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Fractional-Order Composite Adaptive Control of Robot Manipulators

Daniel Gámez [*] **Juan Sifuentes-Mijares** [**] **Víctor Santibáñez** [***] **Miguel A. Llama** [****]

[*] *Tecnológico Nacional de México, Instituto Tecnológico de la Laguna, Torreón, Coahuila, México (e-mail: danielgamezherrera@hotmail.com).*
[**] *Tecnológico Nacional de México, Instituto Tecnológico de la Laguna, Torreón, Coahuila, México (e-mail: jsifuentesm@correo.itlalaguna.edu.mx).*
[***] *Tecnológico Nacional de México, Instituto Tecnológico de la Laguna, Torreón, Coahuila, México (e-mail: vasantibanezd@lalaguna.tecnm.mx).*
[****] *Tecnológico Nacional de México, Instituto Tecnológico de la Laguna, Torreón, Coahuila, México (e-mail: mllama@lalaguna.tecnm.mx).*

Abstract: The branch of system identification and control theory concerned about robot manipulators is adaptive control of non-linear multi-input multi-output dynamic systems which study constitutes a mature and well-founded discipline; nevertheless, there are fundamental problems open to date, one of them is to obtain a control law that is rich enough to guarantee its persistent excitation and consequently that the parametric error converges asymptotically to zero, while the tracking error converges globally asymptotically to zero. An extension of a globally convergent adaptive scheme control for a robot manipulator in the tracking of a determined trajectories with no consideration of the interaction with its environment is proposed, based on the fact that the parameter uncertainty is involved in both the tracking error and the identification error. The first control task is achieved by a feedback linearization technique that takes advantage of the structure of manipulator dynamics. The second task is achieved by a fractional order filtering technique to avoid the joint acceleration and enrich the regressor matrix in such way that is persistently exciting. Finally, the control law is expressed as a feedforward compensation and a simple PD controller.

Keywords: Fractional-order, manipulators, composite adaptive control, persistent excitation, Lyapunov's direct method

1. INTRODUCTION

One of the major areas of identification in control theory is adaptive control based on the reference model approach, although its research is profuse and has been integrated with other techniques to enhance performance, there are still open fundamental problems about its study. One of them is intrinsic to its basic task: *estimate the uncertain parameters of the plant*; that is, to guarantee that the estimate of the parameters converges to the real value of each one of the physical parameters of the plant in practice as discussed in Narendra (1994). In order to achieve this task, adaptive control implements a recursive identification system with an algorithm to minimize some criterion to some other well-known control strategy, assuming the *certainty equivalence principle*, which implication is that the control law considers the parametric estimates as the nominal values of unknown parameters, that is, the uncertainty of the parametric estimate is not considered as reviewed in Sastry and Bodson (1989).

Robot manipulators applications often require full knowledge of the dynamic parameters to design a controller to perform an accurate tracking of desired trajectories, even more, if feedback linearization is used by the *computed torque* approach. Clearly, it is implicitly assumed that the dynamics in the computed torque are exact, which is not the case in reality as mentioned in Lewis et al. (2003). Then, the performance of such controller may be significantly worse than expected or even turn into an unstable system in closed-loop. Besides, as the model of a n-link articulated manipulator is defined by a set of n highly nonlinear and coupled second order differential equations, it is desirable to treat them in more convenient way. The simplest methodology is the introduction of the linear parametrization of robot dynamics to take into consideration these properties by choosing a new set of parameters in terms of the dynamics parameters such that the manipulator model depend linearly on the new set. Then knowing this vector of parameters is equivalent to know the physical parameters as no structural uncertainty is assumed as concluded in Slotine and Li (1989).

Based on the intuition that getting more information about a problem leads to a faster solution in general, it has been proved in Duarte and Narendra (1989) that using all sources of parametric information improved parameter convergence, therefore, a composite adaptation law, as the one used in Slotine and Li (1989), extract this information not only from the tracking errors, but the identification errors to achieve the parameter adaptation.

The key point in this kind of adaptation law is to use the identification errors as free parameter without worrying for the convergence of tracking errors to zero (as it is already guaranteed by the use of tracking errors), but the convergence of identification errors to zero and to extract enough information about the physical parameters to satisfy the persistent excitation condition by searching the proper fractional order of the stable filter.

2. MANIPULATOR DYNAMICS AND IDENTIFICATION ALGORITHM

In this section, the model of the manipulator and the properties of its parameter matrices are provided for the controller design. Then, the identification algorithm for the joint torque is derived, due to fact that the identification error is used in the composite adaptation law to be a free parameter and enhanced performance. Finally, some insights of iso-damping to obtain the proper fractional order of the stable filter that satisfies the persistent excitation condition are mentioned.

