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Distributed Dynamic Consensus Algorithm for Attitude Estimation

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Abstract: This paper proposes a new dynamic consensus algorithm for the attitude estimation of a rigid body using n Measurement Units (MUs) spatially distributed on the body. The MUs are considered as agents in a sensor network, which is modeled using graph theory. It is assumed that the nth MU shares attitude information only with its neighbor agents. Then, a dynamic consensus algorithm is developed to obtain an average quaternion that describes the general attitude of the rigid body. The objective is to get a more accurate and reliable estimation of the attitude, further providing a fault-tolerant system in the event of failure sensors. This new dynamic consensus algorithm is validated in simulation using three MUs in the network. Results show a better precision and demonstrate that the proposed algorithm provides a good estimation of attitude even in the scenario of possible failure of any of the sensors.

Keywords: Dynamic consensus algorithm, sensor network, attitude estimation, fault tolerance.

1. INTRODUCTION

Attitude estimation has been of great interest in the past few decades (Guerrero-Castellanos et al., 2013), due to the wide range of applications in various fields of engineering such as: robotics, ships, spacecraft, biomechanics, etc. However, in some of these applications the accuracy and reliability are of great importance because of the nature of its application (Betanzos-Ramirez et al., 2021).

Since attitude estimation is an issue very important, this has been carried on with inertial sensors (three-axis gyro and three-axis accelerometer) and three axis magnetometers, these sensors form a sensor suite typically called IMU (Inertial Measurement Unit) or MIMU (Magnetic and Inertial Measurement Unit). In this paper, we use MU (Measurement Unit) to refer accelerometer and magnetometer like a sensor suit. Normally deterministic attitude estimators like TRIAD, SVD, ect. are carry out with vectors observations typically obtained from the MU and this is placed so that its reference system is matched with the reference system of the rigid body, therefore the attitude of the rigid body will be given directly by the orientation measured by the MU. In order to improve the accuracy for attitude estimation some algorithms that fuse or averaging the quaternion attitude has been propose such as in (Markley et al., 2007) the authors propose an algorithm based on the minimization of a weighted sum of the Frobenius norm, in (Challis, 2020) the author mentions another form to average quaternion, this consists of an arithmetic average, that is to say, a sum of n quaternions divided by n.

Sensor network is another concept that make possible to obtain measurements of a sensor array and fuse information that each node generates in order to reach an agreement (Kia et al., 2018). The concept of network sensor has been utilized widely in control systems for consensus theory (Olfati-Saber and Shamma, 2006), where the communication between each node of the sensor network is given by a communication graph and each node is capable of develop local computations (Kia et al., 2018).

This paper is organized as follows. In section 2, a mathematical background for distributed attitude parameterization is provided and the statement problem is presented. Section 3, introduces the dynamic consensus algorithm design. The simulation results are presented in section 4. Finally section 5 presents the conclusions.

2. MATHEMATICAL BACKGROUND

2.1 Unit Quaternions

Consider the following, right-handed coordinate frames (see Fig. 1): the inertial coordinate frame, $\mathbf{E}^f = [\boldsymbol{e}_1^f, \boldsymbol{e}_2^f, \boldsymbol{e}_3^f]$, located at some fixed point in space, the body coordinate frame, $\mathbf{E}^b = [\boldsymbol{e}_1^b, \boldsymbol{e}_2^b, \boldsymbol{e}_3^b]$, located at the center of mass of the rigid body, and, the coordinate frame of the MU i denoted $\mathbf{E}^{mu_i} = [\boldsymbol{e}_1^{mu_i}, \boldsymbol{e}_2^{mu_i}, \boldsymbol{e}_3^{mu_i}]$ which is located in the body. It is assumed that the origin of $\mathbf{E}^b$ coincides with the origin of $\mathbf{E}^{mu_i}$. The rotation of a reference frame to another can be represented by the rotation matrix $R \in SO(3) = \{R \in \mathbb{R}^{3\times 3} : R^T R = I, \det R = 1\}$.

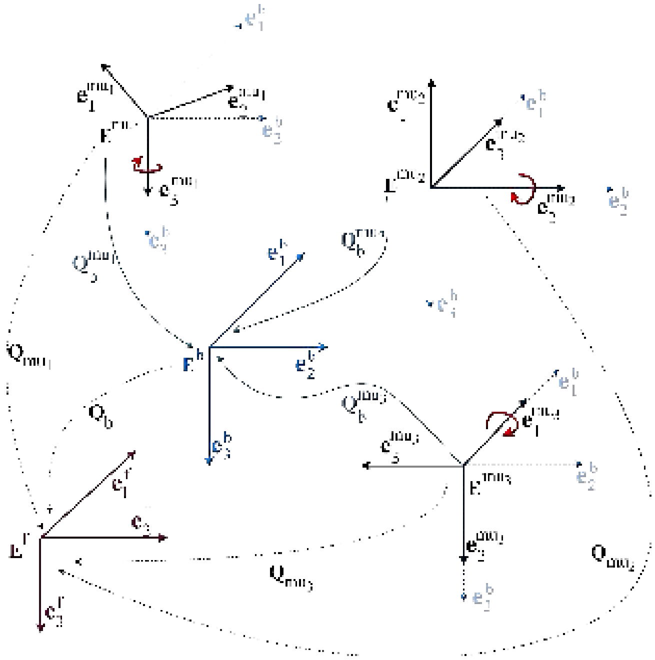

Fig. 1. Relationship between $\mathbf{E}^b$, $\mathbf{E}^f$ and $\mathbf{E}^{mu_i}$ reference systems.

