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Leader-Follower Consensus Formation Control of Nonholonomic Vehicles with Input Constraints

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Abstract: This paper proposes a solution for the leader-follower consensus formation control problem of nonholonomic vehicles that exhibit input constraints. In this consensus problem, the position and the orientation of all the vehicles has to be regulated at a desired equilibrium, hence this pertains to a stabilization scenario. Therefore, in order to satisfy Brockett's theorem, the controller has to be designed to be either discontinuous or time-varying. The proposed scheme is a smooth bounded Proportional plus damping injection controller that incorporates a persistency of excitation term. A comparative simulation analysis with an unbounded control scheme is also provided.

Keywords: Multi-agent systems, formation control, consensus control, nonholonomic systems, persistency of excitation.

1. INTRODUCTION

The collective coordination of multiple mobile agents has received great interest in recent years since the multi-robot system can execute a number of tasks that single systems or fixed-base robots can achieve. A particular coordination problem refers to the formation consensus problem, where all agents agree to a common unspecified value, leaderless consensus, or to a pre-defined value, leader-follower consensus (Ren and Beard, 2008; Cao and Ren, 2011; Wang, 2014; Hatanaka et al., 2015). As the number of robots becomes larger, centralized controllers, where each agent receives the state information of all other agents, becomes problematic due to the heavy communication load. Distributed control, where each agent only uses information from its neighbors to archive the desired formation, becomes more desirable (Ren, 2009).

When working with mobile vehicles the nonholonomic restrictions impose an inconvenient since they cannot reach any position with any velocity (Lin et al., 2005). This problem has been addressed in (Do, 2009) where a partial consensus problem is solved since only translational consensus is reached. In (Dimarogonas and Kyriakopoulos, 2007) a decentralized discontinuous and time-invariant controller is proposed to archive full consensus (both in position and orientation). In (Peng et al., 2015), a decentralized formation control law using a consensus based approach to drive a group of agents exponentially to a desired geometric pattern and reference trajectory is proposed.

In real physical systems, input saturation signals should be considered in the controller design in order to avoid actuator thermal or physical damage in addition to ensuring optimal performance of the controller. Along this line, Ren (2009) uses saturated controllers to ensure the inputs satisfy pre-imposed bounds. In (Zavala-Rio et al., 2011; Kostic et al., 2010) trajectory tracking under bounded inputs is archived. Yu and Liu (2015) proposes a solution for the leader-follower formation problem under velocity constrains. The work of Fu et al. (2021) uses Extended Kalman Filters and a switching control strategy to solve the leader-follower formation and obstacle avoidance with bounded input constrains. Similar techniques have been also used for multiagent systems (Cruz-Zavala et al., 2019), but more scarcely for networked nonholonomic vehicles, *e.g.*, in (Ren and Beard, 2008), without controlling the orientation of the vehicles.

The control design in this paper follows the lines of our previous works (Nuño et al., 2020; Loría et al., 2022; Nuño et al., 2022), but differs from the latter in that here we assume velocity measurements to be available and the network to be delay-free. On the other hand, we design the controller to satisfy pre-imposed bounds, a constraint not considered in these references or any other. We should remark that the consensus formation problem in this paper is inherently a set-point stabilization problem and, due to the nonholonomic constraints, trajectory tracking results, as (Mera et al., 2020), even for a single robot, cannot be directly ported to our scenario (Lizárraga., 2004).

Our main contribution is the solution to the leader-follower consensus formation problem for a network of non-holonomic differential drive robots and a static leader. The interconnection topology of all the followers is undirected, static and connected. The controller is designed to be bounded, smooth and time-varying. Our proposal consists of a Proportional plus damping (P+d) control scheme with a time-varying vanishing term that excites the angular velocity to deal with the nonholonomic restrictions of the vehicles. A comparative simulation analysis with an unbounded control scheme is also provided.

2. PROBLEM SETTING

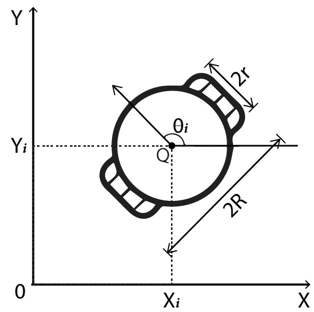

Fig. 1. Geometry of the differential drive vehicle.

