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Non-linear prediction scheme for an omnidirectional mobile robot with disturbed time delay ⋆

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Abstract: This work focuses on the trajectory tracking problem of an input-delayed omnidirectional mobile robot affected by a constant disturbance in the time delay. The trajectory-tracking solution is based on a prediction strategy that represents a generalization of the well-known sub-prediction strategy developed for the linear case. It is formally proven that the prediction scheme provides the future state of the system and that a feedback law based on the future predicted state solves the trajectory tracking problem under some assumptions related to the size of the disturbance and the number of sub-predictors considered. To evaluate the prediction-based strategy, numerical simulations are carried out showing an adequate performance of the overall closed-loop system.

Keywords: Nonlinear systems, trajectory tracking, time-delay, mobile robot.

1. INTRODUCTION

From teleoperation systems to chemical processes, time delays must be taken into account when designing a control law, Niculescu (2001), because they can produce unwanted dynamics in the closed-loop system. This topic has attracted researchers' attention starting with the Smith predictor introduced in Smith (1957) that addressed the case of SISO open-loop stable linear systems and modified later for unstable linear systems, see for instance Palmor (1996). For the nonlinear case, there were also some developments extending the previous linear strategy, for example, Germani et al. (2002) and Hou et al. (2002) that consider a prediction strategy based on state observers presented in Thau (1973). Meanwhile, Mazenc and Bliman (2006) and Krstic (2009) use state feedback methods to solve the nonlinear prediction-based case.

In the field of robotics, delays are frequent, for example, in the context of teleoperation, and many contributions address this problem in a continuous time framework Sira-Ramírez et al. (2010) or in the discrete-time case, Santos et al. (2018) and Velasco-Villa et al. (2007), among others.

The present work considers the trajectory tracking problem for an omnidirectional mobile robot under the effect of a disturbed time delay at the input signal. It is assumed that a constant disturbance affects the time delay so its real value is unknown. It is considered the generalization, to the nonlinear case, of a prediction scheme that is based on the ideas of Fragoso-Rubio et al. (2019), Najafi et al. (2013), and Velasco-Villa et al. (2014) where a Luenberger type sub-predictor observer chain is proposed, Luenberger (1971), for linear systems. Based on the estimated future state of the mobile robot, it is designed a feedback law that solve the mentioned trajectory tracking problem. It is formally proven, that the estimated state converges to a desired future value and that the closed-loop system robot-estimated feedback is able to drive the tracking errors to the origin.

The work is organized as follows. First of all, Section 2 develops the kinematic model that is used throughout the paper, and Section 3 presents the considered non-linear sub-prediction scheme. Section 4 introduces the prediction-based feedback law used to solve the trajectory tracking problem. The stability analysis of the resulting closed-loop system is carried out in Section 5, while numerical simulations that validate the proposed scheme are presented in Section 6. Finally, some concluding remarks are given in Section 7.

2. KINEMATIC MODEL OF AN OMNIDIRECTIONAL MOBILE ROBOT

Considering the configuration of the wheels shown in Figure 1, assuming a rigid body of the robot with non-deformable wheels and that the robot moves on a horizontal plane, Canudas de Wit et al. (1996), it is possible to describe the kinematic model of the robot in the form,

$$\dot{\xi}(t) = -r(J_1 R(\phi(t)))^{-1}\dot{\theta}(t). \tag{1}$$

where

$$\xi(t) = \begin{bmatrix} x(t) \\ y(t) \\ \phi(t) \end{bmatrix}, \theta(t) = \begin{bmatrix} \theta_1(t) \\ \theta_2(t) \\ \theta_3(t) \end{bmatrix}, J_1 = \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} & L \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & L \\ 1 & 0 & L \end{bmatrix}$$

$$R(\phi(t)) = \begin{bmatrix} \cos(\phi(t)) & \sin(\phi(t)) & 0 \\ -\sin(\phi(t)) & \cos(\phi(t)) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

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Notice from Figure 1 that the posture of the robot is given by the state $\xi(t)$, where (x, y) gives the position on the $X-Y$ plane and $\phi(t)$ provide the orientation of the robot with respect to the X axis. $\theta_i(t)$, $i = 1, 2, 3$ represent the rotation or each wheel while $\dot{\theta}_i(t)$ are the angular velocity of the wheels and $R(\phi(t))$ is a rotation matrix. It is assumed that the control signal is given by the rotational velocity of each wheel $u_i(t) = \dot{\theta}_i(t)$ that can be mapped to the classical control signals $(v_1(t), v_2(t), v_3(t))$ represented also in Figure 1.