The n-dimensional unit sphere embedded in $\mathbb{R}^{n+1}$ is denoted as $\mathbb{S}^n = \{x \in \mathbb{R}^{n+1} : x^T x = 1\}$. Members of $SO(3)$ are often parameterized in terms of a rotation $\beta \in \mathbb{R}$ about a fixed axis $\boldsymbol{e} \in \mathbb{S}^2$ by the map $\mathcal{U} : \mathbb{R} \times \mathbb{S}^2 \to SO(3)$ defined as:

$$\mathcal{U}(\beta, \boldsymbol{e}) := I + \sin(\beta)[\boldsymbol{e}^\times] + (1 - \cos(\beta))[\boldsymbol{e}^\times]^2 \quad (1)$$

where $[\boldsymbol{\xi}^\times]$ is the skew symmetric matrix defined by:

$$[\boldsymbol{\xi}^\times] = \begin{pmatrix} 0 & -\xi_3 & \xi_2 \\ \xi_3 & 0 & -\xi_1 \\ -\xi_2 & \xi_1 & 0 \end{pmatrix}$$

Hence, a unit quaternion, $\mathbf{Q} \in \mathbb{S}^3$, is defined as:

$$\mathbf{Q} := \begin{pmatrix} \cos\frac{\beta}{2} \\ \boldsymbol{e}\sin\frac{\beta}{2} \end{pmatrix} = \begin{pmatrix} q_0 \\ \boldsymbol{q} \end{pmatrix} \in \mathbb{S}^3 \quad (2)$$

where $\boldsymbol{q} = [q_1, q_2, q_3]^T \in \mathbb{R}^3$ and $q_0 \in \mathbb{R}$ are known as the vector and scalar parts of the quaternion respectively. The inverse unit quaternion is given by $\mathbf{Q}^{-1} := [q_0, -\boldsymbol{q}^T]^T$, and the quaternion product is defined by (Markley et al., 2007):

$$\mathbf{Q}_1 \odot \mathbf{Q}_2 := \begin{pmatrix} q_{1_0} & -\boldsymbol{q}_1{}^T \\ \boldsymbol{q}_1 & I_3 q_{1_0} + [\boldsymbol{q}_1^\times] \end{pmatrix} \begin{pmatrix} q_{2_0} \\ \boldsymbol{q}_2 \end{pmatrix} \quad (3)$$

$$\mathbf{Q}_1 \otimes \mathbf{Q}_2 := \begin{pmatrix} q_{1_0} & -\boldsymbol{q}_1{}^T \\ \boldsymbol{q}_1 & I_3 q_{1_0} - [\boldsymbol{q}_1^\times] \end{pmatrix} \begin{pmatrix} q_{2_0} \\ \boldsymbol{q}_2 \end{pmatrix} \quad (4)$$

It follows of these definitions that:

$$\mathbf{Q}_1 \otimes \mathbf{Q}_2 = \mathbf{Q}_2 \odot \mathbf{Q}_1 \quad (5)$$

The attitude error is used to quantify the mismatch between two attitudes. If $\mathbf{Q}$ and $\hat{\mathbf{Q}}$ define the true attitude quaternion and the estimated one, respectively, then attitude error is given by:

$$\mathbf{Q}_e := \mathbf{Q} \odot \hat{\mathbf{Q}}^{-1} = [q_{e_0}, \boldsymbol{q}_e{}^T]^T \quad (6)$$

In the case that the true quaternion and the estimated one coincide, the quaternion error becomes:

$$\mathbf{Q}_e = [\pm 1, \mathbf{0}^T]^T \quad (7)$$

where $\mathbf{0}$ is a vector of zeros.

2.2 Attitude Measurement Models

Let us consider the representation of a unit vector $\boldsymbol{x}_j$ with respect to $\mathbf{E}^f$, $\mathbf{E}^b$, and $\mathbf{E}^{mu_i}$ which is denoted by $\boldsymbol{r}_j$, $\boldsymbol{b}_j$, and $\boldsymbol{b}_j^{mu_i}$, respectively. The vectors $\boldsymbol{r}_j$ are called the "reference vectors", and in general, are known quite accurately. The vectors $\boldsymbol{b}_j^{mu_i}$ are known as "observation vectors" and are obtained from the ith measurement unit (in our case, magnetometers and accelerometers).