We consider a set of N differential drive nonholonomic vehicles, like the one depicted in Fig. 1, each of them modeled as the unicycle kinematics (1) under the assumptions that wheels are rolling without slippage, the steering axis is orthogonal to the xy-plane and the geometry center Q coincides with the center of mass.

$$\begin{aligned} \dot{z}_i &= \varphi(\theta_i)v_i, \\ \dot{\theta}_i &= \omega_i; \end{aligned} \tag{1}$$

where $\varphi(\theta_i) := [\cos(\theta_i), \sin(\theta_i)]^\top$, $z_i := [x_i, y_i]^\top \in \mathbb{R}^2$ stands as the Cartesian coordinates and $\theta_i \in \mathbb{R}$ as the orientation with respect to the x-axis, v_i and $\omega_i \in \mathbb{R}$ are the linear and angular velocities, respectively. Subindex $i \in \bar{N} := \{1, \dots, N\}$. The nonholonomic constraint is given by the expression

$$\dot{x}_i \sin(\theta_i) - \dot{y}_i \cos(\theta_i) = 0.$$

The force controlled dynamics of each vehicle is given by

$$\begin{bmatrix} m_i & 0 \\ 0 & I_i \end{bmatrix} \begin{bmatrix} \dot{v}_i \\ \dot{\omega}_i \end{bmatrix} = \frac{1}{r_i} \begin{bmatrix} 1 & 1 \\ 2R_i & -2R_i \end{bmatrix} \begin{bmatrix} \tau_{li} \\ \tau_{ri} \end{bmatrix} \tag{2}$$

where m_i is the mass, I_i is the moment of inertia, R_i is the half distance between the actuated wheels, r_i corresponds to the radius of the actuated wheels, τ_{li} and τ_{ri} stands for the control input torque of the left and right wheels.

The leader-follower consensus formation control problem involves making the vehicles acquire a desired position relative to a constant (desired) formation centre $z_\ell \in \mathbb{R}^2$ and a constant desired orientation $\theta_\ell \in \mathbb{R}$. The formation is determined by defining a constant vector $\delta_i \in \mathbb{R}^2$, $\delta_i := [\delta_{xi}, \delta_{yi}]^\top$, and the relative position error $\bar{z}_i := z_i - \delta_i$ for each follower. Then, the control goal is that all relative positions and orientations of the followers converge to the desired position and the desired orientation, respectively. Hence, for all initial conditions,

$$\lim_{t\to\infty} v_i(t) = 0, \quad \lim_{t\to\infty} \bar{z}_i(t) = \bar{z}_\ell, \tag{3}$$

$$\lim_{t\to\infty} \omega_i(t) = 0, \quad \lim_{t\to\infty} \theta_i(t) = \theta_\ell \qquad \forall\, i \in \bar{N}, \tag{4}$$

In this work we are interested in solving the aforementioned problem under the following constraint.

Constraint 1. Given $\bar{\tau}_{li} > 0$ and $\bar{\tau}_{ri} > 0$, the left and right control input torques must satisfy $|\tau_{li}| \leq \bar{\tau}_{li}$ and $|\tau_{ri}| \leq \bar{\tau}_{ri}$. ◁

We assume that each vehicle possesses position and velocity sensors and is able to communicate them to a number of neighbors through a relatively reliable network. The interconnection of the follower agents is modeled as a undirected, static, connected and weighted graph via the Laplacian matrix $L \in \mathbb{R}^{N\times N}$, such that its elements are defined as

$$\ell_{ij} = \begin{cases} \sum\limits_{k\in\mathcal{N}_i} a_{ik} & i = j \\ -a_{ij} & i \neq j, \end{cases} \tag{5}$$

where $a_{ij} > 0$ if $j \in \mathcal{N}_i$ and $a_{ij} = 0$ otherwise. The set $\mathcal{N}_i$ contains all the neighbors to the ith-node. By construction, $L1_N = 0$, such that $1_N = [1, \cdots, 1]^\top$. Matrix L is symmetric, it has a unique zero eigenvalue, and the rest of its spectrum is strictly positive.

In order to model the leader-follower interconnection, we define a diagonal matrix $A_\ell := \text{diag}\{a_{i\ell}\} \in \mathbb{R}^{N\times N}$, if the ith-vehicle receives the leader pose then $a_{i\ell} > 0$, $a_{i\ell} = 0$ otherwise. The following lemma, which is a special case of Lemma 3 in (Hong et al., 2006) and Lemma 1.6 of (Cao and Ren, 2011), provides a fundamental property of the composed Laplacian $L_\ell := L + A_\ell$.

Lemma 1. Consider a diagonal matrix $A_\ell := \text{diag}\{a_{i\ell}\} \in \mathbb{R}^{N\times N}$ and suppose that, at least, one $a_{i\ell}$ is strictly positive. Assume that the interconnection graph is undirected, static, and connected; then the matrix $L_\ell := L + A_\ell$ is symmetric, positive definite and of full rank.