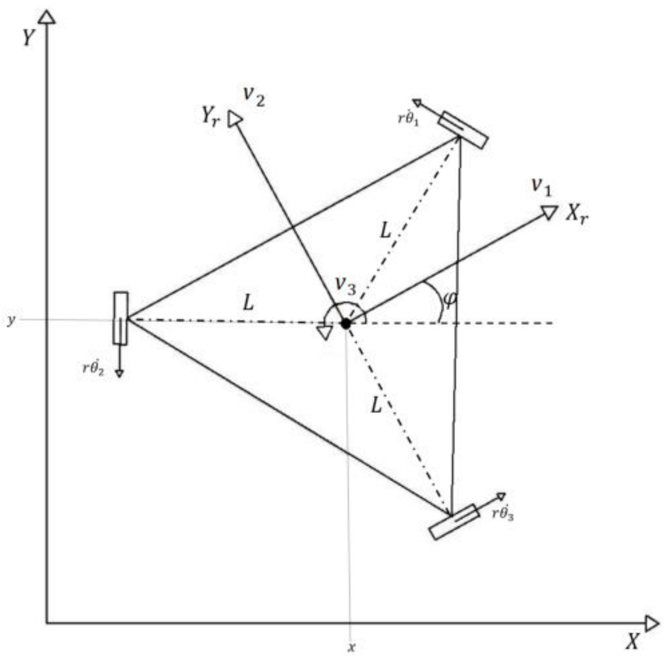

Fig. 1. Omnidirectional mobile robot.

2.1 Time delays at the input signals

Consider a wireless-controlled robot located in a remote position, as is schematically shown in Figure 2. As it is clear, two delays are present in the scheme, a time-delay $\tau_2 > 0$ representing the time it takes the data to go from the robot to the computer, and a time-delay $\tau_1 > 0$ being the time it takes the control signals to go in the opposite direction. Since the time delay occurs at the input of the robot, without loss of generality, it is assumed that there exists a total time delay,

$$\tau = \tau_1 + \tau_2$$

affecting the inputs of the robot. Under these conditions, the kinematic model of the mobile robot (1) can be rewritten as,

$$\dot{\xi}(t) = -r(J_1 R(\phi(t)))^{-1} u(t - \tau). \tag{2}$$

2.2 Delay disturbance

A problem related to the remote control configuration, given the representation (2), is that time delays are usually not exactly known. In an effort to approximate this representation to reality, as a first step, consider that a constant disturbance is included in the total time-delay

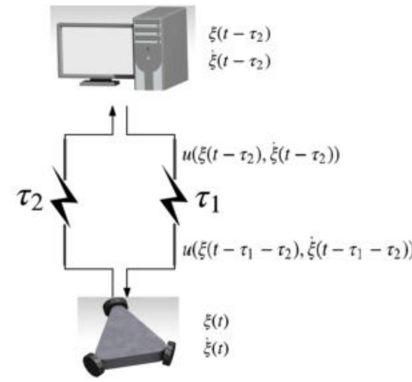

Fig. 2. Delays induced on a remote control scheme.

τ. This fact allows us to rewrite the time delay in the form,

$$\tau = h + \eta \tag{3}$$

with h the known part of the delay and η been the unknown disturbance. Also, it is assumed that,

$$h > |\eta| \in \mathbb{R}.$$

Therefore, it is obtained the following representation,

$$\dot{\xi}(t) = -r(J_1 R(\phi))^{-1} u(t - h - \eta). \tag{4}$$

3. PREDICTION SCHEME

In what follows, the nonlinear prediction scheme for the omnidirectional mobile robot is presented. First of all, it is considered the following assumption.

Assumption 1. There exist a positive integer $n \in \mathbb{N}$ such that the known time-delay h satisfy,

$$\bar{h} = \frac{h}{n} \tag{5}$$

3.1 Advanced system

To design the nonlinear predictor strategy based on an observer chain, consider first the following change of coordinates,

$$\rho_i(t) = \xi(t + i\bar{h} + \eta) = \rho_{i-1}(t + \bar{h} + \eta) \tag{6}$$

for $i = 1, ..., n$.

Considering system (4) and taking the time derivative of the new coordinates (6), it is possible to obtain, after some direct computations,

$$\dot{\rho}_i(t) = -r(J_1 R(\rho_{i3}))^{-1} u(t - (n - i)\bar{h} - \eta) \tag{7}$$

for $i = 1, ..., n$.

Notice that system (7) represents an advance dynamic of the system (4) several times in the future. In particular, for the case of $i = n$, it is obtained,

$$\dot{\rho}_n(t) = -r(J_1 R(\rho_n))^{-1} u(t - \eta)$$

that in the case of a disturbance-free system, it is obtained a system free of delay.