2.1 Robot manipulator model

The joint-space dynamics of any n degrees-of-freedom robot manipulator constituted of rigid links connected by frictionless joints can be written compactly as

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau}\,, \tag{1}$$

where $\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}} \in \mathbb{R}^{n\times 1}$ denote the joint positions, velocities and acceleration vectors, respectively, $\boldsymbol{M} \in \mathbb{R}^{n\times n}$ denotes the inertia matrix while $\boldsymbol{C} \in \mathbb{R}^{n\times n}$ denotes a centripetal-Coriolis matrix. Further, $\boldsymbol{g},\boldsymbol{\tau} \in \mathbb{R}^{n\times 1}$ denote the vectors of generalized forces due to gravity and exogenous torques, respectively.

Basic properties of the manipulator model are revisited in Spong and Vidyasagar (1989) and presented next for its use in the later adaptive control design.

Property 1. By expressing explicitly the dependence of the manipulator model on the dynamic parameters of each of the individual terms, it can be proved that it is linear in terms of a new set of unknown parameters $\boldsymbol{\theta}^* \in \mathbb{R}^{p\times 1}$, therefore, it is linearly parametrizable in the form

$$\boldsymbol{M}(\boldsymbol{q},\boldsymbol{\theta}^*)\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}},\boldsymbol{\theta}^*)\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q},\boldsymbol{\theta}^*) = \boldsymbol{Y}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})\boldsymbol{\theta}^* \tag{2}$$

where $\boldsymbol{Y}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}}) \in \mathbb{R}^{n\times p}$ is the regressor matrix only dependent on the trajectories.

Property 2. The matrix $\dot{\boldsymbol{M}} - 2\boldsymbol{C}$ is skew-symmetric, therefore, the centripetal-Coriolis matrix is related to the inertia matrix by the identity

$$\boldsymbol{x}^{\mathsf{T}}\left(\dot{\boldsymbol{M}} - 2\boldsymbol{C}\right)\boldsymbol{x} = 0, \forall \boldsymbol{x} \in \mathbb{R}^n\,. \tag{3}$$

Property 3. The inertia matrix $\boldsymbol{M} = \boldsymbol{M}^{\mathsf{T}} \in \mathbb{R}^{n\times n}$ is symmetric, uniformly positive definite $\boldsymbol{M}(\boldsymbol{q}) > 0, \forall \boldsymbol{q}$ and bounded above and below by

$$\mu_1\mathbb{I} \leq \boldsymbol{M}(\boldsymbol{q}) \leq \mu_2\mathbb{I}, \qquad \mu_1,\mu_2 > 0\,. \tag{4}$$

2.2 Identification algorithm

In order to derive an algorithm to estimate those unknown parameters of *property 1*, first the model at (1) can be expressed as

$$\boldsymbol{\tau} = \boldsymbol{Y}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})\boldsymbol{\theta}^*\,. \tag{5}$$

Then, realizing the fact that the regressor matrix depends even on the acceleration, the measurement of signals and posterior computation could be simplified using the fractional generalization of the technique used in Hsu et al. (1987) to avoid acceleration, even more, it introduces a way to guarantee the sufficient richness of the signals filtered by using the iso-damping property studied in Chen et al. (2003) which implies that the system will be more robust to gain variations. Specifically, by selecting a stable filter of arbitrary order $\alpha \in (0,1]$ and adaptive gain λ_f, the overshoot is completely controlled, since the overshoot depends directly on the order α chosen and once selected, the overshoots of the closed-loop system remain almost constant for different values of λ_f. Finally, the new regressor matrix is given by

$$\boldsymbol{W}(\boldsymbol{q},\dot{\boldsymbol{q}}) = \mathscr{L}^{-1}\left\{\frac{\lambda_f}{s^\alpha + \lambda_f}\right\} * \boldsymbol{Y}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})\,. \tag{6}$$

Now, filtering both sides of (5) leads to

$$\boldsymbol{y}(t) = \boldsymbol{W}(\boldsymbol{q},\dot{\boldsymbol{q}})\boldsymbol{\theta}^*\,, \tag{7}$$

where $\boldsymbol{y}$ is a new variable called filtered torque. Then, it is possible to define the identification error as the difference between the estimate of this new variable and the actual filtered torque

$$\boldsymbol{e}_i(t) = \boldsymbol{W}\boldsymbol{\theta}(t) - \boldsymbol{y}(t) = \boldsymbol{W}\tilde{\boldsymbol{\theta}}(t)\,. \tag{8}$$

where $\boldsymbol{\theta}(t)$ is the estimate of the nominal unknown vector of parameters $\boldsymbol{\theta}^*$. Furthermore, intuitively, it is known that if the identification error is zero then the estimated parameters will be exactly the nominal parameters since these would exactly generate the response of the plant.