3. PROPOSED CONTROL SCHEME

Let us start the control design by setting the following inner control-loop

$$\begin{bmatrix} \tau_{li} \\ \tau_{ri} \end{bmatrix} = \frac{r_i}{2} \begin{bmatrix} 1 & 1/2R_i \\ 1 & -1/2R_i \end{bmatrix} \begin{bmatrix} u_{vi} \\ u_{\omega i} \end{bmatrix}, \tag{6}$$

where u_{vi} and $u_{\omega i}$ are new control inputs to be designed. After replacing (6) in (2) yields

$$\begin{aligned} \dot{v}_i &= \frac{1}{m_i} u_{vi}, \\ \dot{\omega}_i &= \frac{1}{I_i} u_{\omega i}. \end{aligned}$$

The inputs u_{vi} and $u_{\omega i}$ are composed of two smooth P+d terms that drive the linear and angular accelerations towards the desired control objective. The linear velocity term is given by

$$u_{vi} := -p_{vi}\varphi(\theta_i)^\top e_{zi} - d_{vi}\tanh(v_i), \tag{7}$$

where p_{vi} is a positive proportional gain, d_{vi} is a positive damping gain, and the error e_{zi} is defined as

$$e_{zi} := a_{i\ell}\tanh(\bar{z}_i - \bar{z}_\ell) + \sum_{j\in\mathcal{N}_i} a_{ij}\tanh(\bar{z}_i - \bar{z}_j). \tag{8}$$

For the angular acceleration driving input, we design $u_{\omega i}$ as

$$u_{\omega i} := -p_{\omega i}e_{\theta i} - d_{\omega i}\tanh(\omega_i) + \alpha_i(t, \theta_i, e_{zi}), \tag{9}$$

where $p_{\omega i}$ is a positive proportional gain, $d_{\omega i}$ is a positive damping gain, the error $e_{\theta i}$ is given by

$$e_{\theta i} := a_{i\ell} \tanh(\theta_i - \theta_\ell) + \sum_{j\in\mathcal{N}_i} a_{ij} \tanh(\theta_i - \theta_j), \quad (10)$$

and the α_i function is designed to be a δ-persistently exciting function (Panteley et al. (2001)), as follows

$$\alpha_i(t, \theta_i, e_{zi}) := k_{\alpha i}\psi_i(t)\varphi(\theta_i)^{\perp\top} e_{zi}, \quad (11)$$

where $k_{\alpha i}$ is a positive gain, $\varphi(\theta_i)^\perp := [-\sin(\theta_i), \cos(\theta_i)]^\top$, and the function ψ_i is twice differentiable, bounded, with bounded derivatives. Furthermore, there exist T and μ such that

$$\int_t^{t+T} \dot\psi_i(s)^2 ds \geq \mu, \quad \forall t \geq 0, \quad (12)$$

hence, $\dot\psi_i$ is persistently exciting. Note that $\varphi(\theta_i)^\perp$ is the annihilator of $\varphi(\theta_i)$, i.e., $\varphi(\theta_i)^{\perp\top}\varphi(\theta_i) = 0$. The function α_i is included in the controller in order to overcome the obstacle that have the vehicles' nonholonomy to asymptotic stabilization.

The closed-loop equations are given by two interconnected systems, under the action of the controllers (7) and (9),

$$\Sigma_{vi} : \begin{cases} \dot{\bar z}_i = \varphi(\theta_i) v_i, \\ \dot v_i = -\dfrac{1}{m_i}[p_{vi}\varphi(\theta_i)^\top e_{zi} + d_{vi}\tanh(v_i)], \end{cases} \quad (13)$$

and

$$\Sigma_{\omega i} : \begin{cases} \dot\theta_i = \omega_i, \\ \dot\omega_i = -\dfrac{1}{I_i}[p_{\omega i}e_{\theta i} + d_{\omega i}\tanh(\omega_i) - \alpha_i(t, \theta_i, e_{zi})]. \end{cases} \quad (14)$$

Now, we are ready to formulate our main result.