3.2 Sub-predictors chain

Considering the advanced systems (7), it is possible to propose a Luenberger-type observer that for the case

$i = n$ will provide the estimation of the future state $\xi(t+\tau)$. The Luenberger observer-prediction takes the form,

$$\dot{\hat{\rho}}_i(t) = -r(J_1 R(\hat{\rho}_{i3}))^{-1} u(t-(n-i)\bar{h}) + \lambda e_{\rho_i}(t-\bar{h}) \quad (8)$$

for $i = 1, ..., n$, where the injection errors $e_{\rho_1}(t)$ are defined as,

$$e_{\rho_1}(t) = \rho_1(t) - \hat{\rho}_1(t)$$

for $i = 1$, and

$$e_{\rho_j}(t) = \hat{\rho}_{(j-1)}(t+\bar{h}) - \hat{\rho}_j(t)$$

for $j = 2, 3, ..., n$.

For the design of a feedback law that solves the trajectory tracking problem for the omnidirectional mobile robot, it will be assumed first that,

$$\hat{\rho}_n(t) \to \rho_n(t) = \xi(t+\tau+\eta) \quad (9)$$

thus, it can be considered that the future state $\xi(t+\tau+\eta)$ is available to be used on the feedback design. The convergence to the future value (9) will be proven later.

Remark 1. Notice that the injection error $e_{\rho 1}$ is different from the prediction error that can be defined as

$$\tilde{\rho}_i(t) = \rho_i(t) - \hat{\rho}_i(t) \quad (10)$$

that after direct computations can be rewritten as,

$$\tilde{\rho}_i(t) = \sum_{k=1}^{i} e_{\rho_k}(t+(i-k)\bar{\tau}).$$

4. TRAJECTORY TRACKING PROBLEM

The trajectory tracking problem for the omnidirectional mobile robot free-delay (1), considering $\dot{\theta}(t) = u(t)$, can be solved by using the following feedback law,

$$\begin{aligned} u(t) = &-\frac{1}{r} J_1 R(\rho_{n3})\{\dot{\rho}_{nd}(t) \\ &- k[\rho_n(t) - \rho_{nd}(t)]\} \end{aligned} \quad (11)$$

where $\rho_{nd}(t)$ is the desired trajectory that the system has to follow. In the case of the delayed system (1), notice that that feedback (11) takes the form,

$$\begin{aligned} u(t) = &-\frac{1}{r} J_1 R(\rho_{n3}(t+\tau))\{\dot{\rho}_{nd}(t+\tau) \\ &- k[\rho_n(t+\tau) - \rho_{nd}(t+\tau)]\} \end{aligned} \quad (12)$$

Clearly, it is impossible to directly implement the non-causal feedback (12). However, the use of the estimated future states $\hat{\rho}_n(t)$ obtained by the prediction scheme (8), allows to consider the prediction-based feedback,

$$u(t) = -\frac{1}{r} J_1 R(\hat{\rho}_{n3})\{\dot{\rho}_{nd}(t) - k[\hat{\rho}_n(t) - \rho_{nd}(t)]\}. \quad (13)$$

4.1 Tracking errors

To solve the trajectory tracking problem, define now the trajectory tracking error as,

$$e_s(t) = \rho_n(t) - \rho_{nd}(t). \quad (14)$$

To simplify the representation of the closed loop system consider the next change of variable, for the tracking and injection errors,

$$\begin{aligned} z_s(t) =& e_s(t-n\bar{h}) \\ z_i(t) =& e_{\rho_i}(t-i\bar{h}) \qquad i = 1, 2, ..., n. \end{aligned} \quad (15)$$

Taking the time derivative of the injection and tracking errors and defining,

$$Z_n(t) = [\, z_2 \ \cdots \ z_n \,]^T$$

the closed-loop system is obtained as,

$$\begin{aligned} \begin{bmatrix} \dot{z}_s(t) \\ \dot{z}_1(t) \\ \dot{Z}_n(t) \end{bmatrix} =& \begin{bmatrix} 0 & 0_{1\times n} \\ kI & -(kI)_{1\times n} \\ 0_{(n-1)\times 1} & 0_{(n-1)\times n} \end{bmatrix} \begin{bmatrix} z_s(t) \\ z_1(t) \\ Z_n(t) \end{bmatrix} \\ &+ \begin{bmatrix} 0 & 0_{1\times n} \\ 0_{n\times 1} & -\Lambda_n \end{bmatrix} \begin{bmatrix} z_s(t-\bar{h}) \\ z_1(t-\bar{h}) \\ Z_n(t-\bar{h}) \end{bmatrix} \\ &+ \begin{bmatrix} -(kI)_{2\times 1} & (kI)_{2\times n} \\ 0_{(n-1)\times 1} & 0_{(n-1)\times n} \end{bmatrix} \begin{bmatrix} z_s(t-\eta) \\ z_1(t-\eta) \\ Z_n(t-\eta) \end{bmatrix} \\ &+ \begin{bmatrix} \gamma_t(t-n\bar{h}) \\ \gamma_1(t-\bar{h}) \\ \Gamma_n(t) \end{bmatrix} \end{aligned} \quad (16)$$

