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Fault estimation based on generalized learning observer for a class of nonlinear algebro-differential parameter-varying systems

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Abstract. This paper presents the design of a generalized learning observer (GLO) structure for simultaneous estimation of variable states and actuator faults in a wider class of systems known as descriptor nonlinear parameter varying (D-NLPV) systems. This generalized structure provides additional degrees of freedom in the observer design to improve robustness and reduce the convergence time for fault estimation. Its design is obtained in terms of a set of linear matrix inequalities. The performance of the proposed methodology is evaluated in the model of a heat exchanger with two countercurrent cells with actuator faults.

Keywords: LPV Systems, Generalized dynamic observer, Lipschitz nonlinearities, Descriptor systems.

1. INTRODUCTION

In modern control systems, automation is essential, but it can be vulnerable to sensor and actuator faults. Neglecting to promptly detect and address these faults can lead to various consequences, including plant shutdowns and safety risks Severson et al. (2016). Another significant challenge is the difficulty in measuring system state variables, or the high cost of obtaining suitable sensors.

Descriptor systems, also called generalized, singular, or algebro-differential systems, involve dynamic and algebraic equations. They appear naturally in fields like power systems, electrical networks, and chemical processes. Engineers frequently apply them to model complex systems and use them in various fault detection and isolation (FDI) methods. For nonlinear systems: A method for systematically designing unknown input observers (UIO) for fault estimation is introduced in Venkateswaran and Kravaris (2023). A design of robust residual generator to detect, isolate and identify faults in nonlinear systems is presented in Farooq and Abid (2015). For linear systems: autors in Yeu et al. (2005) present an unknown input observer for fault detection (FD), isolation and reconstruction problem in descriptor systems. Reference Osorio-Gordillo et al. (2018) presents a generalized dynamic observer (GDO) structure for descriptor systems oriented to simultaneous states and faults estimation in descriptor systems. Moreover, these systems can now be used with linear parameter-varying (LPV) models, thanks to the partial linearity present in LPV models. In reference Hamdi et al. (2019) a polytopic sliding mode observer (PSMO) is constructed to simultaneous reconstruction of LPV descriptor system states and actuator faults. Reference Pérez-Estrada et al. (2018) presents a GDO structure for discrete-time LPV systems with unknown inputs and disturbances affecting both the states and outputs. Many fault detection methods usually operate dynamically, implying that fault estimation takes some time to converge. To address this inconvenience, works such as presented by Chen and Chowdhury (2007) introduces a learning observer (LO) that reduces the convergence time by incorporating a delay in the estimated variables. This approach achieves high accuracy even in the presence of abrupt changes. Extentions of this work for FDI are presented in Jia et al. (2012); Chen et al. (2013).

The main motivation for this work is to extend the research presented in Zetina-Rios et al. (2021) considering a wider range of systems beyond LPV systems. This extension aims to capture more complex dynamics in nonlinear systems by employing descriptor nonlinear parameter-varying systems (D-NLPV). Furthermore, a novel GDO structure is proposed to estimate simultaneous variable states and actuator faults in these systems. A model of the heat exchanger with actuator faults is presented to prove the performance of the observer design.

Notation: The symbol $(*)$ denotes the transpose elements on the symmetric positions. The symbol $A^{\perp}$ denotes a maximal row rank matrix such that $A^{\perp}A = 0$, by convention $A^{\perp} = 0$ when A is of full row rank.

2. PRELIMINARIES

Consider the following D-NLPV system in its polytopic form.

$$E\dot{x}(t) = \sum_{i=1}^{k} \mu_i(\varrho(t))\Big[A_i x(t) + B_i u(t) + G f_a(t) + D_i f(t, F_L x)\Big], \quad y(t) = Cx(t), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the known input, $y(t) \in \mathbb{R}^p$ is the measurement output vector and $f_a(t) \in \mathbb{R}^{n_{fa}}$ is the actuator fault vector. Matrix $E \in \mathbb{R}^{n\times n}$ could be singular. $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$ and $D_i \in \mathbb{R}^{n\times n_f}$ are real matrices and $f(t, F_L x)$ is a nonlinearity satisfying the Lipschitz constraint $||\Delta_f(t)|| \leq \lambda||F_L(x_1 - x_2)||$ where $\Delta_f(t) = f(t, F_L x_1) - f(t, F_L x_2)$, λ is a known Lipschitz constant and F_L is real matrix of appropriate dimension.