Thus, it is essential to find an *update law* that minimizes this error according to some criterion. In this work the criterion used is the minimization of the integral of the squared error

$$J\left[u\left(t\right),y\left(t\right),t\right]=\int_0^t {e_i}^2\left(\tau\right)\mathrm{d}\tau\,. \tag{9}$$

then, it can be proved that the update law of the parametric estimates that minimizes the functional in (9), is the gradient:

$$\dot{\boldsymbol{\theta}}\left(t\right)=-\gamma\boldsymbol{W}^{\mathsf{T}}\left(t\right)\boldsymbol{e}_i\left(t\right)\,. \tag{10}$$

Once this update law is derived, conditions that guarantee the asymptotic stability of the identification algorithm that results from (10) and (8) can be obtained:

$$\dot{\boldsymbol{\theta}}\left(t\right)=-\gamma\boldsymbol{W}^{\mathsf{T}}\left(t\right)\boldsymbol{W}\left(t\right)\tilde{\boldsymbol{\theta}}\left(t\right)\,, \tag{11}$$

since $\dot{\tilde{\boldsymbol{\theta}}}\left(t\right)=\dot{\boldsymbol{\theta}}\left(t\right)$ and defining $\boldsymbol{A}\left(t\right)=\gamma\boldsymbol{W}\left(t\right)\boldsymbol{W}^{\mathsf{T}}\left(t\right)$ with $\gamma>0$ and $\boldsymbol{A}\left(t\right)\geq 0\in\mathbb{R}^{p\times p}$, $\forall t>0$. Then, equation (11) can be expressed as the time-varying first order differential equation

$$\dot{\tilde{\boldsymbol{\theta}}}\left(t\right)=-\boldsymbol{A}\left(t\right)\tilde{\boldsymbol{\theta}}\left(t\right)\,. \tag{12}$$

From linear time-varying system theory, it can be concluded that if the time-varying coefficient can be bounded above and below by a positive definite constant matrix over a finite interval, then (12) is asymptotically stable; this leads to the fundamental definition of persistent excitation of a matrix given in Boyd and Sastry (1986).

Definition 1. A matrix $\boldsymbol{W}:[0,\infty)\to\mathbb{R}^{n\times p}$ is persistently exciting (PE) if there exist $\alpha_1,\alpha_2,\delta>0$ such that

$$\alpha_1\mathbb{I}\leq\int_{t_0}^{t_0+\delta}\boldsymbol{W}^{\mathsf{T}}\left(\tau\right)\boldsymbol{W}\left(\tau\right)\mathrm{d}\tau\leq\alpha_2\mathbb{I},\quad\forall t_0\geq 0\,, \tag{13}$$

where $\mathbb{I}$ denotes the proper dimension identity matrix.

To resume, if the regressor matrix in the update law (10) satisfies *definition 1* with a proper fractional order in the stable filter in (6) that avoids the measurement of joint acceleration, but modifies input signals into rich-enough regressor signals by setting different only overshoots or only settling times independently. Then, the parameter uncertainty converges asymptotically to zero. This fractional order is not computed explicitly here, as this paper does not address the algorithm to determine a value to satisfy this linear matrix inequality, but states conditions to draw conclusions in the stability of the identification algorithm.

3. CONTROLLER DESIGN

Given the manipulator model in (1), it is possible to obtain the transformation that leads to linear closed-loop dynamics by using the computed torque technique which is a special application of feedback linearization to the nonlinear coupled structure of a robot manipulator by canceling the non-linearities of the manipulator and taking advantage of *property 3* to assure this transformation is indeed a global diffeomorphism as proven in Gilbert and Ha (1984). A general computed-torque control law is

$$\boldsymbol{\tau}=\boldsymbol{M}\left(\boldsymbol{q}\right)\left(\ddot{\boldsymbol{q}}_d-2\boldsymbol{\Lambda}\dot{\widetilde{\boldsymbol{q}}}-\boldsymbol{\Lambda}^2\widetilde{\boldsymbol{q}}\right)+\boldsymbol{C}\left(\boldsymbol{q},\dot{\boldsymbol{q}}\right)\dot{\boldsymbol{q}}+\boldsymbol{q}\left(\boldsymbol{q}\right)\,, \tag{14}$$

then the tracking error $\widetilde{\boldsymbol{q}}=\boldsymbol{q}-\boldsymbol{q}_d$ satisfies the next closed-loop equation:

$$\ddot{\widetilde{\boldsymbol{q}}}+2\boldsymbol{\Lambda}\dot{\widetilde{\boldsymbol{q}}}+\boldsymbol{\Lambda}^2\widetilde{\boldsymbol{q}}=\boldsymbol{0},\qquad\boldsymbol{\Lambda}\in\mathbb{R}^{n\times n}>0\,, \tag{15}$$

therefore $\widetilde{\boldsymbol{q}}$ converges to zero exponentially. Clearly, it is implicitly assumed that the dynamics in the computed torque at (14) are exact, which is not the case. To deal with this parametric uncertainty, it is possible to apply robust control methodologies.