where $I \in \mathrm{R}^{3\times 3}$ is the identity matrix, $(kI)_{m\times p}$ is a matrix of order $m \times p$ where every element is k, $\Gamma_n(t) \in \mathrm{R}^{(n-1)\times 1}$ is a vector that contains the non-linear terms γ_i and Λ_n is a lower bidiagonal matrix such that,

$$\Lambda_n = \begin{bmatrix} -\lambda I & 0 & 0 & \dots & 0 & 0 \\ \lambda I & -\lambda I & 0 & \dots & 0 & 0 \\ 0 & \lambda I & -\lambda I & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \lambda I & -\lambda I \end{bmatrix}.$$

Under the above conditions, considering the state vector,

$$z(t) = \begin{bmatrix} z_s(t) \\ z_1(t) \\ Z_n(t) \end{bmatrix}.$$

The closed-loop system can be rewritten as,

$$\dot{z}(t) = \bar{A}_0 z(t) + A_1 z(t-\bar{h}) + A_2 z(t-\eta) + \Gamma(t), \quad (17)$$

where,

$$\bar{A}_0 = \begin{bmatrix} 0 & 0_{1\times n} \\ kI & -(kI)_{1\times n} \\ 0_{(n-1)\times 1} & 0_{(n-1)\times n} \end{bmatrix}, \qquad A_1 = \begin{bmatrix} 0 & 0_{1\times n} \\ 0_{n\times 1} & -\Lambda_n \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -(kI)_{2\times 1} & (kI)_{2\times n} \\ 0_{(n-1)\times 1} & 0_{(n-1)\times n} \end{bmatrix}, \qquad \Gamma(t) = \begin{bmatrix} \gamma_t(t-3\bar{h}) \\ \gamma_1(t-\bar{h}) \\ \Gamma_n(t) \end{bmatrix}$$

with

$$\begin{aligned} \gamma_t(t) =& E(\tilde{\rho}_{n3})\,[\dot{\rho}_{nd}(t) + k\tilde{\rho}_n(t) - ke_s(t)] \\ \gamma_1(t) =& [E(\tilde{\rho}_{n3}(t-(n-1)\bar{\tau})) - E(\tilde{\rho}_{n3}(t-(n-1)\bar{\tau}) \\ &- \tilde{\rho}_{13})][\dot{\rho}_{nd}(t-(n-1)\bar{\tau}) + k\tilde{\rho}_n(t-(n-1)\bar{\tau}) \\ &- ke_s(t-(n-1)\bar{\tau})] \\ \gamma_i(t) =& [E(\tilde{\rho}_{n3}(t-(n-i)\bar{\tau}) - \tilde{\rho}_{(i-1)3}(t+\bar{\tau})) \\ &- E(\tilde{\rho}_{n3}(t-(n-i)\bar{\tau}) - \tilde{\rho}_{i3})][\dot{\rho}_{nd}(t-(n-i)\bar{\tau}) \\ &+ k\tilde{\rho}_n(t-(n-i)\bar{\tau}) - ke_s(t-(n-i)\bar{\tau})] \end{aligned}$$

and

$$E(\tilde{\rho}_{n3}) = \begin{bmatrix} -2\sin^2(\frac{\tilde{\rho}_{n3}}{2}) & -\sin(\tilde{\rho}_{n3}) & 0 \\ \sin(\tilde{\rho}_{n3}) & -2\sin^2(\frac{\tilde{\rho}_{n3}}{2}) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Gamma_n(t) = [\gamma_2(t) \ \gamma_3(t) \ \dots \ \gamma_n].$$

In the above representation $\tilde{\rho}_i(t) = \sum_{k=1}^{i} e_{\rho_k}(t+(i-k)\bar{\tau})$ correspond to the prediction error.