These vector representations are linked up through the vector rotation using quaternions. Vectors $\boldsymbol{b}_j^{mu_i}$, $\boldsymbol{b_j}$ and $\boldsymbol{r_j}$ can be written as quaternions as follows:

$$\begin{aligned} &\mathbf{Q}\left(b_j^{mu_i}\right) = [0, (\boldsymbol{b}_j^{mu_i})^T]^T \quad \mathbf{Q}\left(b_j\right) = [0, \boldsymbol{b}_j^T]^T \\ &\mathbf{Q}\left(r_j\right) = [0, \boldsymbol{r}_j^T]^T \end{aligned} \quad (8)$$

Then, these quaternions are related by the rotation quaternions $\mathbf{Q}_b$, $\mathbf{Q}_b^{mu_i}$, and $\mathbf{Q}_{mu_i}$ as follows:

$$\mathbf{Q}\left(b_j\right) = (\mathbf{Q}_b)^{-1} \odot \mathbf{Q}\left(r_j\right) \odot \mathbf{Q}_b \quad (9)$$

$$\mathbf{Q}\left(b_j^{mu_i}\right) = (\mathbf{Q}_b^{mu_i})^{-1} \odot \mathbf{Q}\left(b_j\right) \odot \mathbf{Q}_b^{mu_i} \quad (10)$$

and, as a consequence:

$$\begin{aligned} \mathbf{Q}\left(b_j^{mu_i}\right) &= (\mathbf{Q}_b^{mu_i})^{-1} \odot \left[(\mathbf{Q}_b)^{-1} \odot \mathbf{Q}\left(r_j\right) \odot \mathbf{Q}_b\right] \odot \mathbf{Q}_b^{mu_i} \\ &= (\mathbf{Q}_{mu_i})^{-1} \odot \mathbf{Q}\left(r_j\right) \odot \mathbf{Q}_{mu_i} \end{aligned} \quad (11)$$

where $\mathbf{Q}_{mu_i} = \mathbf{Q}_b \odot \mathbf{Q}_b^{mu_i}$. Note that $\mathbf{Q}_b^{mu_i}$ is quite known by design. Then, if $\mathbf{Q}_{mu_i}$ is estimated, it is possible to know $\mathbf{Q}_b$, *i.e.*, the attitude of $\mathbf{E}^b$ with respect to $\mathbf{E}^f$ by simple quaternion multiplication, that is:

$$\mathbf{Q}_b = \mathbf{Q}_{mu_i} \odot (\mathbf{Q}_b^{mu_i})^{-1} \quad (12)$$

Moreover, from (11) it is easy to prove that:

$$\begin{aligned} \mathbf{Q}_{mu_i} \odot \mathbf{Q}\left(b_j^{mu_i}\right) &= \mathbf{Q}\left(r_j\right) \odot \mathbf{Q}_{mu_i} \\ \mathbf{Q}\left(b_j^{mu_i}\right) \otimes \mathbf{Q}_{mu_i} &= \mathbf{Q}\left(r_j\right) \odot \mathbf{Q}_{mu_i} \end{aligned} \quad (13)$$

Then using the definition of quaternion product (3) and (4), one obtains the following relation:

$$\begin{pmatrix} 0 & -(\boldsymbol{b}_j^{mu_i} - \boldsymbol{r}_j)^T \\ \boldsymbol{b}_j^{mu_i} - \boldsymbol{r}_j & -[(\boldsymbol{b}_j^{mu_i} + \boldsymbol{r}_j)^{\times}] \end{pmatrix} \mathbf{Q}_{mu_i} = H_j^i \mathbf{Q}_{mu_i} = 0 \tag{14}$$

$H_j^i \in \mathbb{R}^{4\times 4}$ is called the observation matrix and one can see that (14) is a well-structured linear system of equations.

2.3 Graph Theory

The interaction of a multi-agent system can be represented by graphs properties. A graph is formally represented by the set $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ consisting of a set of vertices (or nodes) $\mathcal{V} = 1, \ldots, n$ and edges $\mathcal{E} \in \mathcal{V} \times \mathcal{V}$. Each node represents an agent. If there is an edge (i, l) between nodes i and l, with $1 \leq i \leq n$ and $1 \leq l \leq n$, then i and l are called adjacent, *i.e.*, $\mathcal{E} = \{(i, l) \in \mathcal{V} \times \mathcal{V} : i, l \text{ } adjacent\}$. An entry of the adjacency matrix $\mathbf{A}$ is defined by $a_{il} = 1$ if i and l are adjacent and $a_{il} = 0$ otherwise. Note that the diagonal elements of the adjacency matrix are all zero for a graph without any loop (as in the present paper). $\mathcal{G}$ is called undirected if $(i, l) \in \mathcal{E} \Leftrightarrow (l, i) \in \mathcal{E}$. A path from i to l is a sequence of distinct nodes, starting from i and ending with l, such that each pair of consecutive nodes is adjacent. If there is a path from i to l, then i and l are called connected. If all pairs of nodes in $\mathcal{G}$ are connected, then $\mathcal{G}$ is called connected. The distance $d(i, l)$ between two nodes is the number of edges of the shortest path from i to l. The diameter $\mathbf{d}$ of $\mathcal{G}$ is the maximum distance $d(i, l)$ over all pairs of nodes. The degree (or valency) matrix $\mathbf{D}$ of $\mathcal{G}$ is a diagonal matrix whose diagonal elements d_i are equal to the cardinality of node i's neighbor set $\mathcal{N}_i = \{l \in \mathcal{V} : (i, l) \in \mathcal{E}\}$. Let $\mathcal{M}_i = \mathcal{N}_i \cup \{i\}$ denote the set of inclusive neighbors of node i. The Laplacian matrix $\mathcal{L}$ of $\mathcal{G}$ is defined as $\mathcal{L} = \mathbf{D} - \mathbf{A}$. For undirected graphs, $\mathcal{L}$ is symmetric and positive semi-definite, i.e. $\mathcal{L} = \mathcal{L}^T \geq 0$. The row sums of $\mathcal{L}$ are zero.

