

Isolation and identification of periodic disturbances in BLDC motors \star

Alejandra de la Guerra^{*} Axel Coronado-Andrade^{*} Alejandro Gutiérrez-Giles^{**}

 * Instituto de Ingeniería, UNAM, CU, CDMX, (e-mail:ale_delaguerra@comunidad.unam.mx, ACoronadoA@iingen.unam.mx)
 ** Coordinación de Ciencias Computacionales, INAOE, Tonantzintla, Puebla (e-mail: alejandro.giles@inaoep.mx)

Abstract: A high-gain linear observer is proposed for online identification and reconstruction of a periodic disturbance. It is assumed that the frequency of the periodic disturbance is known, but its amplitude and phase are unknown. This is not a stringent assumption since such supposition can be easily characterized for disturbances like cogging torque, or bearing defects, both well-known problems in the control of electric machines. The observer design is based on the internal model principle in combination with a high-gain observer. Numerical results are included to validate the estimation of the periodic disturbance even when another disturbance of a different frequency is added. The estimated disturbance can be used to design a controller for reducing the disturbance effect. Alternatively, the estimated disturbance can be used for fault detection like in the case of gears and bearing faults, or simply to characterize the cogging torque of a particular machine.

Keywords: Observers and filter design, System identification, BLDC motor.

1. INTRODUCTION

Brushless DC motors are mostly used in servo, actuation, positioning, and variable speed applications, where precise motion control and reliable functioning are important for the satisfactory operation of industrial processes. In such a setting, it is common to find all types of disturbances and undesired vibrations of different natures.

There are several examples in the literature regarding torque load estimators for electric motors, as in Ramírez et al. (2013) where a Generalized Proportional Integral Observer (GPIO) is proposed for the induction motor to compensate the external disturbances, or more recently the work by Alonge et al. (2017) where an active disturbance rejection control is proposed for linear induction motors. Another example is the work by Zhu and Li (2019) based on neural networks for electric powertrains. Still, very few works focus on the reconstruction of disturbances for the BLDC motor. Some of the few examples in this topic is the work by Deenadavalan and Ilango (2011) based on Sliding Mode Observers (SMO), where the position and velocity are estimated by using an adaptive gain SMO. Also, the article by Shao et al. (2015), where the load torque is estimated using an SMO based on a linear model of the BLDC motor. Only the works by de la Guerra et al. (2018) and de la Guerra and Alvarez-Icaza (2020) deal with the case of a trapezoidal BLDC, where a GPIO is proposed to compensate variable lumped disturbances, such as load torque and friction terms.

However, in the mentioned works the load torque is estimated or compensated as a whole. In this article, the objective is to estimate and isolate the periodic disturbances that affect the motor shaft. One way to classify these periodic perturbations is by their source. When the periodic perturbation is of internal nature, i.e. it depends on the angular position and/or velocity, it is called a self-excited vibration. A mechanical defect in a rotating system can be interpreted as the presence of a periodic angular perturbation. Examples of this type of disturbance are the cogging torque and the periodic disturbance detected in the case of gears and bearing faults (Bourdon et al., 2014). These types of disturbances are related to machine design and/or operation (Niewiara et al., 2020) and cannot be avoided.

On the other hand, if the perturbation is originated by an external source it is said to be an independent vibration (Alsogkier and Bohn, 2012). Examples of these types of disturbances are the vibrations generated by an inverter (Niewiara et al., 2020).

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In the following lines, a summary of some of the works proposed in the literature to identify or reject periodic disturbances is presented. In Alsogkier and Bohn (2017) a rejection notch filter is proposed to reject a disturbance for Linear Time-Invariant Systems (LTI). In Cortés-Romero et al. (2014) a GPI observer is proposed to identify a periodic disturbance in a DC motor experimental platform. In Yilmaz and Basturk (2019) an output feedback controller is proposed for LTI systems with unknown periodic disturbances. In Niewiara et al. (2020), an application of the extended Kalman filter (EKF) for estimation and attenuation of periodic disturbance in a permanent magnet synchronous motor (PMSM) drive is presented. In Beltran-Carbajal et al. (2021) it is presented an active vibration control technique for direct-current electric motors subjected to harmonic mechanical load torque where vibrating torque disturbances are actively suppressed by the control voltage input.