It has been proved in Slotine and Sastry (1983) that steady-state tracking errors are zero, if a filtered tracking error is defined as the weighted sum of the position error and the velocity error

$$\boldsymbol{s}=\dot{\tilde{\boldsymbol{q}}}+\boldsymbol{\Lambda}\tilde{\boldsymbol{q}},\qquad\boldsymbol{\Lambda}=\boldsymbol{\Lambda}^{\mathsf{T}}>0\,, \tag{16}$$

this new variable is called virtual velocity. It is clear that if $\boldsymbol{s}=\boldsymbol{0}$ is taken as the sliding surface, then $\tilde{\boldsymbol{q}}$ will tend to zero as $t\to\infty$. As $\dot{\boldsymbol{q}}$ will be replaced by $\boldsymbol{s}$ for the design, also the desired trajectory $\boldsymbol{q}_d$ needs to be replaced by the virtual *reference trajectory*:

$$\dot{\boldsymbol{q}}_r=\dot{\boldsymbol{q}}_d-\boldsymbol{\Lambda}\tilde{\boldsymbol{q}}\,. \tag{17}$$

So (16) can be rewritten as $\boldsymbol{s}=\dot{\boldsymbol{q}}-\dot{\boldsymbol{q}}_r$.

3.1 Control law by Lyapunov stability analysis

The design of the adaptive controller involves obtaining a control law for the actuator torques, and an update law for the estimation of the unknown parameters. For this purpose, the Lyapunov method can be applied considering the Lyapunov function candidate

$$V\left(\boldsymbol{s},t\right)=\frac{1}{2}\left[\boldsymbol{s}^{\mathsf{T}}\boldsymbol{M}\boldsymbol{s}+\tilde{\boldsymbol{\theta}}^{\mathsf{T}}\boldsymbol{\Gamma}^{-1}\tilde{\boldsymbol{\theta}}\right],\qquad\boldsymbol{\Gamma}>0\,. \tag{18}$$

Taking the derivative with respect to time of (18), the next equation is obtained:

$$\dot{V}\left(\boldsymbol{s},t\right)=\boldsymbol{s}^{\mathsf{T}}\left(\boldsymbol{M}\ddot{\boldsymbol{q}}-\boldsymbol{M}\ddot{\boldsymbol{q}}_r\right)+\frac{1}{2}\boldsymbol{s}^{\mathsf{T}}\dot{\boldsymbol{M}}\boldsymbol{s}+\dot{\boldsymbol{\theta}}^{\mathsf{T}}\boldsymbol{\Gamma}^{-1}\tilde{\boldsymbol{\theta}}\,. \tag{19}$$

Then, from (1) and from the definition of $\boldsymbol{s}$, it follows that $\boldsymbol{M}\ddot{\boldsymbol{q}}=\boldsymbol{\tau}-\boldsymbol{C}\dot{\boldsymbol{q}}-\boldsymbol{g}=\boldsymbol{\tau}-\boldsymbol{C}\left(\boldsymbol{s}+\dot{\boldsymbol{q}}_r\right)-\boldsymbol{g}$, which leads to

$$\dot{V}=\boldsymbol{s}^{\mathsf{T}}\left(\boldsymbol{\tau}-\boldsymbol{M}\ddot{\boldsymbol{q}}_r-\boldsymbol{C}\dot{\boldsymbol{q}}_r-\boldsymbol{g}+\frac{\dot{\boldsymbol{M}}}{2}-\boldsymbol{C}\right)\boldsymbol{s}+\dot{\boldsymbol{\theta}}^{\mathsf{T}}\boldsymbol{\Gamma}^{-1}\tilde{\boldsymbol{\theta}}\,,$$

but by *property 2*,

$$\dot{V}\left(\boldsymbol{s},t\right)=\boldsymbol{s}^{\mathsf{T}}\left(\boldsymbol{\tau}-\boldsymbol{M}\ddot{\boldsymbol{q}}_r-\boldsymbol{C}\dot{\boldsymbol{q}}_r-\boldsymbol{g}\right)+\dot{\boldsymbol{\theta}}^{\mathsf{T}}\boldsymbol{\Gamma}^{-1}\tilde{\boldsymbol{\theta}}\,. \tag{20}$$

Choosing a control law constituted by a precompensation $\hat{\boldsymbol{\tau}}$ formed by the pair at (1), but using the estimates of the matrices $\boldsymbol{M}$ and $\boldsymbol{C}$, and the vector $\boldsymbol{g}$; and a simple proportional derivative controller with derivative gain $\boldsymbol{K}_D=\boldsymbol{K}_D^{\mathsf{T}}>0$:

$$\boldsymbol{\tau}=\hat{\boldsymbol{\tau}}-\boldsymbol{K}_D\boldsymbol{s}=\hat{\boldsymbol{M}}\ddot{\boldsymbol{q}}+\hat{\boldsymbol{C}}\dot{\boldsymbol{q}}+\hat{\boldsymbol{g}}-\boldsymbol{K}_D\boldsymbol{s}\,. \tag{21}$$