2.4 Problem Statement

According to (9)-(10), the goal of the attitude estimation process is to estimate the attitude of $\mathbf{E}^b$ with respect to $\mathbf{E}^f$, *i.e.* quaternion $\mathbf{Q}_b$. This process will be carried out via the estimation of quaternion $\mathbf{Q}_{mu_i}$, which maps the observation vectors into the known reference vectors. That is, given the observation vectors $\boldsymbol{b}_j^{mu_i}$ and the reference vectors $\boldsymbol{r}_j$, with $j \in \{1, 2 \cdots m\}$ the number of reference vectors, and $i \in \{1, 2 \cdots n\}$ the number of measurement units (denoted hereafter agents), the objective is to find $\mathbf{Q}_{mu_i}$ in each agent i such that:

$$H_j^i \mathbf{Q}_{mu_i} = 0 \tag{15}$$

Once the attitude quaternion $\mathbf{Q}_{mu_i}$ is estimated in each agent, we will propose an algorithm that averages these quaternions dynamically in the $\mathbf{E}^b$ frame.

3. DESIGN OF THE DYNAMIC CONSENSUS ALGORITHM

Consider a set of n agents, each one conformed by a measurement unit (MU). One assumes that each agent is capable of providing a unit quaternion of attitude, i.e. $\hat{\mathbf{Q}}_{mu_i}$ with $i \in \{1, 2 \cdots n\}$. Then, in this section one explains how this quaternion is estimated. After that, the dynamic average in the $\mathbf{E}^b$ frame will be exposed.

From (14), the observation matrix for the ith agent in function of m pair vectors $[\boldsymbol{b}_j^{mu_i}, \boldsymbol{r}_j]$ with $j \in \{1, 2, ..., m\}$ is written as follows:

$$H_m^i = \begin{pmatrix} 0 & -(\boldsymbol{b}_1^{mu_i} - \boldsymbol{r}_1)^T \\ \boldsymbol{b}_1^{mu_i} - \boldsymbol{r}_1 & -[(\boldsymbol{b}_1^{mu_i} + \boldsymbol{r}_1)^{\times}] \\ & \vdots \\ 0 & -(\boldsymbol{b}_m^{mu_i} - \boldsymbol{r}_m)^T \\ \boldsymbol{b}_m^{mu_i} - \boldsymbol{r}_m & -[(\boldsymbol{b}_m^{mu_i} + \boldsymbol{r}_m)^{\times}] \end{pmatrix} \in \mathbb{R}^{4m\times 4} \tag{16}$$

such that:

$$H_m^i \mathbf{Q}_{mu_i} = \begin{pmatrix} \varphi_1^T(t) \\ \vdots \\ \varphi_{4m}^T(t) \end{pmatrix} \mathbf{Q}_{mu_i} = 0 \tag{17}$$

Quaternion $\mathbf{Q}_{mu_i}$ can be obtained by a deterministic method. In (Guerrero-Castellanos et al., 2013) a deterministic attitude estimator which we coin here as "*Learning Quaternion Sequence*" (LQS) is reported and its implementation with others deterministic attitude estimators are reported in (Betanzos-Ramirez et al., 2021). LQS is key in the distributed dynamic consensus for attitude estimation proposed here and then it is briefly presented. Consider a matrix observation $H_m^i(t)$ in the instant t and an initial quaternion denoted $\mathbf{Q}_{ps}^0$. Then, a estimation of the attitude of the ith agent is given by

$$\hat{\mathbf{Q}}_{mu_i} = \left[\prod_{\kappa=1}^{4m} \left(\boldsymbol{I}_4 - \frac{\gamma \varphi_\kappa(t) \varphi_\kappa^T(t)}{\alpha + \varphi_\kappa^T(t) \varphi_\kappa(t)} \right) \right] \mathbf{Q}_{ps}^0 \tag{18}$$

where $\alpha \geq 0$ and $0 < \gamma < 2$. Once (18) is performed, one enforces the quaternion norm

$$\hat{\mathbf{Q}}_{mu_i} = \frac{\mathbf{Q}_{ps}^{4m}}{\left\| \mathbf{Q}_{ps}^{4m} \right\|} \tag{19}$$

