\omega}} = \sum_{i=1}^{r}\sum_{j=1}^{\rho}\mathbf{w}_i(y)\boldsymbol{\omega}_j(x)C_{ij}.$$

It is important to stress that convex model (6) is exact in $\mathbb{R}^n$ but only convex in Ω_x.

2.2 Useful lemmas

As shown in (Estrada-Manzo et al., 2015), the use of Finsler's Lemma helps to separate the Lyapunov function and the controller; thus, it allows achieving LMI conditions. In consequence, the equality conditions in (Kau et al., 2007) as well as ILMI conditions in (Huang and Nguang, 2007) are avoided.

Lemma 1. (Finsler's Lemma). (Oliveira and Skelton, 2001) Let $\xi \in \mathbb{R}^n$, $\mathcal{Q} = \mathcal{Q}^T \in \mathbb{R}^{n\times n}$, and $\mathcal{R} \in \mathbb{R}^{m\times n}$ such that $\text{rank}(\mathcal{R}) < n$; the next expressions are equivalents

- $\xi^T \mathcal{Q}\xi < 0, \forall \xi \in \{\xi \in \mathbb{R}^n : \xi \neq 0, \mathcal{R}\xi = 0\}$.
- $\exists \mathcal{M} \in \mathbb{R}^{n\times m} : \mathcal{M}\mathcal{R} + \mathcal{R}^T\mathcal{M}^T + \mathcal{Q} < 0$.

Lemma 2. (Tuan et al., 2001): Let $\Upsilon_{ik}^j = (\Upsilon_{ik}^j)^T$, $i, k \in \{1, 2, \ldots, r\}$, $j \in \{1, 2, \ldots, \rho\}$ be matrices of appropriate dimensions. Then

$$\Upsilon_{\mathbf{ww}}^{\boldsymbol{\omega}} = \sum_{i=1}^{r}\sum_{k=1}^{r}\sum_{j=1}^{\rho} \mathbf{w}_i(y)\mathbf{w}_k(y)\boldsymbol{\omega}_j(x)\Upsilon_{ik}^j < 0,$$

holds if the following set of LMIs

$$\frac{2}{r-1}\Upsilon_{ii}^j + \Upsilon_{ik}^j + \Upsilon_{ki}^j < 0, \quad \forall i, j, k, \tag{7}$$

holds too.

Notation: In matrix expressions, an asterisk $(*)$ denotes the transpose of the symmetric element, this is:

$$\begin{bmatrix} A & B^T \\ B & C \end{bmatrix} = \begin{bmatrix} A & (*) \\ B & C \end{bmatrix};$$

for in-line expressions, it stands for the transpose of terms on its left side, i.e., $A+B+A^T+B^T+C = A+B+(*)+C$.

3. MAIN RESULTS

Based on the idea of (Estrada-Manzo et al., 2015), let us propose the the following control law:

$$u(t) = H_{\mathbf{ww}}^{-1}K_{\mathbf{w}}y(t), \tag{8}$$

where

$$H_{\mathbf{ww}} = \sum_{i=1}^{r}\sum_{k=1}^{r}\mathbf{w}_i(y)\mathbf{w}_k(y)H_{ik}, \quad H_{ik} \in \mathbb{R}^{m\times m},$$

$$K_{\mathbf{w}} = \sum_{i=1}^{r}\mathbf{w}_i(y)K_i, \quad K_i \in \mathbb{R}^{m\times o},$$

are nonlinear gains with convex structures depending only on available signals (via the premise vector $z(y)$). In this sense, (8) is a generalization of the controller given in (Estrada-Manzo et al., 2015).