However, it is not feasible to estimate each of the matrices, but apply property 1 and obtain a linear relationship in terms of its parameters

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}}_r + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}_r + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{Y}(\boldsymbol{q},\dot{\boldsymbol{q}},\boldsymbol{q}_r,\dot{\boldsymbol{q}}_r)\boldsymbol{\theta}^*. \quad (22)$$

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \widehat{\boldsymbol{\tau}} = \boldsymbol{Y}(\boldsymbol{q},\dot{\boldsymbol{q}})\boldsymbol{\theta}. \quad (23)$$

Finally, substituting (23) in (21), the control law that determines the torques of the actuators of the robot manipulator is

$$\boldsymbol{\tau} = \boldsymbol{Y}\boldsymbol{\theta} - \boldsymbol{K}_D\boldsymbol{s}. \quad (24)$$

The control law (24) leads to the derivative of the Lyapunov function (20) resulting in

$$\dot{V}(\boldsymbol{s},t) = \boldsymbol{s}^{\top}\boldsymbol{Y}\widetilde{\boldsymbol{\theta}} - \boldsymbol{s}^{\top}\boldsymbol{K}_D\boldsymbol{s} + \dot{\boldsymbol{\theta}}^{\top}\boldsymbol{\Gamma}^{-1}\widetilde{\boldsymbol{\theta}}. \quad (25)$$

3.2 Composite adaptation

A composite adaptation law extract information about parameters not only from the tracking errors but the identification errors to achieve the parameter adaptation, such law is the one proposed in Slotine and Li (1989), that is

$$\dot{\boldsymbol{\theta}} = -\boldsymbol{P}(t)\left[\boldsymbol{Y}^{\top}\boldsymbol{s} + \boldsymbol{W}^{\top}\boldsymbol{R}(t)\boldsymbol{e}_i\right], \quad (26)$$

where $\boldsymbol{R} = \boldsymbol{R}^{\top} > 0 \in \mathbb{R}^{n\times n}$ denotes the weighting matrix on identification information, $\boldsymbol{P} = \boldsymbol{P}^{\top} > 0 \in \mathbb{R}^{n\times n}$ denotes the gain matrix of the composite gradient algorithm.

Nevertheless, to simplify the analysis, it will be assumed that $\boldsymbol{P}(t) = \boldsymbol{\Gamma}$ y $\boldsymbol{R}(t) = \mathbb{I}_{n\times n}$. Then, substituting (26) with the previous considerations in (25) leads to

$$\dot{V}(\boldsymbol{s},t) = -\boldsymbol{s}^{\top}\boldsymbol{K}_D\boldsymbol{s} - \boldsymbol{e}_i{}^{\top}\boldsymbol{W}\tilde{\boldsymbol{\theta}}. \quad (27)$$

Since $\boldsymbol{e}_i = \boldsymbol{W}\tilde{\boldsymbol{\theta}} \Rightarrow \boldsymbol{e}_i{}^{\top} = \tilde{\boldsymbol{\theta}}^{\top}\boldsymbol{W}^{\top}$, then the derivative of the Lyapunov function becomes

$$\dot{V}(\boldsymbol{s},t) = -\boldsymbol{s}^{\top}\boldsymbol{K}_D\boldsymbol{s} - \boldsymbol{e}_i{}^{\top}\boldsymbol{e}_i < 0, \quad \forall \boldsymbol{s} \neq \boldsymbol{0}, \forall \boldsymbol{e}_i \neq \boldsymbol{0}. \quad (28)$$

This implies that $V(\boldsymbol{s}(t),t) \leq V(\boldsymbol{s}(0),0)$, therefore, $\boldsymbol{s}$ y $\tilde{\boldsymbol{\theta}}$ are bounded above by the construction of (18). Which indicates that $V(t)$ will decrement whenever either the tracking error $\boldsymbol{s}$ or the identification error $\boldsymbol{e}_i$ is non-zero.