5. ERRORS CONVERGENCE ANALYSIS

To carry out the stability analysis of the closed-loop system (17), a null term is introduced in the system in the form,

$$\begin{aligned}\dot{z}(t) =&(\bar{A}_0 + A_2)z(t) + A_1 z(t-\bar{h}) \\ &- A_2[z(t) - z(t-\eta)] + \Gamma(t) \\ =&\underline{A_0 z(t)} + A_1 z(t-\bar{h}) - A_2[z(t) - z(t-\eta)] + \Gamma(t) \\ =&A_0 z(t) + A_1 z(t-\bar{h}) - A_2 \int_{-\eta}^{0} \dot{z}(t+\theta)d\theta + \Gamma(t)\end{aligned} \tag{18}$$

that can be rewritten in the form,

$$\begin{aligned}\dot{z}(t) =&A_0 z(t) + A_1 z(t-\bar{h}) - F(z_t) + \Gamma(t) \\ &- A_2 \int_{-\eta}^{0} \Gamma(t+\theta)d\theta\end{aligned} \tag{19}$$

with

$$\begin{aligned}F(z_t) =&A_2 \int_{-\eta}^{0} \left[\bar{A}_0 z(t+\theta) + A_1 z(t-\bar{h}+\theta)\right. \\ &\left.+A_2 z(t-\eta+\theta)\right] d\theta\end{aligned} \tag{20}$$

and

$$A_0 = \bar{A}_0 + A_2 = \begin{bmatrix} -kI & (kI)_{1\times n} \\ 0_{n\times 1} & 0_{n\times n}\end{bmatrix}.$$

Following the ideas in Egorov and Mondié (2015), the next candidate Lyapunov-Krasovskii functional is proposed:

$$\begin{aligned}\tilde{V}(z_t) =& v_0(z_t) + \int_{-h}^{0} z^T(t+\theta)R_0 z(t+\theta)d\theta \\ &+ \int_{-h}^{0} z^T(t+\theta)A_1^T(\theta+\bar{h})R_1 A_1 z(t+\theta)d\theta \\ &+ |\text{sign}(\eta)| \left(\frac{1+h}{\epsilon}\right) \times \\ &\left[\int_{-\eta-\eta\chi(\eta)}^{0} z^T(t+\theta)A_0^T A_0^T(\theta+\eta+\eta\chi(\eta))U(0)A_0 A_0 z(t+\theta)d\theta\right. \\ &\left.+ \int_{-h-\eta\chi(\eta)}^{0} z^T(t+\theta)A_1^T A_0^T(\theta+h+\eta\chi(\eta))U(0)A_0 A_1 z(t+\theta)d\theta\right]\end{aligned}$$

where $U(t)$ is the Lyapunov matrix, Kharitonov (2013), $R_1 > 0$, $M > 0$,

$$\begin{aligned}v_0(z_t) =& z^T(t)U(0)z(t) + 2z^T(t)\int_{-h}^{0} U(-\theta-h)A_1 z(t+\theta)d\theta \\ &+ \int_{-h}^{0} z^T(t+\theta_1)A_1^T \int_{-h}^{0} U(\theta_1-\theta_2)A_1 z(t+\theta_2)d\theta_2 d\theta_1 \\ R_0 =&W_0 - 2U(0)|\eta|M^{-1}U(0) > 0 \\ W_0 =&W - hA_1^T R_1 A_1 \\ &- [1+h]|\text{sign}(\eta)|\left[(\eta+\eta\chi(\eta))A_0^T A_0^T U(0)A_0 A_0\right. \\ &\left.+ (h+\eta\chi(\eta))A_1^T A_0^T U(0)A_0 A_1\right] \\ -W =&A_0^T U(0) + U(0)A_0 + A_1^T U(h) + U^T(h)A_1\end{aligned}$$

and

$$\chi(\eta) = \begin{cases} 0 & \text{if } \eta < 0 \\ 1 & \text{if } \eta \geq 0.\end{cases}$$

after some tedious computations, the time derivative of the functional $\tilde{V}(z_t)$ along the solution of the system is,

$$\begin{aligned}\frac{d}{dt}\tilde{V}(z_t) =&z^T(t)[R_0 - W_0]z(t) \\ &+ 2\left[\Gamma(t) - F(z_t) - A_2\int_{-\eta}^{0}\Gamma(t+\theta)d\theta\right]^T \times \\ &\left[U(0)z(t) + \int_{-h}^{0} U(-\theta-h)A_1 z(t+\theta)d\theta\right] \\ &- z^T(t-h)R_0 z(t-h) \\ &- \int_{t-h}^{t} z^T(\theta)A_1^T R_1 A_1 z(\theta)d\theta \\ &- |\text{sign}(\eta)|\left(\frac{1+h}{\epsilon}\right) \times \\ &\left[\int_{t-\eta-\eta\chi(\eta)}^{t} z^T(\theta)A_0^T A_0^T A_0 A_0 z(\theta)d\theta\right. \\ &\left.+ \int_{t-h-\eta\chi(\eta)}^{t} z^T(\theta)A_1^T A_0^T A_0 A_1 z(\theta)d\theta\right].\end{aligned} \tag{21}$$