Traditional approaches compute the closed-loop system between (6) and (8), i.e.,

$$E_{\mathbf{w}\boldsymbol{\omega}}\dot{x}(t) = \left(A_{\mathbf{w}\boldsymbol{\omega}} + B_{\mathbf{w}\boldsymbol{\omega}}H_{\mathbf{ww}}^{-1}K_{\mathbf{w}}C_{\mathbf{w}\boldsymbol{\omega}}\right)x(t),$$

from which the task of finding LMI conditions is impossible because the designing gains $H_{\mathbf{ww}}$ and $K_{\mathbf{w}}$ are "trapped" in the middle of known matrices. In order to avoid this issue, Lemma 1 comes at hand. Then, the convex model together with the control law are put together as an equality constraint:

$$\underbrace{\begin{bmatrix} A_{\mathbf{w}\boldsymbol{\omega}} & -E_{\mathbf{w}\boldsymbol{\omega}} & B_{\mathbf{w}\boldsymbol{\omega}} \\ H_{\mathbf{ww}}^{-1}K_{\mathbf{w}}C_{\mathbf{w}\boldsymbol{\omega}} & 0 & -I \end{bmatrix}}_{\mathcal{R}}\underbrace{\begin{bmatrix} x \\ \dot{x} \\ u \end{bmatrix}}_{\xi} = 0. \tag{9}$$

The Lyapunov function candidate to be considered is

$$V(x) = x^TPx, \quad P = P^T > 0; \tag{10}$$

its time-derivative is

$$\dot{V} = \dot{x}^TPx + x^TP\dot{x}. \tag{11}$$

Without replacing the dynamics of the system and incorporating the term $u^T0_m\,u = 0$ in (11) gives $\dot{V} = \dot{x}^TPx + x^TP\dot{x} + u^T0u$, which leads to the following:

$$\dot{V}(x) = \begin{bmatrix} x \\ \dot{x} \\ u \end{bmatrix}^T \underbrace{\begin{bmatrix} 0 & P & 0 \\ P & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\mathcal{Q}} \begin{bmatrix} x \\ \dot{x} \\ u \end{bmatrix} < 0. \tag{12}$$

Considering the previous developments, the following result is stated:

Theorem 3. The origin $x = 0$ of the nonlinear descriptor model (1), under the convex control law (8), is asymptotically stable if there exist matrices $P = P^T > 0$, $M_{1k}, M_{3k} \in \mathbb{R}^{n\times n}$, $M_{5k} \in \mathbb{R}^{m\times n}$, $H_{ik} \in \mathbb{R}^{m\times m}$, $K_k \in \mathbb{R}^{m\times o}$ $i, k \in \{1, 2, \ldots, r\}$, $j \in \{1, 2, \ldots, \rho\}$ such that LMIs in (7) hold with

$$\Upsilon_{ik}^j = \begin{bmatrix} \Gamma^{(1,1)} & (*) & (*) \\ \Gamma^{(2,1)} & -M_{3k}E_{ij} - E_{ij}^TM_{3k}^T & (*) \\ \Gamma^{(3,1)} & -M_{5k}E_{ij} + (M_{3k}B_{ij} - \eta_2H_{ik})^T & \Gamma^{(3,3)} \end{bmatrix},$$

where

$$\Gamma^{(1,1)} = M_{1k}A_{ij} + \eta_1K_kC_{ij} + (*),$$
$$\Gamma^{(2,1)} = M_{3k}A_{ij} + \eta_2K_kC_{ij} + (P - M_{1k}E_{ij})^T,$$
$$\Gamma^{(3,1)} = M_{5k}A_{ij} + K_kC_{ij} + (M_{1k}B_{ij} - \eta_1H_{ik})^T,$$
$$\Gamma^{(3,3)} = M_{5k}B_{ij} - H_{ik} + (*).$$