Remark 3.1. Note that (18) represents an iterative algorithm to be performed for each instant t. Then, for $t = 0$ the initial quaternion $\mathbf{Q}_{ps}^0$ in (18) can be chosen arbitrarily. In the sequel, for all $t > 0$, $\mathbf{Q}_{ps}^0$ will be obtained from the output of the dynamic consensus. In the present collaborative framework, $\mathbf{Q}_{ps}^0$ it can be chosen as:

$$\mathbf{Q}_{ps}^0 = \begin{cases} \text{arbitrary unit quaternion, for } t = 0 \\ \bar{\mathbf{Q}}_i, \text{for } t > 0 \end{cases} \tag{20}$$

Remark 3.2. Quaternion $\hat{\mathbf{Q}}_{mu_i}$ represents the attitude estimation of the $\mathbf{E}^{mu_i}$ frame. This attitude can be represented in the $\mathbf{E}^b$ frame by means of (12), that is:

$$\hat{\mathbf{Q}}_{b_i} = \hat{\mathbf{Q}}_{mu_i} \odot (\mathbf{Q}_b^{mu_i})^{-1} \tag{21}$$

It is deserving to be noted that $\mathbf{Q}_b^{mu_i}$ are known.

Each agent is formed by a MU whose sensors account with their own noise characteristic. As a consequence, the quaternion estimated by each agent will be different in term of reliability. The idea is to take each estimated quaternion $\hat{\mathbf{Q}}_{b_1}, \hat{\mathbf{Q}}_{b_2}, ..., \hat{\mathbf{Q}}_{b_n}$ and perform a distributed dynamic consensus (average) in the $\mathbf{E}^b$ frame.

Let us define the following state vectors $\bar{\mathbf{Q}} = [\bar{\mathbf{Q}}_1^T, \ldots, \bar{\mathbf{Q}}_n^T]^T$ and $\Upsilon = [(\hat{\mathbf{Q}}_{b_1})^T, (\hat{\mathbf{Q}}_{b_2})^T, \ldots, (\hat{\mathbf{Q}}_{b_n})^T]^T$. Thus, we propose the following Dynamic Consensus Algorithm (DCA):

$$\dot{\bar{\mathbf{Q}}}_i(t) = \sum_{l\in\mathcal{N}_i} a_{il}\Big(\bar{\mathbf{Q}}_l(t) - \bar{\mathbf{Q}}_i(t)\Big) + \sum_{l\in\mathcal{M}_i} a_{il}\Big(\hat{\mathbf{Q}}_{b_l}(t) - \bar{\mathbf{Q}}_i(t)\Big) \tag{22}$$

where a_{il} are elements of the adjacency matrix $\mathbf{A}$ of the sensor network, $\mathcal{N}_i$ and $\mathcal{M}_i$ are the set of neighbors and the set of inclusive neighbors of node i, respectively, as already defined in subsection 2.3. Then, one has the following results.

Proposition 3.1. The distributive algorithm in (22) gives a consensus filter with the following collective dynamics:

$$\dot{\bar{\mathbf{Q}}} = -\Xi\bar{\mathbf{Q}} + B\Upsilon \tag{23}$$

where $\Xi = (\boldsymbol{I}_{4n} + \mathbf{D} + \mathcal{L})$ and $B = \boldsymbol{I}_{4n} + \mathbf{A}$, with $\mathbf{D}$ and $\mathcal{L}$ the degree and Laplacian matrices respectively, $\boldsymbol{I}_{4n}$ is the identity matrix.

Proof 3.1. The proof follows from the definition of graph Laplacian reviewed in subsection 2.3. For further details, see (Olfati-Saber and Shamma, 2006).

Proposition 3.2. The consensus filter described in (23) is a distributed stable low-pass filter with transfer function given by:

$$H(s) = (s\boldsymbol{I}_{4n} + \Xi)^{-1}B \tag{24}$$

Proof 3.2. Applying Gersgorin theorem to matrix Ξ guarantees that all poles of $H(s)$ are strictly negative. Then the filter is stable. Furthermore, $H(s)$ is a proper MIMO transfer function, which means that it is a low-pass filter (Olfati-Saber and Shamma, 2006).

Figure 2 depicts a schema of the proposed approach. This configuration offers certain advantages, such as fault tolerance if any of the sensors associated with each MU fails, resulting in a fault-tolerant attitude estimator.