Proposition 1. Consider a swarm of N differential drive nonholonomic vehicles, each one of them modeled by (1) and (2) under Constraint 1, such the interconnection is modeled by (5). Suppose that at least one vehicle has access to the desired leader position and orientation of the centre of the formation. Defining maximum inputs as $\bar u_{vi} := d_{vi} + \sqrt{2}p_{vi}(a_{i\ell} + \ell_{ii})$ and $\bar u_{\omega i} := d_{\omega i} + p_{\omega i}(a_{i\ell} + \ell_{ii}) + \sqrt{2}k_{\alpha i}\bar\psi_i(a_{i\ell} + \ell_{ii})$ such that the following inequalities are satisfied

$$d_{\omega i} \geq \frac{\sqrt{2}k_{\alpha i}\bar\psi_i(a_{i\ell} + \ell_{ii})}{\tanh(1)}, \quad (15)$$

and

$$2R_i\bar u_{vi} + \bar u_{\omega i} < \frac{4R_i}{r_i}\min\{\bar\tau_{ri}, \bar\tau_{li}\}. \quad (16)$$

Then, the control scheme described by (6) with (7) and (9) ensures that the control goal (3)–(4) is achieved. ◃

Remark 1. Before going through the proof, let us remark that the control objective is ensured if we can prove that the errors e_{zi} and $e_{\theta i}$ globally, converge to zero. In order to see this fact, let us invoke Lemma 2.4 in (Ren, 2009), to write

$$\sum_{i=1}^N (\bar z_i - \bar z_\ell)^\top e_{zi} = \frac{1}{2}\sum_{i=1}^N \sum_{j\in\mathcal{N}_i} a_{ij}(\bar z_i - \bar z_j)^\top \tanh(\bar z_i - \bar z_j) + \sum_{i=1}^N a_{i\ell}(\bar z_i - \bar z_\ell)^\top \tanh(\bar z_i - \bar z_\ell).$$

Suppose now that $e_{zi} = 0$, since $(\bar z_i - \bar z_j)^\top \tanh(\bar z_i - \bar z_j)$ vanishes only when $\bar z_i - \bar z_j = 0$, it follows that $\bar z_i - \bar z_j = 0$ and $\bar z_i - \bar z_\ell = 0$, for all $j \in \mathcal{N}_i$ and $i \in \bar N$. Hence, it is also true that

$$\frac{1}{2}\sum_{i=1}^N \sum_{j\in\mathcal{N}_i} a_{ij}(\bar z_i - \bar z_j)^\top(\bar z_i - \bar z_j) + \sum_{i=1}^N a_{i\ell}|\bar z_i - \bar z_\ell|^2 = 0.$$

Using the composed Laplacian matrix properties, this last equation can be written as

$$(\bar z - 1_N \otimes \bar z_\ell)^\top (L_\ell \otimes I_2)(\bar z - 1_N \otimes \bar z_\ell) = 0,$$

where $\bar z = \mathrm{col}(\bar z_i)$. Thus, invoking Lemma 1, it holds that $\bar z = 1_N \otimes \bar z_\ell$ and thus, $e_{zi} = 0$ implies that $\bar z_i = \bar z_\ell$, for all $i \in \bar N$.

Following these steps we can also show that $e_{\theta i} = 0$ implies that $\theta_i = \theta_\ell$, for all $i \in \bar N$. △

Proof of Proposition 1. The proof is divided in three steps, one to prove boundedness of all trajectories, another to establish convergence of the position and the orientation errors and, finally, in the last step we show that the designed controller satisfies Constraint 1.

(*Boundedness of Trajectories*). For the Σ_{vi} system consider the following Lyapunov candidate function

$$V := \frac{1}{2}\sum_{i=1}^N \Bigg[\frac{m_i}{p_{vi}}v_i^2 + 2a_{i\ell}\Big(\ln(\cosh(\bar x_i - \bar x_\ell)) + \ln(\cosh(\bar y_i - \bar y_\ell))\Big) + \sum_{j\in\mathcal{N}_i} a_{ij}\ln(\cosh(\bar x_i - \bar x_j)) + \sum_{j\in\mathcal{N}_i} a_{ij}\ln(\cosh(\bar y_i - \bar y_j))\Bigg] \quad (17)$$

which is positive definite and radially unbounded with respect to v_i, $\bar z_i - \bar z_\ell$, and $\bar z_i - \bar z_j$, for all $i \in \bar N$ and $j \in \mathcal{N}_i$. Evaluating the total derivative $\dot V$ along the trajectories in (13), and after applying (Ren, 2009, Lemma 2.4), we obtain

$$\dot V = -\sum_{i=1}^N \frac{d_{vi}}{p_{vi}} v_i \tanh(v_i) \leq 0. \quad (18)$$

Since $V \geq 0$ and $\dot V \leq 0$ then v_i, $\bar z_i - \bar z_\ell$, and $\bar z_i - \bar z_j \in \mathcal{L}_\infty$. Moreover, $v_i \in \mathcal{L}_2$. Since the right-hand side of (13) is uniformly bounded, then $\dot v_i \in \mathcal{L}_\infty$.