Proof. Due to Lemma 1, the inequality (12) under the equality constraint (9) can be put together yielding:

$$\mathcal{M}\begin{bmatrix} A_{\mathbf{w}\boldsymbol{\omega}} & -E_{\mathbf{w}\boldsymbol{\omega}} & B_{\mathbf{w}\boldsymbol{\omega}} \\ H_{\mathbf{ww}}^{-1}K_{\mathbf{w}}C_{\mathbf{w}\boldsymbol{\omega}} & 0 & -I \end{bmatrix} + (*) + \begin{bmatrix} 0 & P & 0 \\ P & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} < 0, \tag{13}$$

with $\mathcal{M} \in \mathbb{R}^{(2n+m)\times(n+m)}$ being a free matrix; thus, in order to obtain LMI conditions and inspired by (Chadli and Guerra, 2012; Estrada-Manzo et al., 2015) the structure of the slack matrix $\mathcal{M}$ is as follows

$$\mathcal{M} = \begin{bmatrix} M_{1\mathbf{w}} & \eta_1H_{\mathbf{ww}} \\ M_{3\mathbf{w}} & \eta_2H_{\mathbf{ww}} \\ M_{5\mathbf{w}} & H_{\mathbf{ww}} \end{bmatrix},$$

where $M_{1\mathbf{w}}, M_{3\mathbf{w}} \in \mathbb{R}^{n\times n}$, $M_{5\mathbf{w}} \in \mathbb{R}^{m\times n}$, and $\eta_1, \eta_2 \in \mathbb{R}^{n\times m}$ being slack matrices [2], yields

$$\Upsilon_{\mathbf{ww}}^{\boldsymbol{\omega}} = \begin{bmatrix} \Gamma^{(1,1)} & (*) & (*) \\ \Gamma^{(2,1)} & -M_{3\mathbf{w}}E_{\mathbf{w}\boldsymbol{\omega}} - E_{\mathbf{w}\boldsymbol{\omega}}^TM_{3\mathbf{w}}^T & (*) \\ \Gamma^{(3,1)} & -M_{5\mathbf{w}}E_{\mathbf{w}\boldsymbol{\omega}} + (M_{3\mathbf{w}}B_{\mathbf{w}\boldsymbol{\omega}} - \eta_2H_{\mathbf{ww}})^T & \Gamma^{(3,3)} \end{bmatrix} < 0$$

where

[2] Matrices η_1 and η_2 are employed to adjust the dimensions of the inequality, they have to be fixed in advance.

$$
\begin{aligned}
\Gamma^{(1,1)} &= M_{1\mathbf{w}} A_{\mathbf{w}\boldsymbol{\omega}} + \eta_1 K_{\mathbf{w}} C_{\mathbf{w}\boldsymbol{\omega}} + (*)\,,\\
\Gamma^{(2,1)} &= M_{3\mathbf{w}} A_{\mathbf{w}\boldsymbol{\omega}} + \eta_2 K_{\mathbf{w}} C_{\mathbf{w}\boldsymbol{\omega}} + (P - M_{1\mathbf{w}} E_{\mathbf{w}\boldsymbol{\omega}})^T\,,\\
\Gamma^{(3,1)} &= M_{5\mathbf{w}} A_{\mathbf{w}\boldsymbol{\omega}} + K_{\mathbf{w}} C_{\mathbf{w}\boldsymbol{\omega}} + (M_{1\mathbf{w}} B_{\mathbf{w}\boldsymbol{\omega}} - \eta_1 H_{\mathbf{w}\mathbf{w}})^T\,,\\
\Gamma^{(3,3)} &= M_{5\mathbf{w}} B_{\mathbf{w}\boldsymbol{\omega}} - H_{\mathbf{w}\mathbf{w}} + (*)\,.
\end{aligned}
$$

Finally, applying Lemma 2 concludes the proof.

Remark 4. Note that from (13) an LMI problem cannot be achieved only by dropping off the convex functions $\mathbf{w}$ and $\boldsymbol{\omega}$; thus the structure of the slack matrix $\mathcal{M}$ becomes important, it should be such that an LMI problem can be computed (Chadli and Guerra, 2012).

Remark 5. Conditions in Theorem 3 generalize those in Theorem 1 from (Estrada-Manzo et al., 2015), this can be seen from two points of view: 1) the family of systems that can be considered, i.e., while in (Estrada-Manzo et al., 2015) the premise vector depends only on measurable signals, our proposal is able to take into account unmeasurable ones via the premise vector $\zeta(x)$; 2) $\mathcal{M}$ contains two different matrices for adjusting dimensions, that is, η_1 and η_2 which provides more degree of freedom than (Estrada-Manzo et al., 2015).