Let $rank(E) = r < n$ and $E^{\perp} \in \mathbb{R}^{s\times n}$ be a full row matrix such that $E^{\perp}E = 0$, in this case $s = n - r$.

Consider $\mu_i(\varrho(t))$ as the membership functions formed with known variant parameters $\varrho(t) \in \mathbb{R}^l$. The membership functions have the following properties:

$$\sum_{i=1}^{k} \mu_i(\varrho(t)) = 1, \quad \mu_i(\varrho(t)) \geq 0, \tag{2}$$

for $i = 1, \dots, k = 2^l$.

The following definitions and theorem will be used in the sequel of the paper.

Definition 1 (Yip and Sincovec (1981b)). For a linear descriptor system described by the pair (E, A_i), there exist a unique solution if there exist a constant scalar $s \in \mathbb{C}$ such that

$$det(sE - A_i) \neq 0, \tag{3}$$

Definition 2 (Yip and Sincovec (1981a)). If the state response of system (1), starting from an arbitrary initial state $x(0)$, does not contain impulse terms, then system (1) or the pair (E, A_i) $\forall i = 1, \dots, k$ is called impulse-free.

Definition 3 (Darouach (2009)). The following condition is a generalization of the impulse observability

$$\text{rank}\begin{bmatrix} E & A_i \\ 0 & C \\ 0 & E \end{bmatrix} = \text{rank}(E) + n, \ \ \forall i = 1, \dots, k. \tag{4}$$

Definition 4 (Duan (2010)). The descriptor system (1) is R-observable if and only if

$$\text{rank}\begin{bmatrix} sE - A_i \\ C \end{bmatrix} = n, \forall s \in \mathbb{C}, s \text{ finite}, \ \ \forall i = 1, \dots, k. \tag{5}$$

Assumption 1. It is assumed that system (1) is regular, Impulse observable and Reachable-observable.

Assumption 2. It is assumed that system rank(B_i) = rank$[B_i \ G]$.

Assumption 3. The actuator fault behavior is assumed to be constant, i.e. $\dot{f}_a(t) = 0$.

Lemma 2 (Xu (2002)). Let $\mathcal{M}$ and $\mathcal{N}$ be two constant matrices of appropriate dimensions. Then, the following inequality:

$$\mathcal{M}^T\mathcal{N} + \mathcal{N}^T\mathcal{M} \leq \gamma\mathcal{M}^T\mathcal{M} + \frac{1}{\gamma}\mathcal{N}^T\mathcal{N} \tag{6}$$

holds for any scalar $\gamma > 0$.

3. PROBLEM STATEMENT.

Let us consider the following generalized nonlinear observer for system (1)

$$\dot{\zeta}(t) = \sum_{i=1}^{k} \mu_i(\varrho(t))\Big[N_i\zeta(t) + H_i v(t) + F_i y(t) + J_i u(t) + TD_i f(t, F_L\hat{x}) + TG\hat{f}_a(t)\Big], \tag{7}$$

$$\dot{v}(t) = \sum_{i=1}^{k} \mu_i(\varrho(t))\Big[S_i\zeta(t) + L_i v(t) + M_i y(t)\Big], \tag{8}$$

$$\hat{x}(t) = P\zeta(t) + Qy(t), \tag{9}$$

$$\hat{f}_a(t) = \hat{f}_a(t-\tau) + \Phi_{fa}(C\hat{x}(t) - y(t)), \tag{10}$$

where τ denotes the sampling time interval, $\zeta(t) \in \mathbb{R}^{q_0}$ represents the state vector of the observer, $v(t) \in \mathbb{R}^{q_1}$ is an auxiliar vector and $\hat{x}(t) \in \mathbb{R}^n$ is the estimate of $x(t)$. The matrices N_i, H_i, F_i, S_i, L_i, M_i, J_i, P, Q and T are unknown matrices of appropriate dimensions, which must be determined such that $\hat{x}(t)$ and $\hat{f}_a(t)$ converges asymptotically to $x(t)$ and $f_a(t)$, respectively.