3.1 Fault-Tolerant Robustness

For simulations purpose, the reference attitude (true attitude) and the sensor measurement on each MU will be generated by the following system:

$$\dot{\boldsymbol{\omega}} = J^{-1}\left(-\boldsymbol{\omega}\times J\boldsymbol{\omega} + \Gamma\right) \tag{25}$$

$$\dot{\mathbf{Q}}_b(t) = \frac{1}{2}\begin{pmatrix} -\boldsymbol{q}^T \\ q_0\boldsymbol{I}_3 + [\boldsymbol{q}^\times] \end{pmatrix}\boldsymbol{\omega} \tag{26}$$

When solving the dynamic equation (25), an angular velocity $\boldsymbol{\omega}$ expressed in the $\mathbf{E}^b$ reference system is obtained, with respect to $\Gamma \in \mathbb{R}^3$ which is the vector of input

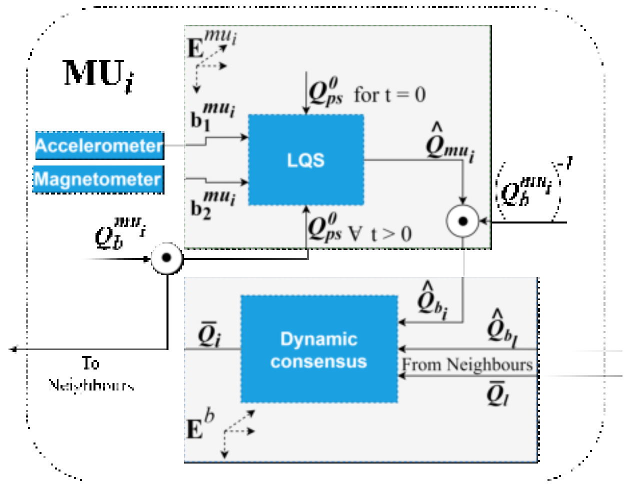

Fig. 2. Block diagram of an agent in the sensor network. ⊙ represents the quaternion product.

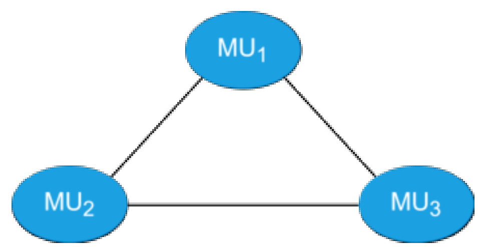

Fig. 3. Communication graph between each MU_i.

torque for each axis of the rigid body. J represents the rigid body's inertia matrix. Then $\boldsymbol{\omega}$ is an input signal for the kinematic equation (26), that gives a quaternion expressed in the $\mathbf{E}^b$ reference system when it is solved, i.e. $\mathbf{Q}_b$, with $\boldsymbol{q}$ and q_0 its vector and scalar parts. Both $\boldsymbol{\omega}$ and $\mathbf{Q}_b$ are taken as reference (true) signals for the simulation process.

Now consider the following unit reference vectors written as quaternions: $\mathbf{Q}(r_1) = [0;\ 0;\ 0;\ 1]$ (gravitational acceleration) and $\mathbf{Q}(r_2) = [0;\ 0.337;\ 0;\ 0.348]$ (earth's magnetic field), which are described in the $\mathbf{E}^f$ reference system, these new quaternions can be mapped to the $\mathbf{E}^{mu_i}$ reference system through the equation (11). It is worthy to note that $\mathbf{Q}_b^{mu_i}$ is defined by the designer and $\mathbf{Q}_b$ is given by the system (26), therefore it is possible to obtain the quaternions $\mathbf{Q}\left(b_1^{mu_i}\right)$ and $\mathbf{Q}\left(b_2^{mu_i}\right)$. So the simulated measurements for ith accelerometer and ith magnetometer can be taken from vector part of $\mathbf{Q}\left(b_1^{mu_i}\right)$ and $\mathbf{Q}\left(b_2^{mu_i}\right)$ respectively. Next LQS algorithm and dynamic consensus can be executed like in Figure 2.

Finally in order to obtain the mismatch between $\hat{\mathbf{Q}}_{mu_i}$ and $\mathbf{Q}_b$ (reference/true quaternion) and between $\bar{\mathbf{Q}}_i$ and $\mathbf{Q}_b$, equation (6) is applied to these quaternions. Next the inverse cosine of the scalar part of the quaternion $\mathbf{Q}_e$ is calculated, *i.e.*, $\beta_e = \arccos q_{e_0}$.

It is important to highlight that the LQS algorithm makes use of well-known reference vectors $\boldsymbol{r}_j$ and observation vectors $\boldsymbol{b}_j^{mu_i}$, as defined in the observation matrix H_m^i in (16). However, for each agent this matrix is function of data that are sensed. If any of the sensors fail in MU i, the matrix changes with a pair vector $[\boldsymbol{b}_j^{mu_i}, \boldsymbol{r}_j]$ that falls.

Obviously, in a practical application it will be necessary to develop a fault detection algorithm in order to identify the sensor that has failed and make the necessary changes. This is not considered here.