In several articles, the disturbance models are based on Fourier series, e.g. in Chu et al. (2016), where a reduced-order nonlinear observer is designed to estimate the cogging torque for a Permanent Magnet Direct Current (PMDC) driver. In the mentioned design it is assumed that the angular speed $\omega \neq 0$ and that ω^2 is bounded. With a different approach, in Reyna et al. (2018) a Fourier Series Controller is proposed to reduce the torque ripple for PMSM and BLDC drivers. In this context, it is important to mention the work by Ruderman et al. (2012) where an observer-based drive control can efficiently reject the harmonic torque disturbances. The design is based on the state-space torque harmonics representation and a Luenberger observer using the first two harmonics.

In this article, we address the problem of isolating and identifying a periodic disturbance in a BLDC motor. The design is based on the internal model principle and a harmonics representation, as proposed in Ruderman et al. (2012), but incorporating a linear time-varying observer. The goal of this design is to reconstruct the disturbance even in the presence of another periodic disturbance.

The estimated disturbance can be used to design a condition monitoring technique to identify vibration features in the case of bearing faults (Zarei et al., 2014). Alternatively, the estimated disturbance can be employed for the characterization of the cogging torque in BLDC motors. Moreover, as mentioned in Fico et al. (2019), the cogging torque monitoring can be used to detect stator demagnetization, because due to the missing magnet, the distribution of the cogging torque around a revolution is uneven, and its magnitude increases.

The paper is organized as follows, Section 2 recalls the mathematical model of the BLDC motor and the disturbance model. Section 3 presents the observer design. Section 4 includes a numerical validation of the observer. Finally, Section 5 includes the conclusions of this work.

2. PRELIMINARIES

2.1 Brushless DC motor mathematical model

In a BLDC motor, with Y-configuration, the windings are connected to a central point and power is applied to the remaining end of each winding. This means that the stator windings phases are balanced, which implies that the stator currents satisfy the following expression

$$i_1 + i_2 + \dots + i_m = 0, (1)$$

with m the number of stator phases. The motor has N_s stator slots and N_p rotor poles. It is assumed that stator windings are identical, i.e. the resistance and inductance parameters are the same for each phase.

The mathematical model of a balanced m-phases brushless DC motor as described by Chiasson (2005) is

$$\mathbf{D}\frac{d\mathbf{i}}{dt} = k_{\mathrm{e}}\omega\mathbf{E}(\theta) - \mathbf{R}\mathbf{i} + \mathbf{u}$$
(2a)

$$\frac{d\theta}{dt} = \omega \tag{2b}$$

$$J\frac{d\omega}{dt} = -k_{\rm m}\mathbf{E}^T(\theta)\mathbf{i} + \tau_{\rm p}(t) + \delta(t) \qquad (2c)$$

where $\mathbf{i} \in \mathbb{R}^m$ is the vector of stator currents, $\mathbf{u} \in \mathbb{R}^m$ is the vector of voltage inputs, $\omega \in \mathbb{R}$ is the angular velocity, $k_{\mathbf{e}} \in \mathbb{R}$ the back electromotive force constant, $k_{\mathbf{m}} \in \mathbb{R}$ the torque constant, $\mathbf{R} \in \mathbb{R}^{m \times m}$ is a diagonal matrix accounting for the winding resistances, $\tau_{\mathbf{p}}(t) \in \mathbb{R}$ is the periodic part of the load torque, $\delta(t)$ represents the unmodeled external disturbances, $J \in \mathbb{R}$ is the rotor inertia, and $\mathbf{D} \in \mathbb{R}^{3 \times 3}$ is the inductance matrix is defined as

$$\mathbf{D} = \begin{bmatrix} L + M & 0 & 0 \\ 0 & L + M & 0 \\ 0 & 0 & L + M \end{bmatrix}, \quad (3)$$

with $L \in \mathbb{R}$ the stator phase inductance an $M \in \mathbb{R}$ the mutual inductance between stator phases. The back–electromotive force vector is defined as

$$\mathbf{E} = \begin{bmatrix} e\left(\theta\right)\\ e\left(\theta - \frac{2\pi}{3}\right)\\ e\left(\theta - \frac{4\pi}{3}\right) \end{bmatrix}, \qquad (4)$$

with $e(\theta)$ given by

$$e(\theta) = \begin{cases} \frac{6\theta}{\pi} & \text{if} - \pi/6 \le \theta \le \pi/6\\ 1 & \text{if} & \pi/6 \le \theta \le 5\pi/6\\ \frac{-6(\theta - \pi)}{\pi} & \text{if} & 5\pi/6 \le \theta \le 7\pi/6\\ -1 & \text{if} & 7\pi/6 \le \theta \le 11\pi/6 \end{cases}$$
(5)

where $e\left(\theta - \frac{2\pi}{3}\right)$ and $e\left(\theta - \frac{4\pi}{3}\right)$ are defined using (5) with the respective phase displacement.