Let a matrix $T \in \mathbb{R}^{q_0\times n}$ to consider the following transformed error

$$\epsilon(t) = \zeta(t) - TEx(t), \tag{11}$$

whose derivative is

$$\dot{\epsilon}(t) = \sum_{i=1}^{k} \mu_i(\varrho(t))\Big[N_i\epsilon(t) + H_i v(t) + (N_i TE - TA_i + F_i C)x(t) + (J_i - TB_i)u(t) + TGe_f(t) + TD_i\Delta_f(t)\Big] \tag{12}$$

where $e_f(t) = \hat{f}_a(t) - f_a(t)$ and $\Delta_f(t) = f(t, F_L\hat{x}) - f(t, F_L x)$. Equation (12) is independent of $x(t)$ and $u(t)$, if the following equations are satisfied:

$$(a) \ \ N_i TE + F_i C - TA_i = 0, \tag{13}$$

$$(b) \ \ J_i = TB_i. \tag{14}$$

Then equation (12) becomes:

$$\dot{\epsilon}(t) = \sum_{i=1}^{k} \mu_i(\varrho(t))\Big[N_i\epsilon(t) + H_i v(t) + TGe_f(t) + TD_i\Delta_f(t)\Big]. \tag{15}$$

By using equation (11), equations (8) and (9) can be written as

$$\dot{v}(t) = \sum_{i=1}^{k} \mu_i(\varrho(t))\Big[S_i\epsilon(t) + L_i v(t) + (S_i TE + M_i C)x(t)\Big], \tag{16}$$

$$\hat{x}(t) = P\epsilon(t) + (PTE + QC)x(t), \tag{17}$$

Now if the following conditions are satisfied

$$(c) \ \ S_i TE + M_i C = 0, \tag{18}$$

$$(d) \ \ PTE + QC = I_n, \tag{19}$$

then, equation (16) becomes

$$\dot{v}(t) = \sum_{i=1}^{k} \mu_i(\varrho(t)\Big[S_i\epsilon(t) + L_i v(t)\Big], \tag{20}$$

and the state estimation error becomes

$$\hat{x}(t) - x(t) = e(t) = P\epsilon(t). \tag{21}$$

If conditions *(a)–(d)* are satisfied, the following observer error dynamics equation is obtained from (15) and (20)

$$\dot{\varphi}(t) = \sum_{i=1}^{k} \mu_i(\varrho(t))\Big[\mathbb{A}_i\varphi(t) + \mathbb{B}e_f(t) + \mathbb{C}_i\Delta_f(t)\Big], \tag{22}$$

where $\varphi(t) = \begin{bmatrix}\epsilon(t)\\ v(t)\end{bmatrix}$, $\mathbb{A}_i = \begin{bmatrix}N_i & H_i\\ S_i & L_i\end{bmatrix}$, $\mathbb{B} = \begin{bmatrix}TG\\ 0\end{bmatrix}$ and $\mathbb{C}_i = \begin{bmatrix}TD_i\\ 0\end{bmatrix}$.

In this case, if $e_f(t) = 0$, $\Delta_f(t) = 0$ and $\begin{bmatrix}N_i & H_i\\ S_i & L_i\end{bmatrix}$ is stable, then $lim_{t\to\infty}\ e(t) = 0$.

Now, the problem of the GLDO (7)-(10) design is reduced to find matrices N_i, F_i, J_i, H_i, L_i, M_i, S_i, P, Q, T and Φ_{fa}, such that the error dynamic (18) is asymptotically stable.