4. SIMULATIONS RESULTS

Since one is interested in distributed attitude estimation, a simulation was developed making use of three agents, so, the algorithms described in the block diagram of Figure 2 are simulated, with a graph communication given by Figure 3, where each node accounts with one MU with its own noise characteristics. First, two observation vectors are available, gravitational and magnetic vectors, which are obtained from $\mathbf{Q}\left(b_1^{mu_i}\right)$ and $\mathbf{Q}\left(b_2^{mu_i}\right)$, i.e. $\boldsymbol{b}_1^{mu_i}$, $\boldsymbol{b}_2^{mu_i}$, respectively, for $i \in \{1, 2, 3\}$. Then, the matrix H_m^i of the LQS algorithm, previously defined in its general form in (16), takes the form:

$$H_m^i = \begin{pmatrix} 0 & -(\boldsymbol{b}_1^{mu_i} - \boldsymbol{r}_1^{mu_i})^T \\ \boldsymbol{b}_1^{mu_i} - \boldsymbol{r}_1^{mu_i} & -[(\boldsymbol{b}_1^{mu_i} + \boldsymbol{r}_1^{mu_i}) \times] \\ 0 & -(\boldsymbol{b}_2^{mu_i} - \boldsymbol{r}_2^{mu_i})^T \\ \boldsymbol{b}_2^{mu_i} - \boldsymbol{r}_2^{mu_i} & -[(\boldsymbol{b}_2^{mu_i} + \boldsymbol{r}_2^{mu_i}) \times] \end{pmatrix} \in \mathbf{R}^{8\times 4} \quad (27)$$

Remind that LQS algorithm is developed for each node, this block provides $\mathbf{Q}_{mu_i}$, *i.e.*, attitude of the MU with respect to inertial frame. So, a transformation is required in order to obtain $\hat{\mathbf{Q}}_b$ which parameterizes the attitude of the rigid body. Each node, represented in Figures 3 and 2, runs the dynamic consensus algorithm (22).

Table 1 shows the quaternion $\mathbf{Q}_b^{mu_i}$ associated to each node of the network that relates the $\mathbf{E}^{mu_i}$ and $\mathbf{E}^b$ reference systems through a rotation. The resulting allocation of each MU over the rigid body can be translated to Figure 1, where the origin of $\mathbf{E}^b$ and $\mathbf{E}^{mu_i}$ reference systems are coincident, but their components are not aligned.

Table 1. Rotation applied to each MU with respect to $\mathbf{E}^b$ reference system.

$\mathbf{Q}_b^{mu_i}$ associated to each MU		
i	$\mathbf{Q}_b^{mu_i}$	Euler angles
1	$[0.3827; 0.9239; 0; 0]^T$	$\phi = 135^o, \theta = 0^o, \psi = 0^o$
2	$[0.3827; 0; 0.9239; 0]^T$	$\phi = 0^o, \theta = 135^o, \psi = 0^o$
3	$[0.3827; 0; 0; 0.9239]^T$	$\phi = 0^o, \theta = 0^o, \psi = 135^o$

Euler angles will be depicted and analysed since they are more intuitive. However, the algorithm is quaternion-based. The dynamic consensus algorithm is simulated together with the LQS algorithm in order to obtain a single estimate of the attitude of the rigid body. The initial condition for $t = 0$ are: $\bar{\mathbf{Q}}^0 = [0.933; 0.25; 0.25; -0.06699]$ and $\mathbf{Q}_{ps}^0 = [1; 0; 0; 0]$. Figure 4 shows the result obtained from this simulation, where the continuous lines represent the Euler angles obtained from reference $\mathbf{Q}_b$, and the dashed lines the Euler angles obtained from the dynamic consensus algorithm. In this case, a torque in axis *yaw* of rigid body is applied.

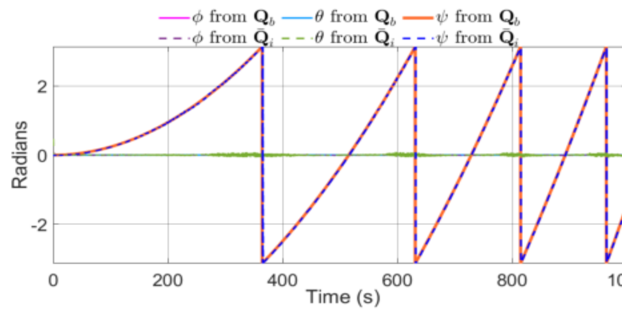

Fig. 4. Comparison between Euler angles from reference and Euler angles from dynamic consensus.

If the gravitational vector fails, in any of the nodes, the matrix (16) could be modified at that node. This matrix becomes:

$$H_m^i = \begin{pmatrix} 0 & -(\boldsymbol{b}_2^{mu_i} - \boldsymbol{r}_2^{mu_i})^T \\ \boldsymbol{b}_2^{mu_i} - \boldsymbol{r}_2^{mu_i} & -[(\boldsymbol{b}_2^{mu_i} + \boldsymbol{r}_2^{mu_i}) \times] \end{pmatrix} \in \mathbf{R}^{4\times 4} \quad (28)$$

When the magnetic vector fails, the matrix (16) becomes:

$$H_m^i = \begin{pmatrix} 0 & -(\boldsymbol{b}_1^{mu_i} - \boldsymbol{r}_1^{mu_i})^T \\ \boldsymbol{b}_1^{mu_i} - \boldsymbol{r}_1^{mu_i} & -[(\boldsymbol{b}_1^{mu_i} + \boldsymbol{r}_1^{mu_i}) \times] \end{pmatrix} \in \mathbf{R}^{4\times 4} \quad (29)$$