In view of the definition of $F(z_t)$ in (20), consider the following expression,

$$\begin{aligned}2F^T(z_t)\left[U(0)z(t) + \int_{-h}^{0} U(-\theta-h)A_1 z(t+\theta)d\theta\right] \\ = 2F_1(z_t) + 2F_2(z_t)\end{aligned}$$

with,

$$\begin{aligned}F_1(z_t) =&\left[A_2\int_{-\eta}^{0}[\bar{A}_0 z(t+\theta) + A_1 z(t-h+\theta)\right. \\ &\left.+ A_2 z(t-\eta+\theta)]d\theta\right]^T \left[U(0)z(t)\right] \\ F_2(z_t) =&\left[A_2\int_{-\eta}^{0}[\bar{A}_0 z(t+\theta) + A_1 z(t-h+\theta)\right. \\ &\left.+ A_2 z(t-\eta+\theta)]d\theta\right]^T \left[\int_{-h}^{0} U(-\theta-h)A_1 z(t+\theta)d\theta\right].\end{aligned}$$

It is possible to upper-bound functions F_1, F_2 using the identity $2a^T b \leq a^T M a + b^T M^{-1} b \mid M > 0$ as follows,

$$\begin{aligned}|2F_1(z_t)| \leq&\text{sign}(\eta)\int_{-\eta}^{0} F^T(z_t,\theta)MF(z_t,\theta)d\theta, \\ &+ |\eta|z^T(t)U(0)M^{-1}U(0)z(t),\end{aligned}$$

and

$$\begin{aligned}&|2F_2(z_t) \leq sign(\eta)\int_{-\eta}^{0} F^T(z_t,\theta)MF(z_t,\theta)d\theta \\ &+|\eta|\int_{-h}^{0} z^T(t+\theta)A_1^T U(h+\theta)M^{-1}U(h+\theta)A_1 z(t+\theta)d\theta,\end{aligned}$$

where,

$$M = \frac{1}{\epsilon} U(0).$$

As a consequence,

$$\begin{aligned}|2F_1(z_t)+2F_2(z_t)| \leq & \; z^T(t) W z(t) \\ & + \int_{-h}^{0} z^T(t+\theta) A_1^T R_1 A_1 z(t+\theta) d\theta \\ & + |\text{sign}(\eta)| \left(\frac{1+h}{\epsilon} \right) \times \\ & \left[\int_{t-\eta-\eta\chi(\eta)}^{t} z^T(\theta) A_0^T A_0^T A_0 A_0 z(\theta) d\theta \right. \\ & \left. + \int_{t-h-\eta\chi(\eta)}^{t} z^T(\theta) A_1^T A_0^T A_0 A_1 z(\theta) d\theta \right].\end{aligned} \tag{22}$$

This, in turn, allows bounding the functional derivative as,

$$\begin{aligned}\frac{d}{dt}\tilde{V}(z_t) \leq & - z^T(t) R_0 z(t) + [2 + a_1 h + a_2 \eta] \nu L_c ||z(t)||^2 \\ & + [1 + a_2 \eta] \nu a_1 L_c \int_{-h}^{0} ||z(t+\theta)||^2 d\theta \\ & + [1 + a_1 h] \nu a_2 L_c \int_{\eta}^{0} ||z(t+\theta)||^2 d\theta\end{aligned} \tag{23}$$

where, $a_1 = ||A_1||$, $a_2 = ||A_2||$, $\nu = \sup_{\tau \in [0,h]} ||U(\tau)||$ and $||\Gamma(t)|| \leq L_c ||z(t)||$.

Bounds on equation (23) allows modifying the functional $\tilde{V}(z_t)$ as follows in order to assure the negative definitiveness of the functional,

$$\begin{aligned}\tilde{V}_p(z_t) = & \tilde{V}(z_t) + 2\frac{\mu}{h} \int_{t-h}^{t} z^T(\theta)[h - t + \theta] U^T(h) A_1 z(\theta) d\theta \\ & + 2\frac{\mu}{\eta} \int_{t-\eta}^{t} z^T(\theta)[\eta - t + \theta] U(0) A_0 z(\theta) d\theta\end{aligned} \tag{24}$$

with a $\mu > 0$.

The time derivative of $\tilde{V}_p(z_t)$ yields,

$$\begin{aligned}\frac{d}{dt}\tilde{V}_p(z_t) = & \frac{d}{dt}\tilde{V}(z_t) - \mu z^T(t) W z(t) \\ & - 2\frac{\mu}{h} \int_{t-h}^{t} z^T(\theta) U^T(h) A_1 z(\theta) d\theta \\ & - 2\frac{\mu}{\eta} \int_{t-\eta}^{t} z^T(\theta) U(0) A_0 z(\theta) d\theta.\end{aligned} \tag{25}$$

Equation (25) implies that $\frac{d}{dt}\left(\tilde{V}_p(z_t)\right) < 0$ if the following conditions are satisfied,

i) $R1 \geq 2U(0)|\eta| > 0$
ii) $\mu \lambda_{min}(W) \geq [2 + a_1 h + a_2 \eta] \nu L_c$
iii) $2\mu[\nu_{min}] \geq [1 + a_2 \eta] \nu h L_c$
iv) $2\mu a_0 \geq [1 + a_1 h] a_2 \eta L_c$.