Now for the $\Sigma_{\omega i}$ system consider the following Lyapunov candidate function

$$W := \frac{1}{2}\sum_{i=1}^N \Bigg[\frac{I_i}{p_{\omega i}}\omega_i^2 + 2a_{i\ell}\ln(\cosh(\theta_i - \theta_\ell)) + \sum_{j\in\mathcal{N}_i} a_{ij}\ln(\cosh(\theta_i - \theta_j))\Bigg], \quad (19)$$

which is positive definite and radially unbounded with respect to ω_i, $\theta_i - \theta_\ell$, and $\theta_i - \theta_j$, for all $i \in \bar N$ and $j \in \mathcal{N}_i$. The time derivative of W along the trajectories in (14) and, once again, applying (Ren, 2009, Lemma 2.4), yields

$$\dot W = -\sum_{i=1}^N \frac{1}{p_{\omega i}}[d_{\omega i}\omega_i\tanh(\omega_i) - \omega_i\alpha_i(t, \theta_i, e_{zi})]. \quad (20)$$

The δ-persistently exciting function is bounded by

$$|\alpha_i| \leq \sqrt{2}k_{\alpha i}\bar\psi_i(a_{i\ell} + \ell_{ii}),$$

where $|\psi_i| \leq \bar{\psi}_i$. Consequently

$$\dot{W} \leq -\sum_{i=1}^{N} \frac{1}{p_{\omega i}}[d_{\omega i}\omega_i \tanh(\omega_i) - \omega_i\sqrt{2}k_{\alpha i}\bar{\psi}_i(a_{i\ell} + \ell_{ii})],$$

given that $\tanh(\cdot)$ is strictly increasing, for all $|\omega_i| \geq 1$ we have

$$\dot{W} \leq -\sum_{i=1}^{N} \frac{1}{p_{\omega i}}[d_{\omega i}\tanh(1) - \sqrt{2}k_{\alpha i}\bar{\psi}_i(a_{i\ell} + \ell_{ii})]|\omega_i|.$$

Satisfying the inequality (15), then we have that $\dot{W} \leq 0$. Hence, for all $t \geq 0$ such that $|\omega_i(t)| \geq 1$ we have $\dot{W}(\omega(t), \theta(t)) \leq 0$ so $|\omega_i(t)|$ is bounded. For any other t, $|\omega_i(t)| \leq 1$. This implies that $\omega_i \in \mathcal{L}_\infty$, for all $t \geq 0$. Since the right-hand side of (14) is uniformly bounded, this implies that $\dot{\omega}_i \in \mathcal{L}_\infty$.

(*Convergence of Position Error*). Since $v_i \in \mathcal{L}_\infty \cap \mathcal{L}_2$ and $\dot{v}_i \in \mathcal{L}_\infty$, we have that $\lim_{t\to\infty} v_i(t) = 0$. This, in turn, implies that

$$\lim_{t\to\infty} \int_0^t \dot{v}_i(t) = -v_i(0).$$

Hence, to prove that $\lim_{t\to\infty} \dot{v}_i(t) = 0$ by Barbălat's Lemma, we need to show that $\dot{v}_i$ is uniformly continuous. For, consider

$$\begin{aligned}\ddot{v}_i = &-\frac{1}{m_i}\left[d_{vi}\dot{v}_i \mathrm{sech}^2(v_i) + p_{vi}\omega_i\varphi(\theta_i)^{\top\perp}e_{zi}\right] \\ &+ \frac{1}{m_i}p_{vi}\varphi(\theta_i)^\top \dot{e}_{zi}.\end{aligned} \tag{21}$$

Observe that all the terms on the right-hand-side in (21) are bounded. Thus, $\dot{v}_i \to 0$, so from (13) we have

$$\lim_{t\to\infty} \varphi(\theta_i(t))^\top e_{zi}(t) = 0. \tag{22}$$

Invoking Barbalăt's Lemma, it also follows that $\lim_{t\to\infty} \ddot{v}_i(t) = 0$. From $\dot{v}_i \to 0$, we have that $\dot{e}_{zi} \to 0$. Thus, the first and last terms on the right-hand side of (21), as well as $\ddot{v}_i$, vanish individually. Consequently,

$$\lim_{t\to\infty} \omega_i(t)\varphi(\theta_i(t))^{\top\perp}e_{zi}(t) = 0. \tag{23}$$