As mentioned before, the mismatch between $\mathbf{Q}_b$ and $\hat{\mathbf{Q}}_i$ and $\bar{\mathbf{Q}}_i$ is $\beta_e = \arccos q_{e_0}$, in order to analyze this mismatch the ISE index defined in (30) was used (Guerrero-Castellanos et al., 2021).

$$ISE = \int_0^T \tilde{x}^2(t)dt \quad (30)$$

Therefore, the ISE criterion was applied to angle β_e as follows,

$$ISE_{\beta_e} = \int_0^T \tilde{\beta}_e^2(t)dt \quad (31)$$

Table 2. Sensor operation during the simulation process of Figure 5

scene	time (s)	MU_1		MU_2		MU_3		MU	
–	–	$\boldsymbol{b}_1$	$\boldsymbol{b}_2$	$\boldsymbol{b}_1$	$\boldsymbol{b}_2$	$\boldsymbol{b}_1$	$\boldsymbol{b}_2$	$\boldsymbol{b}_1$	$\boldsymbol{b}_2$
1	$0-200$	✓	✓	✓	✓	✓	✓	✓	✓
2	$200-400$	✓	X	✓	✓	✓	✓	✓	✓
3	$400-500$	X	✓	X	✓	✓	✓	X	✓
4	$500-700$	✓	✓	✓	X	X	✓	✓	✓
5	$700-900$	X	✓	✓	X	✓	X	✓	X
6	$900-1100$	X	✓	✓	X	X	✓	✓	✓
7	$1100-1300$	✓	✓	X	X	X	✓	X	✓
8	$1300-1500$	X	X	X	X	✓	✓	✓	✓

Table 2 shows the operation of sensors for the sensor network, where MU_1, MU_2 and MU_3 are nodes or agents of

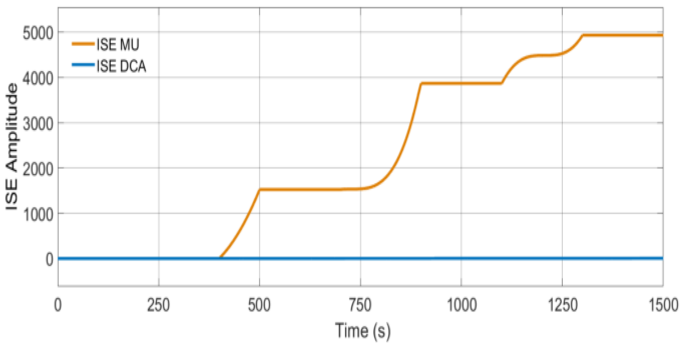

Fig. 5. ISE index of the quaternion error when a sensor failure occurs

the network and MU is independent of the sensor network. For this case $\boldsymbol{b}_1$ and $\boldsymbol{b}_2$ represent the vector observations obtained from accelerometer and magnetometer respectively, a ✓ indicates that the correspond sensor is working correctly, X represent the case of failure of any sensor.

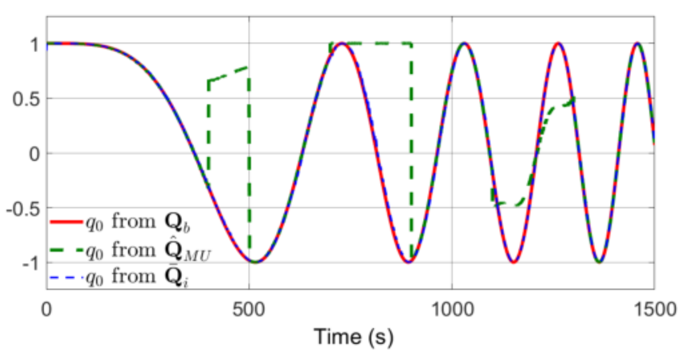

Fig. 6. Time evolution of the scalar part of $\mathbf{Q}_b$, $\hat{\mathbf{Q}}_{MU}$ and $\bar{\mathbf{Q}}_i$

Figure 5, shows the results of simulation associated to Table 2. Figure 6 shows the behavior of $q_0(t)$ for the $\mathbf{Q}_b$ (true attitude), $\hat{\mathbf{Q}}_{MU}$ (single measurement unit) and for $\bar{\mathbf{Q}}_i$ (average quaternion), these behaviors are related with simulation of Figure 5.

5. CONCLUSION

The paper presents a dynamic consensus algorithm for distributed attitude estimation. Attitude estimation is fundamental in many applications, such as robotics, biomedical engineering, and aerospace. In most of them, knowing an accurate value of the attitude is critical. Performing a distributed estimation and developing the dynamic consensus algorithm ensures a more precise attitude estimation. Furthermore, with the proposed approach, the system remains robust in the event of any sensor failure. Simulation results were presented. In the future, real-time experiments will be performed.

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