Therefore, it is possible to conclude that if the above conditions are satisfied, then, the closed-loop system (17) will be asymptotically stable. In other words, it is possible to assure the stability of the system if there exists a μ large enough to bound the positive terms of $\frac{d}{dt}\tilde{V}(z_t)$. It is also noticeable that condition (i) exists to maintain the positive definiteness of $\tilde{V}_p(z_t)$.

6. NUMERICAL SIMULATION

To show the performance of the prediction-based trajectory tracking strategy developed in this work, some numerical simulations will be carried out by considering a desired trajectory generated by a lemniscate type given by the parametric equations,

$$\rho_{nd1} = A\cos(pt)$$
$$\rho_{nd2} = B\sin((2pt))$$

with $A = 1$, $B = 0.4$ and $p = \frac{\pi}{20}$.

The parameters for the omnidirectional robot have been set as $L = 0.1877$ m and $r = 0.0381$ m, using as reference a robot built in our laboratory. The considered time delay is $h = 1.5$ s and $\eta = 0.03$ s. The predictor considers, $n = 3$, $\lambda_i = 0.5$, and for the feedback law, $k = 1$.

It is important to remark that the parameters λ_i and η were chosen such that both satisfy the inequalities (i)-(iv) in order to make system (17) asymptotically stable. The details of the computations are omitted due to space reasons, however, in Figure 3 the stability region is depicted.

As is shown in the theoretical developments, the convergence of the injection errors implies the convergence of the prediction errors $\tilde{\rho}_3$ as is presented in Figure 4.

The trajectory tracking solution is based on the estimated predicted state, since the prediction errors converge, it is possible to show the adequate convergence on the tracking errors in Figure 5. Finally, the evolution of the omnidirectional delayed mobile robot in the $X - Y$ plane is depicted in Figure 6.

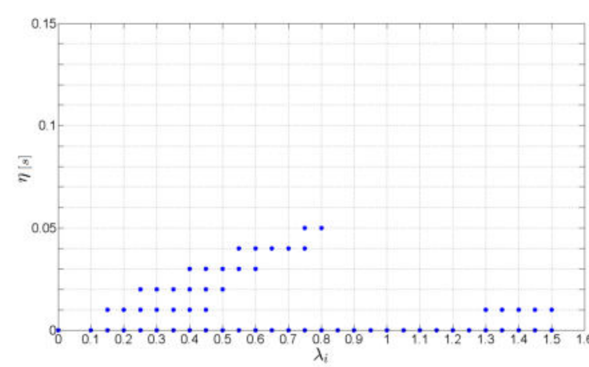

Fig. 3. Stability region of the system with a time delay of $h = 1.5$ s .

7. CONCLUSION

This paper presents a strategy to solve the problem of time delay in the input of an omnidirectional mobile robot. It also shows that the prediction scheme considered

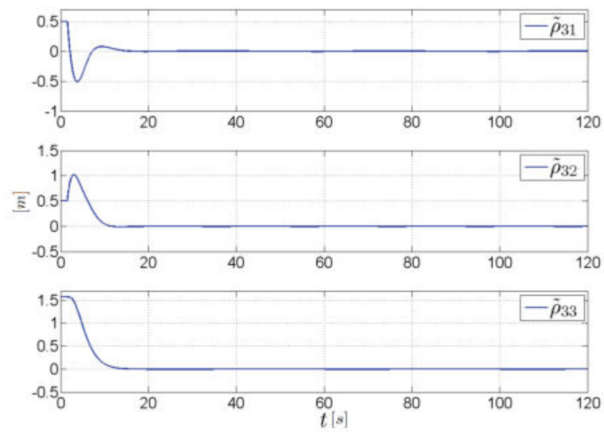

Fig. 4. Prediction error $\tilde{\rho}_3$.

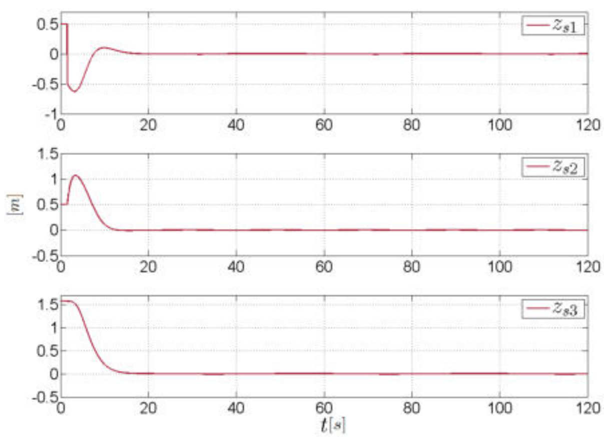

Fig. 5. Trajectory tracking error z_s.