4. CASE STUDY

Consider a robot manipulator with two degrees of freedom which position can be described by a vector $\boldsymbol{q}$ of the measurement of the angles associated with each joint, and which inputs are given by a vector $\boldsymbol{\tau}$ of torques, applied to the joints of the manipulator. Suppose the robot manipulator is in the horizontal plane, this situation implies that $\boldsymbol{g}(\boldsymbol{q}) \equiv \boldsymbol{0}$ then a modeling methodology, such as the one based in Euler Lagrange can be used to derive the dynamics of this manipulator. Now, after choosing a proper set of parameters model to apply *property 1*, the manipulator model can be written explicitly as in Slotine and Li (1991), that is:

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} -h\dot{q}_2 & -h(\dot{q}_1+\dot{q}_2) \\ h\dot{q}_1 & 0 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

where

$$\begin{aligned}
M_{11} &= \theta_1 + 2\theta_3\cos q_2 + 2\theta_4 \sin q_2 \\
M_{12} &= M_{21} = \theta_2 + \theta_3\cos q_2 + \theta_4\sin q_2 \\
M_{22} &= \theta_2 \\
h &= \theta_3 \sin q_2 - \theta_4\cos q_2 \\
\theta_1 &= I_1 + m_1 l_{c_1}^2 + I_e + m_e l_{c_e}^2 + m_e l_1^2 \\
\theta_2 &= I_e + m_e l_{c_e}^2 \\
\theta_3 &= m_e l_1 l_{c_e}\cos\delta_e \\
\theta_4 &= m_e l_1 l_{c_e}\sin\delta_e
\end{aligned}$$

The physical parameters of the manipulator used in the simulation are shown in table 1. The manipulator is initially at rest at ($q_1(0) = 0$, $q_2(0) = 0$), and the desired trajectories are specified by a vector $\boldsymbol{q}_d(t)$ given by:

$$\boldsymbol{q}_d(t) = \begin{bmatrix} q_{d_1} \\ q_{d_2} \end{bmatrix} = \begin{bmatrix} \frac{\pi}{6} - \frac{\pi}{6}\cos(2\pi t) \\ \frac{\pi}{4} - \frac{\pi}{4}\cos(2\pi t) \end{bmatrix}. \quad (29)$$

Table 1. Physical parameters of the manipulator

Parameter	Notation	Value
Mass of link 1	m_1	1 kg
Length of link 1	l_1	1 m
Equivalent mass of link 2	m_2	2 kg
Equivalent angle of link 2	δ_e	$\frac{\pi}{6}$ rad
Moment of inertia of link 1	I_1	$0.12\ \mathrm{kg}\cdot\mathrm{m}^2$
Distance to the center of mass of link 1	l_{c_1}	0.5 m
Equivalent moment of inertia of link 2	I_e	$0.25\ \mathrm{kg}\cdot\mathrm{m}^2$
Distance to the center of mass of link 2	l_{c_e}	0.6 m

In order to design the composite adaptive trajectory controller, the linear parameterization of the robot model described in (1) must be obtained first. The components of the matrix $\boldsymbol{Y} \in \mathbb{R}^{2\times 4}$ can be written explicitly as

$$Y_{11} = \ddot{q}_{r_1} \qquad Y_{21} = 0 \qquad Y_{12} = \ddot{q}_{r_2} \qquad Y_{22} = \ddot{q}_{r_1} + \ddot{q}_{r_2}$$

$$Y_{13} = -(\dot{q}_2\dot{q}_{r_1} + \dot{q}_1\dot{q}_{r_2} + \dot{q}_2\dot{q}_{r_2})\sin q_2 + (2\ddot{q}_{r_1} + \ddot{q}_{r_2})\cos q_2$$

$$Y_{14} = (\dot{q}_2\dot{q}_{r_1} + \dot{q}_1\dot{q}_{r_2} + \dot{q}_2\dot{q}_{r_2})\cos q_2 + (2\ddot{q}_{r_1} + \ddot{q}_{r_2})\sin q_2$$

$$Y_{23} = \dot{q}_1\dot{q}_{r_1}\sin q_2 + \ddot{q}_{r_1}\cos q_2$$

$$Y_{24} = -\dot{q}_1\dot{q}_{r_1}\cos q_2 + \ddot{q}_{r_1}\sin q_2$$

While the ideal parameters, after substituting the values used in simulation (from table 1) are determined to be:

$$\boldsymbol{\theta}^* = [\theta_1\ \theta_2\ \theta_3\ \theta_4]^{\top} = [3.34\ 0.97\ 1.0392\ 0.6]^{\top}.$$

This ideal parameters are expected to be the ones that the identification algorithm estimate as time increases, of course, this parameters are the set of elements that constitutes the linear parametrization, and not the physical ones, which can be obtained easily substituting a nominal structure of the with the physical parameters that are already know and solving for the unknown ones.

The matrix gain for the filter in virtual velocity is $\Lambda = \mathrm{diag}\{20, 20\}$. Besides, the reference trajectory is $\dot{\boldsymbol{q}}_r = \dot{\boldsymbol{q}}_d - \Lambda\tilde{\boldsymbol{q}}$, so $\ddot{\boldsymbol{q}}_r = \ddot{\boldsymbol{q}}_d - \Lambda\dot{\tilde{\boldsymbol{q}}}$. The derivative gain matrix is $\boldsymbol{K}_D = \mathrm{diag}\{300, 300\}$. The weighting matrix on identification information is $\boldsymbol{P}(t) \equiv \boldsymbol{\Gamma} = \mathrm{diag}\{3.423525, 0.86080156, 0.83288, 0.24402\}$, where a metaheuristic algorithm has been used to computed them. On the other hand, $\boldsymbol{W}$ and $\boldsymbol{e}$ are generated after applying a filter of arbitrary order as described in (6), with $\lambda_f = 120$ as base and the variable order $\alpha(t)$ such that it satisfies the persistent excitation condition at (13).