Now, because φ and $\varphi^\perp$ take values in orthogonal spaces, if (22) and (23) hold simultaneously whereas $\omega_i \not\to 0$, then $\lim_{t\to\infty} e_{zi}(t) = 0$. If, alternatively, (22) and (23) hold simultaneously because $\lim_{t\to\infty} \omega_i(t) = 0$, then $\lim_{t\to\infty} \int_0^t \dot{\omega}_i(\sigma)d\sigma = -\omega_i(0)$. Moreover,

$$\ddot{\omega}_i = -\frac{1}{I_i}\left[d_{\omega i}\dot{\omega}_i \mathrm{sech}^2(\omega_i) + p_{\omega i}\dot{e}_{\theta i} - \dot{\alpha}_i\right], \tag{24}$$

where

$$\begin{aligned}\dot{\alpha}_i = &k_{\alpha i}\dot{\psi}_i(t)\varphi(\theta_i)^{\perp\top}e_{zi} - k_{\alpha i}\omega_i\psi_i(t)\varphi(\theta_i)^\top e_{zi} \\ &+ k_{\alpha i}\psi_i(t)\varphi(\theta_i)^{\perp\top}\dot{e}_{zi}\end{aligned} \tag{25}$$

is bounded because so are all the terms on the respective right-hand sides of (25) and (24). Hence $\lim_{t\to\infty} \dot{\omega}_i(t) = 0$ and $\lim_{t\to\infty} \int_0^t \ddot{\omega}_i(\sigma)d\sigma = -\dot{\omega}_i(0)$. A direct similar computation shows that, also, $\omega_i^{(3)} \in \mathcal{L}_\infty$, so, by Barbalăt's Lemma, it follows that $\lim_{t\to\infty} \ddot{\omega}_i(t) = 0$. From this and (24) it follows that $\dot{\alpha}_i \to 0$, so from (25) we conclude that

$$\lim_{t\to\infty} \dot{\psi}_i(t)\varphi(\theta_i(t))^{\perp\top}e_{zi}(t) = 0.$$

However, since $\dot{\psi}_i(t)$ satisfies (12), $\dot{\psi}_i(t) \not\to 0$, so the last limit holds only if $\lim_{t\to\infty} \varphi(\theta_i(t))^{\perp\top}e_{zi}(t) = 0$, which together with (22), implies that $\lim_{t\to\infty} e_{zi}(t) = 0$. Following Remark 1, we conclude that $\lim_{t\to\infty} e_{zi}(t) = 0$ implies the second limit in (3).

(*Convergence of Orientation Error*). Assume, for now, that $\alpha_i \equiv 0$ for all $i \in \bar{N}$. Then, a simple inspection of (20) shows that $\ddot{W} \in \mathcal{L}_\infty$ and, invoking Barbalăt's Lemma, we conclude that $\dot{W} \to 0$, which implies in turn that $\lim_{t\to\infty} \omega_i(t) = 0$. The same conclusion is drawn for $\dot{\omega}_i$, after differentiating on both sides of (14) and observing that $\ddot{\omega}_i$ is uniformly bounded. Hence, under the condition that $\alpha_i \equiv 0$, we see from (14) that $\dot{\omega}_i \to 0$ and $\omega_i \to 0$ imply that $\lim_{t\to\infty} e_{\theta i}(t) = 0$. Now, $e_{\theta i} = 0$ and the conclusions in Remark 1, imply that $\theta_i = \theta_\ell$. The limits in (4) follow. Furthermore, because the closed-loop solutions are uniformly globally bounded under any bounded $\alpha_i \not\equiv 0$, the same conclusion follows using a cascades argument (Loría, 2008), provided that $\alpha_i \to 0$, which we have established with the fact that $\lim_{t\to\infty} e_{zi}(t) = 0$.

(*Saturation Avoidance*). In this final part, we show that the actuators do not saturate if the inequality (16) holds. For this purpose, note that, from (7) and (9) with the fact that $|\tanh(\cdot)| < 1$, results

$$|u_{vi}| \leq d_{vi} + \sqrt{2}p_{vi}(a_{i\ell} + \ell_{ii}) := \bar{u}_{vi},$$

and

$$|u_{\omega i}| \leq d_{\omega i} + p_{\omega i}a_{i\ell} + (p_{\omega i} + \sqrt{2}k_{\alpha i}\bar{\psi}_i)\ell_{ii} := \bar{u}_{\omega i}.$$

Hence, from (6) we have

$$\max\{|\tau_{ri}|, |\tau_{li}|\} \leq \frac{r_i}{2}[\bar{u}_{vi} + \frac{1}{2R_i}\bar{u}_{\omega i}]$$

and from (16), it is fulfilled that $|\tau_{li}| \leq \bar{\tau}_{li}$ and $|\tau_{ri}| \leq \bar{\tau}_{ri}$. Thus Constraint 1 holds. □