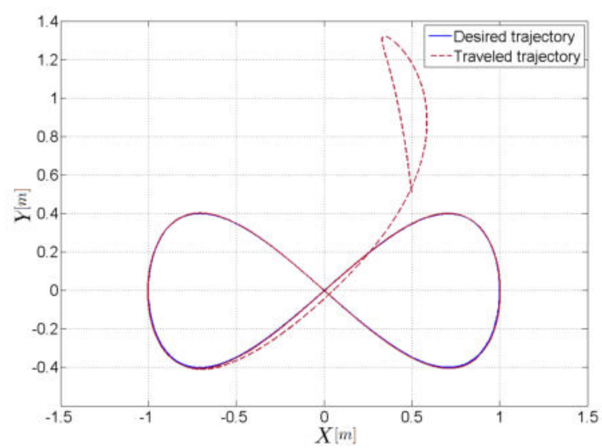

Fig. 6. Trajectory depicted by the omnidirectional robot.

is robust with respect to a constant perturbation in the time delay. However, as can be seen in Sections 5 and 6, η needs to be very small compared to h to ensure the predictions converge to the real states. Future work aims at the modification of the present scheme to improve robustness with respect to time delay disturbances.

REFERENCES

Canudas de Wit, C., Bastin, G., and Siciliano, B. (1996). *Theory of Robot Control*. Springer-Verlag, Berlin, Heidelberg, 1st edition.

Egorov, A.V. and Mondié, S. (2015). The delay lyapunov matrix in robust stability analysis of time-delay systems.

Fragoso-Rubio, V., Velasco-Villa, M., Hernández-Pérez, M.A., del Muro-Cuéllar, B., and Márquez-Rubio, J.F. (2019). Prediction–observer scheme for linear systems with input–output time delay. *International Journal of Control, Automation and System*, 17.

Germani, A., Manes, C., and Pepe, P. (2002). A new approach to state observation of nonlinear systems with delayed output. *IEEE Transactions on Automatic Control*, 47, 96–101.

Hou, M., Ztek, P., and Patton, R.J. (2002). An observer design for linear time-delay systems. *IEEE Transactions on Automatic Control*, 47, 121–125.

Kharitonov, V.L. (2013). *Time-Delay Systems: Lyapunov Functionals and Matrices*. Birkhäuser.

Krstic, M. (2009). *Delay compensation for nonlinear, adaptive, and PDE systems*. Springer.

Luenberger, D. (1971). An introduction to observers. *IEEE Transactions on Automatic Control*, 16, 596–602.

Mazenc, F. and Bliman, P.A. (2006). Backstepping design for time delay nonlinear systems. *IEEE Transactions on Automatic Control*, 51, 149–154.

Najafi, M., Hosseinnia, S., Sheikholeslam, F., and Karimadini, M. (2013). Closed-loop control of dead time systems via sequential sub-predictors. *International Journal of Control*, 86.

Niculescu, S.I. (2001). *Delay Effects on Stability, A Robust Control Approach*. Springer.

Palmor, Z.J. (1996). Time-delay compensation smith predictor and its modifications. *The Control Handbook*, 1, 224–229.

Santos, J., Conceiçao, A., Santos, T., and Araújo, H. (2018). Remote control of an omnidirectional mobile robot with time-varying delay and noise attenuation. *Mechatronics*, 52, 7–21.

Sira-Ramírez, H., López-Uribe, C., and Velasco-Villa, M. (2010). Trajectory-tracking control of an input delayed omnidirectional mobile robot. In *International Conference on Electrical Engineering, Computing Science and Automatic Control(CCE)*.

Smith, O.J.M. (1957). Closer control of loops with deadtime. *Chem. Eng. Prog.*, 53(5), 217–219.

Thau, F.E. (1973). Observing the state of non-linear dynamic systems. *International Journal of Control*, 17(3), 471–479.

Velasco-Villa, M., del Muro-Cuellar, B., and Alvarez-Aguirre, A. (2007). Smith-predictor compensator for a delayed omnidirectional mobile robot. In *Mediterranean Conference on Control and Automation*.

Velasco-Villa, M., Mandujano-García, E., Estrada-Sanchez, I., Rodríguez-Cortés, H., and Sira-Ramírez, H. (2014). Esquema predictor-observador para el control de un robot móvil omnidireccional con retardos de tiempo. In *Congreso Latinoamericano de Control Automático*.