4. NUMERICAL EXAMPLES

This section is devoted to illustrate the advantages of the proposal via numerical examples. All the tests have been performed in SeDuMi (Sturm, 1999) within YALMIP (Lofberg, 2004) for MATLAB R2015a.

Example 6. Consider a nonlinear descriptor (1) with

$$
E(x)=\begin{bmatrix} 2 & -(x_1^2+1)^{-1} \\ (x_1^2+1)^{-1} & 1 \end{bmatrix}, g(x)=\begin{bmatrix} 1 \\ -1 \end{bmatrix}, h(x)=x_1,
$$

$$
f(x)=\begin{bmatrix} -7\sin x_1 - 2x_2\cos x_2 \\ -0.5x_1^3 + 0.5x_2 \end{bmatrix}=\begin{bmatrix} -7\dfrac{\sin x_1}{x_1} & -2\cos x_2 \\ -0.5x_1^2 & 0.5 \end{bmatrix}x.
$$

Figure 1 shows that the system trajectories diverge from the origin as time goes to infinity. Thus, the task is to asymptotically stabilize the origin within the region of interest $\Omega_x = \{x : |x_1| \le 2, x_2 \in \mathbb{R}\}$. The nonlinearities have been selected as shown in Table 1, there are three depending on available signals while only one depends on unavailable ones.

Table 1. Nonlinearities and their bounds in Example 6

$z(y)$ / $\zeta(x)$	z_i^0 / ζ_i^0	z_i^1 / ζ_i^1
$z_1(x_1) = (x_1^2+1)^{-1}$	0.2	1
$z_2(x_1) = \sin(x_1)/x_1$	0.4546	1
$z_3(x_1) = x_1^2$	0	4
$\zeta_1(x_2) = \cos x_2$	-1	1

Thus, an exact convex descriptor model of the form (6) is computed, with vertex

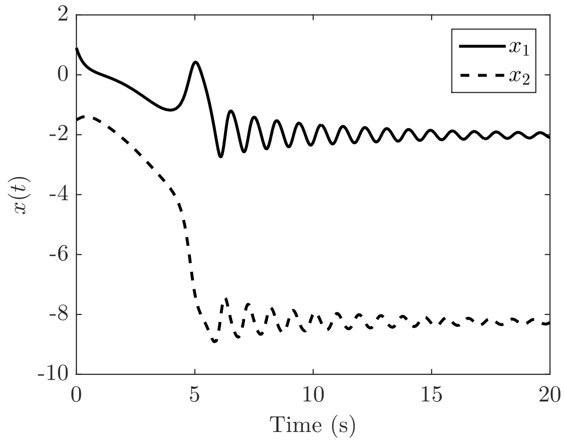

Fig. 1. State trajectories in open-loop for Example 6.

$$
E_1 = E_2 = E_3 = E_4 = \begin{bmatrix} 2 & -0.2 \\ 0.2 & 1 \end{bmatrix}, B=\begin{bmatrix} 1 \\ -1 \end{bmatrix},
$$

$$
E_5 = E_6 = E_7 = E_8 = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}, C=\begin{bmatrix} 1 \\ 0 \end{bmatrix}^T,
$$

$$
A_{1,1}=A_{5,1}=\begin{bmatrix} -3.1825 & 2 \\ 0 & 0.5 \end{bmatrix}, A_{2,1}=A_{6,1}=\begin{bmatrix} -3.1825 & 2 \\ -2 & 0.5 \end{bmatrix},
$$

$$
A_{3,1} = A_{7,1} = \begin{bmatrix} -7 & 2 \\ 0 & 0.5 \end{bmatrix}, A_{4,1} = A_{8,1}=\begin{bmatrix} -7 & 2 \\ -2 & 0.5 \end{bmatrix},
$$