2.2 Disturbance model

It would be assumed that there is an independent periodic disturbance defined as

$$\tau_{\rm p}(t) = \sum_{n=1}^{N} (a_n \cos(\omega_0 t) + b_n \sin(\omega_0 t)) \tag{6}$$

which is a Fourier series without constant term, with $0 < N < \infty a_n$, b_n the Fourier coefficients, and ω_0 a known frequency. The constant term will be omitted since the DC-bias can be attributed to the Coulomb friction and therefore it is not a part of the periodic disturbance (Ruderman et al., 2012).

This kind of perturbation is not related to the control system variables, as opposed to the self-excited vibration perturbations which are commonly position-dependent. However, the next result can be applied to the self-excited perturbation with minimal changes.

3. OBSERVER DESIGN

This section presents the observer for the perturbation. To design this observer, the next assumptions are made.

Assumption 1. The time constant of the mechanical subsystem is much larger than the time constant of the electrical subsystem, i.e. only equation (2c) is considered for the observer design.

Assumption 2. The stator current vector **i**, angular position θ , and angular velocity ω are measured. Also, the input torque is defined as

$$\tau_{\rm e} = -k_{\rm m} \mathbf{E}^T(\theta) \mathbf{i}$$

Taking into account the above assumptions, and defining the speed estimation error

$$\tilde{\omega} = \omega - \hat{\omega},$$

the observer proposed to estimate the motor speed and external periodic disturbance is defined as

$$\hat{\omega} = \tau_{\rm e}/J + \hat{\tau}_{\rm p}/J + K_2 \tilde{\omega} \tag{7a}$$

$$\dot{\hat{\tau}}_{\rm p} = -\omega_0 \hat{a}_1 \sin(\omega_0 t) + \omega_0 \hat{b}_1 \cos(\omega_0 t) + J K_1 \tilde{\omega} \qquad (7b)$$

$$\dot{\hat{a}}_1 = -\frac{J}{\omega_0}\sin(\omega_0 t)K_0\tilde{\omega}$$
(7c)

$$\dot{\hat{b}}_1 = \frac{J}{\omega_0} \cos(\omega_0 t) K_0 \tilde{\omega}$$
(7d)

The first two equations are based on the motor and perturbations models. On the other hand, the last two equations estimate the Fourier coefficients that can be used to obtain the reconstructed external disturbance, $\hat{\tau}_r$, with two Fourier terms as

$$\hat{\tau}_{\rm r} = \hat{a}_1 \cos(\omega_0 t) + \hat{b}_1 \sin(\omega_0 t) \,. \tag{8}$$

Alternatively, with these two coefficients, it is possible to calculate the amplitude and phase of the reconstructed signal,

$$\hat{\tau}_{\rm c} = \hat{A}\cos(\omega_0 t + \hat{\phi}), \qquad (9)$$

using the expressions,

$$\hat{A} = \sqrt{\hat{a}_1^2 + \hat{b}_1^2} \tag{10a}$$

$$\hat{\phi} = \operatorname{atan2}(\hat{b}_1, \hat{a}_1) \,. \tag{10b}$$

3.1 Error dynamics

To obtain the estimation error dynamics it is necessary to derive some auxiliary expressions first. With respect to the periodic perturbation, the first time-derivative is

$$\mathbf{F}_{\mathbf{p}} = -\omega_0 a_1 \sin(\omega_0 t) + \omega_0 b_1 \cos(\omega_0 t) \,. \tag{11}$$

Therefore, from the estimation speed error definition, the estimation error dynamics can be written as

$$\dot{\tilde{\omega}} = \dot{\omega} - \dot{\hat{\omega}} \tag{12}$$

Substituting equations (2c) and (7) in the last expression it is obtained

$$\dot{\tilde{\omega}} = -\tilde{\tau}_p / J - K_2 \tilde{\omega} + \delta(t) / J , \qquad (13)$$

where $\tilde{\tau}_p = \tau_p - \hat{\tau}_p$. Therefore the dynamics of $\tilde{\tau}_p$ is