4. SIMULATIONS

In order to show the effectiveness of the proposed scheme we performed some numerical simulations, we compare the controller of Proposition 1 with one in $\tanh(s)$ is replaced by its argument s. The simulation setup consists in six differential drive vehicles, whose communication topology is depicted in Fig. 2. All interconnection weights are equal to one except the leader connections whose weight is set to 10. The physical parameters and actuators bounds are shown in Table 1, while the initial conditions and the offsets that define a triangular formation with center at the coordinates $(x_\ell, y_\ell) = (-25, -25)$ are shown in Table 2.

Table 1. Physical parameters and actuators bounds

index	m_i [Kg]	I_i [Kg m^2]	R_i [m]	r_i [m]	$\bar{\tau}_i$ [N]
1, 2	10	3	0.3	0.05	55
3, 4	36	15.625	0.35	0.15	160
5, 6	23	9.3125	0.225	0.1	56

The control gains for each robot are set to satisfy the conditions (15) and (16), i.e., $\mathbf{p}_v = 56 \otimes 1_6$, $\mathbf{d}_v = 840 \otimes 1_6$,

Table 2. Initial conditions

index	$x_i(0)$	$y_i(0)$	$\theta_i(0)$	δ_{xi}	δ_{yi}
1	-3	0	π	-5	-3
2	0	5	$\pi/2$	-2.5	2
3	-1	5	$\pi/2$	0	7
4	2	0	0	2.5	2
5	-2	7.5	$-\pi/4$	5	-3
6	-2	-5	$-3\pi/4$	0	-3

$\mathbf{d}_\omega = [28\ 59\ 28\ 59\ 28\ 28]^\top$, $\mathbf{p}_\omega = [2.8\ 5.9\ 2.8\ 5.9\ 2.8\ 2.8]^\top$ $\mathbf{k}_{\alpha i} = [3\ 1\ 3\ 1\ 3\ 3]^\top$. The function $t \mapsto \psi_i(t)$ is defined as

$$\psi_i(t) \ := \frac{5}{4} + \sum_{k=1}^{5} \frac{4}{(2k-1)}\pi \sin\Big(\frac{2k-1}{2}t\Big).$$

For a fair comparison we used identical initial conditions, gains and δ-persistently exciting functions for both control schemes, constrained and unscontrained one.

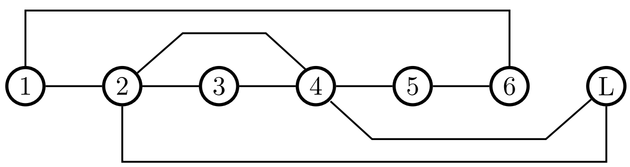

Fig. 2. Undirected-graph topology used in the numerical simulations

The paths described by the vehicles on the plane are illustrated in Figures 3–4. In both cases the formation goal is achieved, and the final orientations are depicted by pointing arrows. Note that all orientations match the leader's $\theta_\ell = 5\pi/4$ rad. From the Figures 5–6 we can realize that it takes more time for the bounded scheme to achieve the control goal, which is an expected behavior considering that the actuators are bounded. In fact, we can observe the bounds of the actuators in Fig. 7, on the left we present the unbounded scheme, while on the right we observe the proposed scheme.

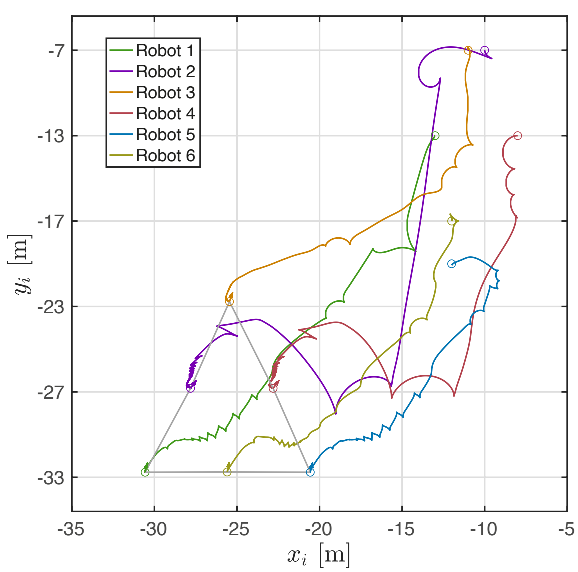

Fig. 3. Paths followed by the agents under the unbounded scheme (orientation of each agent is represented by an arrow.