$$
A_{1,2}=A_{5,2}=\begin{bmatrix} -3.1825 & -2 \\ 0 & 0.5 \end{bmatrix}, A_{2,2}=A_{6,2}=\begin{bmatrix} -3.1825 & -2 \\ -2 & 0.5 \end{bmatrix},
$$

$$
A_{3,2}=A_{7,2}=\begin{bmatrix} -7 & -2 \\ 0 & 0.5 \end{bmatrix}, A_{4,2}=A_{8,2}=\begin{bmatrix} -7 & -2 \\ -2 & 0.5 \end{bmatrix};
$$

the convex functions are constructed as (4); they form the scheduling functions (5). The conditions in Theorem 3 are tested for several selections of η_1 and η_2; among them the ones suggested by Estrada-Manzo et al. (2015), that is,

$$
1)\ \eta_1=\eta_2=\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad 2)\ \eta_1=\eta_2=\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad 3)\ \eta_1=\eta_2=B;
$$

none of them are feasible. However, our proposal allows more flexibility; thus, LMIs are obtained with

$$
\eta_1 = B \quad \text{and} \quad \eta_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
$$

and found feasible. Some of the computed values are $K_1 = -0.3422$, $K_3 = -0.4373$, $K_6 = -0.7868$, $K_8 = 0.9750$, $H_{1,1} = 0.0847$, $H_{3,2} = 0.1839$, $H_{7,5} = 0.3015$, $H_{5,4} = 0.2299$, $H_{8,1} = 0.2575$, $H_{7,7} = 1473$. The closed-loop system is run for initial conditions $x(0) = [0.9\ \ -1.5]^T$, Figure 2 shows the states converging towards the origin while Figure 3 plots the control signal.

It is important to stress that previous approaches cannot be applied because they only considered systems with measurable premise variables; additionally, conditions (Chadli and Guerra, 2012; Estrada-Manzo et al., 2019) are not in the descriptor form.

Example 7. This example is intended to show how the extra flexibility on the selection of η_1 and η_2 helps to provide a better solution set than a single matrix η as in (Estrada-Manzo et al., 2015), see Remark 5. To this end, consider example 1 in (Estrada-Manzo et al., 2015), where the system has the form

$$E_{\mathbf{w}}\dot{x}(t) = A_{\mathbf{w}}x(t) + B_{\mathbf{w}}u(t), \quad y(t) = C_{\mathbf{w}}x(t),$$

note that the convex functions only depend on available signals ($r = 2$ and $\rho = 0$). The vertex matrices are:

$$E_1 = \begin{bmatrix} 1.05 & 0.7 & 0.7 \\ -0.1 & 1.1 & -0.2 \\ 0.1 & 0.5 & 0.9-a \end{bmatrix}, \; E_2 = \begin{bmatrix} 0.9+b & 0.8 & 0.77 \\ -0.9 & 1.1 & -0.2 \\ 0.4 & 0.5 & 0.6 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} -1.15 & 0.1 & 1.8+b \\ 0.3 & -1.3 & -0.5 \\ -0.1 & 0.8 & -0.8 \end{bmatrix}, \; A_2 = \begin{bmatrix} -1.2 & -0.3 & -0.1 \\ 0.4 & -0.6 & 0.3 \\ -0.2 & -0.2 & -0.2-a \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.6 & 1.2 \\ 0.3 & 1.5-a \\ -0.6 & 1.3 \end{bmatrix}, \; B_2 = \begin{bmatrix} -1.3 & 2.1 \\ -2.7 & 0.5 \\ 1.5 & 1.6 \end{bmatrix},$$