$$\dot{\tilde{\tau}}_p = \dot{\tau}_p - \dot{\tilde{\tau}}_p \,, \tag{14}$$

Substituting (11) and (7b) in the last expression it is obtained

$$\dot{\tilde{\tau}}_p = -\omega_0 \sin(\omega_0 t) \tilde{a}_1 + \omega_0 \cos(\omega_0 t) \tilde{b}_1 - J K_1 \tilde{\omega} , \qquad (15)$$

where $\tilde{a}_1 = a_1 - \hat{a}_1$ and $\tilde{b}_1 = b_1 - \hat{b}_1$. The second derivative of (15) can be written as

$$\begin{aligned} \ddot{\tilde{\tau}}_p &= -\omega_0 \sin(\omega_0 t) (-\dot{\hat{a}}_1) - \omega_0^2 \cos(\omega_0 t) \tilde{a}_1 \\ &+ \omega_0 \cos(\omega_0 t) (-\dot{\hat{b}}_1) - \omega_0^2 \sin(\omega_0 t) \tilde{b}_1 - J K_1 \dot{\tilde{\omega}} . \end{aligned}$$
(16)

By taking two time-derivatives of (13) and subtituting (15)-(16) one obtains

$$\tilde{\omega}^{(3)} + K_2 \ddot{\tilde{\omega}} + K_1 \dot{\tilde{\omega}} + K_0 \tilde{\omega} = (17)$$
$$- \frac{1}{I} \left(\omega_0^2 \cos(\omega_0 t) \tilde{a}_1 + \omega_0^2 \sin(\omega_0 t) \tilde{b}_1 - \ddot{\delta}(t) \right).$$

In turn, the Fourier coefficients estimation dynamics are governed by

$$\dot{\tilde{a}}_1 = \frac{J}{\omega_0} \sin(\omega_0 t) K_0 \tilde{\omega} \tag{18}$$

$$\dot{\tilde{b}}_1 = -\frac{J}{\omega_0} \cos(\omega_0 t) K_0 \tilde{\omega} \,. \tag{19}$$

The error dynamics is completely described by the vector $\boldsymbol{x} \in \mathbb{R}^n$, where *n* is the dimension of the state space (in the present case n = 5), such that

$$\boldsymbol{x} = \begin{bmatrix} \tilde{\omega} \ \dot{\tilde{\omega}} \ \ddot{\tilde{\omega}} \ \tilde{a}_1 \ \tilde{b}_1 \end{bmatrix}^{\mathrm{T}} .$$
 (20)

Care should be taken since the closed-loop estimation error dynamics given by (17)–(19) represents a linear time-varying system, for which asymptotic stability is not easily obtained, i.e. by computing its eigenvalues, but by some other techniques such as Floquet theory (Rugh Wilson, 1993) and Averaging (Khalil, 2002). In our case, we are aiming only for the ultimate boundedness of the estimation errors, not asymptotic convergence. Notice also that if the second time derivative of the unknown perturbation $\delta(t)$ is zero, e.g. for constant or ramp disturbances, the error dynamics analysis could be simplified.

4. SIMULATION

In this section, a numerical validation of the observer is presented. Simulations were performed in Simulink-Matlab using the parameters of the Anaheim BLDC BLY-344S-240V motor, which can be found in Table 1.



Fig. 1. Block diagram of FOC controller and perturbation observer

The Field Oriented Control technique (FOC) was used to control currents and speed of the motor and the space vector modulation technique was used for inverter switching as shown in Figure 1. The sampling time of the observer was $T_s = 0.025$ [ms]. It was also assumed that continuous measurement of the angular position and currents was available.

Table 1. DLDC motor paramete	Table 1	1. BLDC	' motor	parameter
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Parameter	Value
Rated Voltage	240 [V]
Rated Torque	2.1 [N m]
Rated Power	600 [W]
Resistance	$1.2 \ [\Omega]$
Inductance	$0.00475 \ [mH]$
Electric constant	$0.3455 \; [V/rad/s]$
Mechanical constant	$0.3811 \ [Nm/A]$
Inertia	$0.0002618 \ [kgm^2]$
Viscous friction coefficient	$0.000095 [\mathrm{Nms}]$

Simulations were conducted with three different periodic load torque signals while the rotor was spinning at a constant speed of 80 [rad/s], using the gain settings $K_0 = 2.1 \times 10^3, K_1 = 1.47 \times 10^6, K_2 = 3.43 \times 10^8$.