5. SIMULATION RESULTS

The proposed controller is simulated with all the information in the previous section, the results of each of the estimated parameters are presented in Figures 1-4

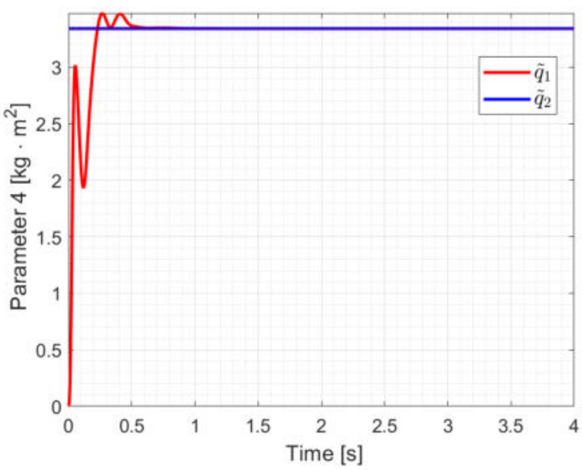

Fig. 1. Parameter 1, θ_1

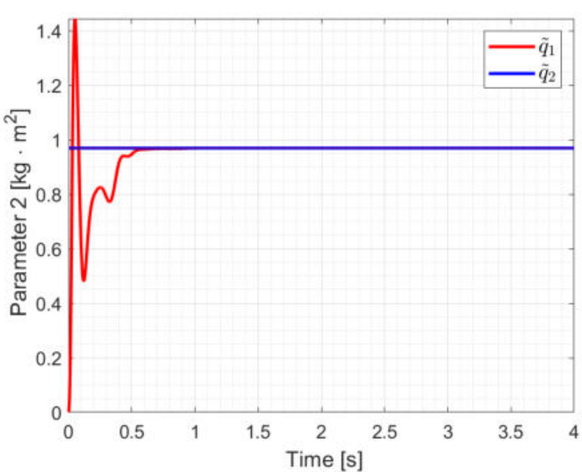

Fig. 2. Parameter 2, θ_2

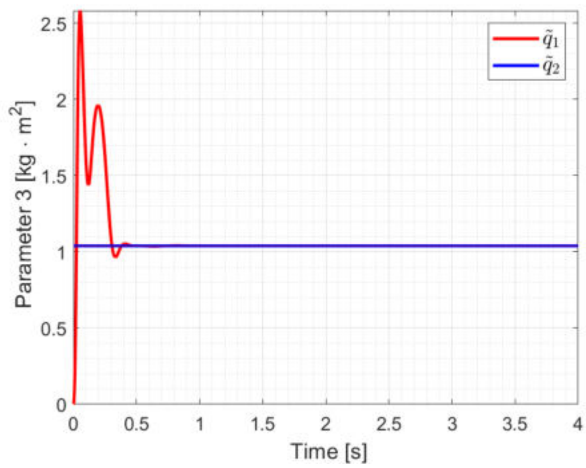

Fig. 3. Parameter 3, θ_3

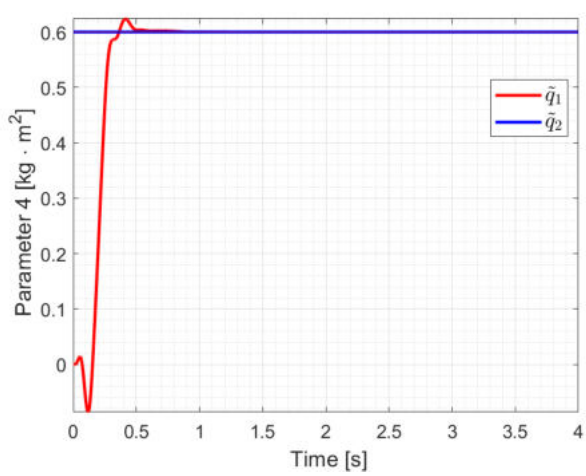

Fig. 4. Parameter 4, θ_4

As it can be confirmed by the previous figures, each of the parameters of $\boldsymbol{\theta}$ converge to the nominal parameters of the manipulator, so parameter convergence is guaranteed. Moreover, both tracking errors converge to zero as shown in Figures 5-6, so the control objective is achieved too.