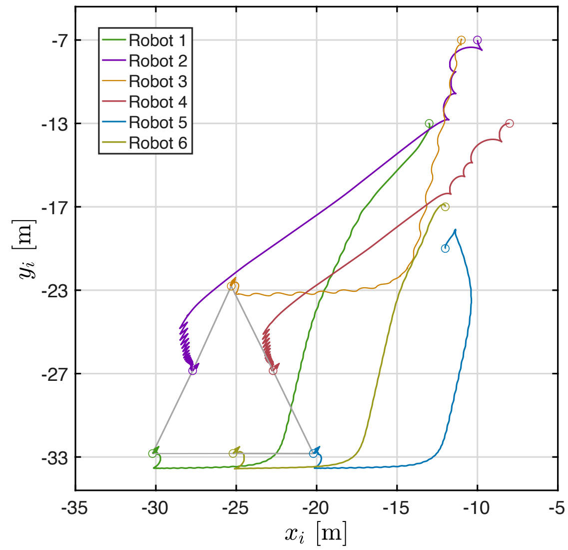

Fig. 4. Paths followed by the agents under the bounded scheme (orientation of each agent is represented by an arrow.

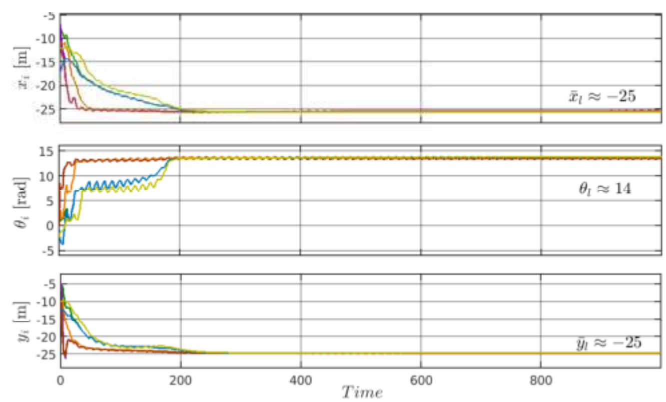

Fig. 5. Pose consensus for the unbounded scheme.

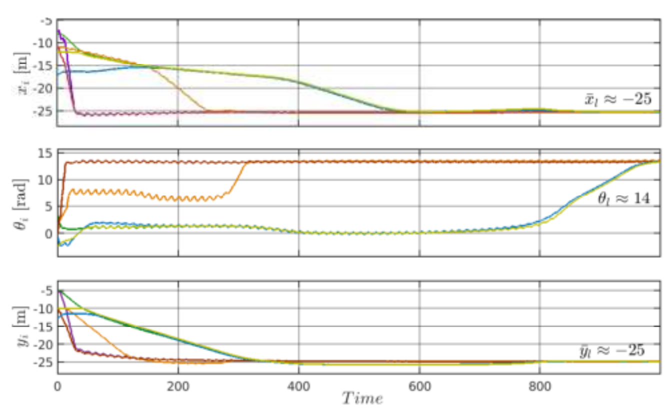

Fig. 6. Pose consensus for the bounded scheme.

5. CONCLUSIONS

In this work we solve the leader-follower consensus formation problem for force-controlled differential drive vehicles. To cope with the difficulty imposed by the nonholonomic constraints, our controller is time-varying. At the same time, the control inputs are guaranteed to respect bounds imposed by the actuators. Assuming that the

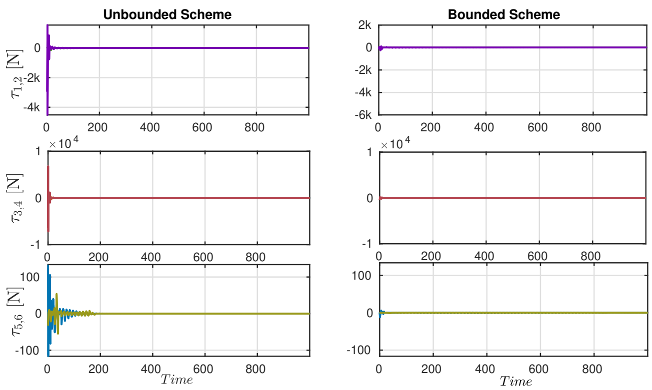

Fig. 7. Comparison of torque results for both schemes.

interconnection graph is undirected, static and connected, the proposed scheme ensures consensus and stabilization both in the Cartesian positions and orientations. Further research is aimed at relaxing the assumption that the complete state is available for measurement.

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