4. OBSERVER DESIGN

This section will be devoted to the parameterization of all the matrices of the observer

4.1 *Observer parameterization*

The parameterization can be obtained by first considering constraints *(c)* and *(d)* which can be written as

$$\begin{bmatrix}S_i & M_i\\ P & Q\end{bmatrix}\begin{bmatrix}TE\\ C\end{bmatrix} = \begin{bmatrix}0\\ I_n\end{bmatrix}, \tag{23}$$

the necessary and sufficient condition for (23) to have a solution is

$$\text{rank}\begin{bmatrix}TE\\ C\\ I_n\end{bmatrix} = \text{rank}\begin{bmatrix}TE\\ C\end{bmatrix} = n. \tag{24}$$

From equation (24) there always exist two matrices $T \in \mathbb{R}^{q_0\times n}$ and $K \in \mathbb{R}^{q_0\times p}$ such that

$$TE + KC = R, \tag{25}$$

where $R \in \mathbb{R}^{q_0\times n}$ is an arbitrary full row rank matrix such that rank $\begin{bmatrix}R\\ C\end{bmatrix} = n$.

Equation (25) can be also written as

$$\begin{bmatrix}T & K\end{bmatrix}\underbrace{\begin{bmatrix}E\\ C\end{bmatrix}}_{\Omega} = R, \tag{26}$$

and since rank $\begin{bmatrix}\Omega\\ R\end{bmatrix} = \text{rank}(\Omega)$, the solution of (26) is given by

$$[T\ \ K] = R\Omega^{+} - Z_1(I_{n+p} - \Omega\Omega^{+}), \tag{27}$$

where Ω^{+} denotes the generalized inverse matrix such that $\Omega\Omega^{+}\Omega = \Omega$. Therefore

$$T = \underbrace{R\Omega^{+}\begin{bmatrix}I_n\\ 0\end{bmatrix}}_{T_1} - Z_1\underbrace{(I_{n+p} - \Omega\Omega^{+})\begin{bmatrix}I_n\\ 0\end{bmatrix}}_{T_2}, \tag{28}$$

$$K = \underbrace{R\Omega^{+}\begin{bmatrix}0\\ I_p\end{bmatrix}}_{K_1} - Z_1\underbrace{(I_{n+p} - \Omega\Omega^{+})\begin{bmatrix}0\\ I_p\end{bmatrix}}_{K_2}, \tag{29}$$

where Z_1 is an arbitrary matrix. By replacing $TE = R - KC$ into condition *(a)*, we obtain

$$N_iR + \underbrace{(F_i - N_iK)}_{\bar{K}_i}C = TA_i, \tag{30}$$

$$\begin{bmatrix}N_i & \bar{K}_i\end{bmatrix}\underbrace{\begin{bmatrix}R\\ C\end{bmatrix}}_{\Sigma} = TA_i, \tag{31}$$

the general solution to equation (31) is given by

$$\begin{bmatrix}N_i & \bar{K}_i\end{bmatrix} = TA_i\Sigma^{+} - Y_{1i}(I_{q_o+p} - \Sigma\Sigma^{+}), \tag{32}$$

by replacing T from (28) into (32) we have

$$\begin{bmatrix}N_i & \bar{K}_i\end{bmatrix} = T_1A_i\Sigma^{+} - Z_1T_2A_i\Sigma^{+} - Y_{1i}(I_{q_0+p} - \Sigma\Sigma^{+}),$$

$$N_i = N_{1i} - Z_1N_{2i} - Y_{1i}N_3, \tag{33}$$

$$\bar{K}_i = \bar{K}_{1i} - Z_1\bar{K}_{2i} - Y_{1i}\bar{K}_3, \tag{34}$$

where $N_{1i} = T_1A_i\Sigma^{+}\begin{bmatrix}I_{q_0}\\ 0\end{bmatrix}$, $N_{2i} = T_2A_i\Sigma^{+}\begin{bmatrix}I_{q_0}\\ 0\end{bmatrix}$, $N_3 = (I_{q_0+p} - \Sigma\Sigma^{+})\begin{bmatrix}I_{q_0}\\ 0\end{bmatrix}$, $\bar{K}_{1i} = T_1A_i\Sigma^{+}\begin{bmatrix}0\\ I_p\end{bmatrix}$, $\bar{K}_{2i} = T_2A_i\Sigma^{+}\begin{bmatrix}0\\ I_p\end{bmatrix}$, $\bar{K}_3 = (I - \Sigma\Sigma^{+})\begin{bmatrix}0\\ I_p\end{bmatrix}$and Y_{1i} is an arbitrary matrix of appropriate dimension.