4.1 Two periodic disturbances

In the first simulation, the load torque is given by the expression

$$\tau_{\rm p} = 0.1\sin(w_1 t) + 0.05\sin(w_2 t), \qquad (21)$$

where $w_1 = 2\pi \cdot 60$ is the known frequency of the periodic signal of interest and $w_2 = 2\pi \cdot 6$ is the frequency of another periodic signal that is not necessarily known.

Figure (2a) shows the velocity estimated by the observer, Figure (2b) displays the periodic disturbance, τ_p , the estimated periodic disturbance, $\hat{\tau}_p$, and the reconstructed periodic disturbance, $\hat{\tau}_r$. It must be noted that the reconstructed disturbance recovers only the signal of interest while the estimated disturbance estimates the sum of all the vibrations. Lastly, figure (2c) shows the estimated Fourier coefficients \hat{a}_1 and \hat{b}_1 used to calculate $\hat{\tau}_r$.





4.2 External periodic disturbance

In this simulation the load torque is given by

$$\tau_{\rm p} = 0.1 \sin(\omega_1 t) \,. \tag{22}$$

This signal represents an external periodic disturbance and a high-frequency example.

Figure (3a) shows the the periodic disturbance, τ_p and the reconstructed periodic disturbance, $\hat{\tau}_r$. Figure (3b) shows the estimated Fourier coefficients \hat{a}_1 and \hat{b}_1 used to calculate $\hat{\tau}_r$. Using equations (10) with the estimated Fourier coefficients \hat{a}_1 and \hat{b}_1 it is obtained

$$\hat{A} = \sqrt{\hat{a}_1^2 + \hat{b}_1^2} = 0.1$$

 $\hat{\phi} = \operatorname{atan2}(\hat{b}_1, \hat{a}_1) = \frac{\pi}{2},$

therefore

$$\tau_{\rm c} = 0.1 \cos\left(\omega_1 t + \frac{\pi}{2}\right) = 0.1 \sin(\omega_1 t) + 0.1 \sin(\omega_1 t)$$

In this manner, it is possible to reconstruct the signal of interest by recovering the phase and amplitude which are the most commonly unknown signal characteristics.



(a) Periodic disturbance, τ_p (–), $\hat{\tau}_r$ (–)



(b) Fourier coefficients, \hat{a}_1 (-), \hat{b}_1 (-)

Fig. 3. Simulation results example 60 [Hz] signal.

4.3 Internal periodic disturbance

In the last simulation, the load torque is not a function of time but a function of angular position and is given by the expression,

$$\tau_{\rm p} = 0.1 \sin(N_c \theta) \,, \tag{24}$$

where $N_c = 8$ is the minimum common multiple of the number of poles and slots. This signal represents the cogging torque which is an undesirable disturbance that produces speed ripple. This disturbance is especially prominent at lower speeds, with the symptom of jerkiness as mentioned in Chu et al. (2016). To implement this example, it is necessary to use the rotor's angular position to generate the external periodic disturbance.

Figure (4a) shows the periodic disturbance, τ_p and the reconstructed periodic disturbance, $\hat{\tau}_r$. Figure (4b) shows

the estimated Fourier coefficients a_1 and b_1 used to calculate $\hat{\tau}_r$. In this case, the reconstructed disturbance has a negative coefficient, which may be mathematically sound but not realistic.



Fig. 4. Simulation results for example with cogging torque.

4.4 Discussion

As a metric for the error between the periodic disturbance τ_p and the estimation $\hat{\tau}_r$, the root mean square error (RMSE) expressed by equation (25) was used.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N} (x_i - \hat{x}_i)}{N}}.$$
 (25)

The RMSE for the previous cases is presented in Table 2.

Table 2. RMS error values.

Case 1	0.005770
Case 2	0.005089
Case 3	0.004631

The three cases presented in the simulations show that the proposed observer:

- can reconstruct the disturbance of interest even in the presence of other unknown periodic disturbances,
- it works for low and high frequencies,
- it can reconstruct internal and external disturbances,
- $\hat{\tau}_r$ and τ_c can reconstruct the disturbance without introducing delays in contrast with the schemes based on low pass filters.

Therefore, the proposed observer can detect and isolate the periodic disturbance without introducing delays.

5. CONCLUSION

The proposed observer can identify and isolate periodic disturbances of known frequency even when a vibration of unknown frequency is added. The design is simple and easy to tune. The numerical validation shows that it can reconstruct periodic disturbances of high and low frequency and internal and external nature.

The future work includes the experimental validation of the proposed observer and the closed loop stability analysis. It will also include the use of this observer in a fault diagnosis scheme for BLDC motors.

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