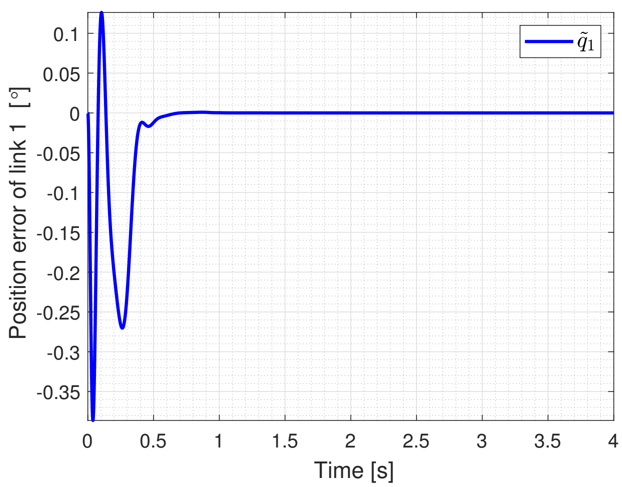

Fig. 5. Tracking error of link 1, $\tilde{q}_1$.

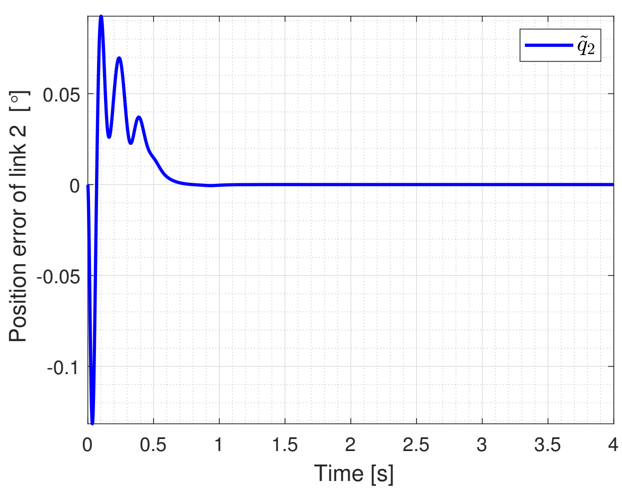

Fig. 6. Tracking error of link 2, $\tilde{q}_2$.

6. CONCLUSION

This work proposes an extension of the composite adaptive control for robot manipulators, in which the controller is formed by a feedforward compensation and a simple PD controller, but the regressor matrix is persistently exciting by an adequate fractional order stable filter to add enough richness to the signal being filtered, these changes in the reggresor matrix can be done thanks to the fact that the identification errors are used as free parameter with an indirect care for the convergence of tracking errors to zero, since it is already guaranteed by the use of tracking errors, so identification errors dynamics can be manipulated to extract enough information about the physical parameters and to converge to zero, which implies estimating the set of parameters linearly related to the torque and with prior knowledge of some physical parameters, determine the unknown ones.

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REFERENCES

Boyd, S. and Sastry, S. (1986). Necessary and sufficient conditions for parameter convergence in adaptive control. *Automatica*, 22(6), 629–639.

Chen, Y., Hu, C., and Moore, K. (2003). Relay feedback tuning of robust pid controllers with iso-damping property. In *42nd IEEE International Conference on Decision and Control (IEEE Cat. No.03CH37475)*, volume 3, 2180–2185.

Duarte, M. and Narendra, K. (1989). Combined direct and indirect approach to adaptive control. *IEEE Transactions on Automatic Control*, 34(10), 1071–1075.

Gilbert, E.G. and Ha, I.J. (1984). An approach to nonlinear feedback control with applications to robotics. *IEEE Transactions on Systems, Man, and Cybernetics*, SMC-14(6), 879–884.

Hsu, P., Bodson, M., Sastry, S., and Paden, B. (1987). Adaptive identification and control for manipulators without using joint accelerations. In *Proceedings. 1987 IEEE International Conference on Robotics and Automation*, volume 4, 1210–1215.

Lewis, F., Dawson, D., and Abdallah, C. (2003). *Robot Manipulator Control: Theory and Practice*. Automation and Control Engineering. CRC Press.

Narendra, K. (1994). Parameter adaptive control-the end...or the beginning? In *Proceedings of 1994 33rd IEEE Conference on Decision and Control*, volume 3, 2117–2125.

Sastry, S. and Bodson, M. (1989). *Adaptive Control: Stability, Convergence and Robustness*. Dover Books on Electrical Engineering Series. Dover Publications.

Slotine, J.J. and Sastry, S.S. (1983). Tracking control of non-linear systems using sliding surfaces with application to robot manipulators. In *1983 American Control Conference*, 132–135.

Slotine, J.J.E. and Li, W. (1989). Composite adaptive control of robot manipulators. *Automatica*, 25(4), 509–519.

Slotine, J. and Li, W. (1991). *Applied Nonlinear Control*. Prentice Hall.

Spong, M. and Vidyasagar, M. (1989). *Robot Dynamics and Control*. Wiley.