$C_1 = [0.4\ 1\ 0]$, and $C_2 = [0.8\ 1\ 0]$; the parameters vary as $a \in [-0.5\ \ 1]$ and $b \in [-0.5\ \ 1]$. Conditions in Theorem 3 have been tested under the following different choices for η_1 and η_2:

$$\text{Op}_1\text{: } \eta_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \eta_2 = B_{\mathbf{w}}; \; \text{Op}_2\text{: } \eta_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \eta_2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix};$$

$$\text{Op}_3\text{: } \eta_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \eta_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \text{Op}_4\text{: } \eta_1 = B_{\mathbf{w}}, \eta_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix};$$

$$\text{Op}_5\text{: } \eta_1 = B_{\mathbf{w}}, \eta_2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}, \text{Op}_6\text{: } \eta_1 = B_{\mathbf{w}}, \eta_2 = B_{\mathbf{w}};$$

$$\text{Op}_7\text{: } \eta_1 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}, \eta_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \text{Op}_8\text{: } \eta_1 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}, \eta_2 = B_{\mathbf{w}};$$

$$\text{Op}_9\text{: } \eta_1 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}, \eta_2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}.$$

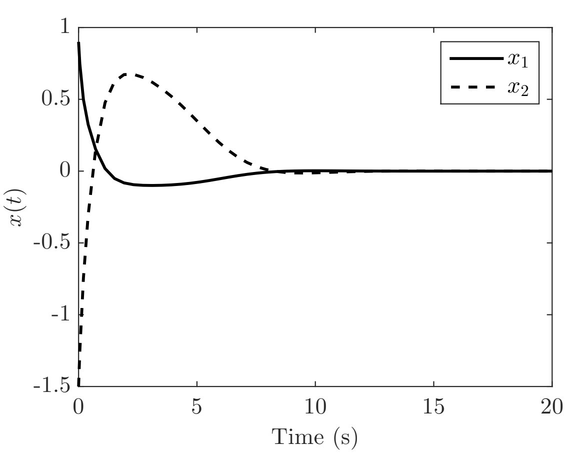

Fig. 2. State trajectories in closed-loop for Example 6.

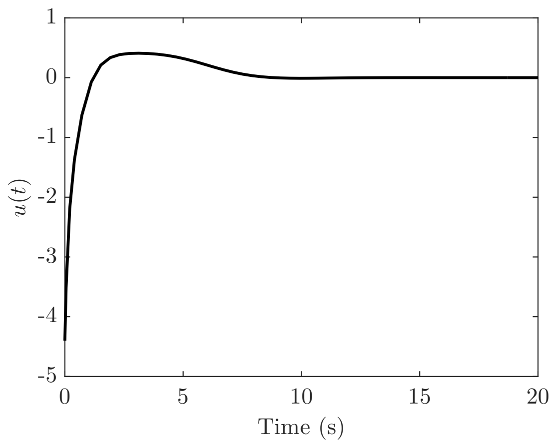

Fig. 3. Control signal in Example 6.

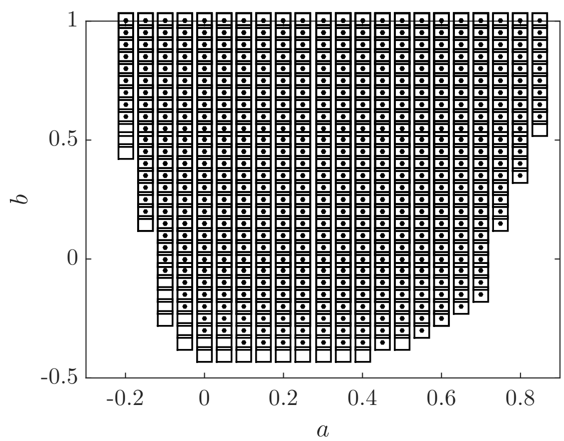

Fig. 4. Feasible set for configurations in (Estrada-Manzo et al., 2015) (●) and 9 configurations for Theorem 3 (□).

Figure 4 shows a clear superiority in terms of feasible points, this is, using η_1 and η_2 helps reducing conservativeness in comparison with a single η.

5. CONCLUSION

It has been presented a less conservative set of LMIs for the design of static output feedback controllers. Furthermore, it has been shown that convex modeling can be applied such that available and unavailable signals are split in order to use only the former ones in the controller structure; thus allowing a greater family of nonlinear systems than previous approaches. As a shortcoming, a question arises regarding the right selection for variables η_1 and η_2; this is left as future work.

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