Now, from (30) we can deduce the value of F_i as

$$F_i = \bar{K}_i + N_iK,$$

$$F_i = F_{1i} - Z_1F_{2i} - Y_{1i}F_3, \tag{35}$$

where $F_{1i} = T_1A_i\Sigma^{+}\begin{bmatrix}K\\ I_p\end{bmatrix}$, $F_{2i} = T_2A_i\Sigma^{+}\begin{bmatrix}K\\ I_p\end{bmatrix}$ and $F_3 = (I_{q_0+p} - \Sigma\Sigma^{+})\begin{bmatrix}K\\ I_p\end{bmatrix}$.

On the other hand, from equation (25) we get

$$\begin{bmatrix}TE\\ C\end{bmatrix} = \begin{bmatrix}I_{q_0} & -K\\ 0 & I_p\end{bmatrix}\Sigma, \tag{36}$$

by using (23) and (36) we obtain

$$\begin{bmatrix}S_i & M_i\\ P & Q\end{bmatrix}\begin{bmatrix}I_{q_0} & -K\\ 0 & I_{p,}\end{bmatrix}\Sigma = \begin{bmatrix}0\\ I_n\end{bmatrix}, \tag{37}$$

which leads the following solution

$$\begin{bmatrix}S_i & M_i\\ P & Q\end{bmatrix} = \left(\begin{bmatrix}0\\ I_n\end{bmatrix}\Sigma^{+} - \begin{bmatrix}Y_{2i}\\ Y_{3i}\end{bmatrix}(I_{q_0+p} - \Sigma\Sigma^{+})\right)\begin{bmatrix}I_{q_0} & K\\ 0 & I_p\end{bmatrix}, \tag{38}$$

where we have used the fact that

$$\begin{bmatrix}I_{q_0} & -K\\ 0 & I_p\end{bmatrix}^{-1} = \begin{bmatrix}I_{q_0} & K\\ 0 & I_p\end{bmatrix}, \tag{39}$$

and where Y_{2i} and Y_{3i} are arbitrary matrices of appropriate dimensions. From (38) we can deduce the general form of matrices S_i, M_i, P and Q as follows

$$S_i = -Y_{2i}N_3, \tag{40}$$

$$M_i = -Y_{2i}F_3, \tag{41}$$

$$P = P_1 - Y_3N_3, \tag{42}$$

$$Q = Q_1 - Y_3F_3, \tag{43}$$

where $F_3 = (I_{q_0+p} - \Sigma\Sigma^{+})\begin{bmatrix}K\\ I_p\end{bmatrix}$, $N_3 = (I_{q_0+p} - \Sigma\Sigma^{+})\begin{bmatrix}I_{q_0}\\ 0\end{bmatrix}$, $Q_1 = \Sigma^{+}\begin{bmatrix}K\\ I_p\end{bmatrix}$ and $P_1 = \Sigma^{+}\begin{bmatrix}I_{q_0}\\ 0\end{bmatrix}$.

Now, by using the value of matrices N_i, S_i and T given by (33), (40) and (28), respectively, the observer error dynamics (22) can be written as

$$\dot{\varphi}(t) = \sum_{i=1}^{k} \mu_i(\varrho(t)) \left[\bar{\mathbb{A}}_i \varphi(t) + \mathbb{B} e_f(t) + \mathbb{C}_i \Delta_f(t) \right], \tag{44}$$

where $\bar{\mathbb{A}}_i = \mathbb{A}_{1i} - \mathbb{Y}_i \mathbb{A}_2$, $\mathbb{A}_{1i} = \begin{bmatrix} N_{1i} - Z_1 N_{2i} & 0 \\ 0 & 0 \end{bmatrix}$, $\mathbb{A}_2 = \begin{bmatrix} N_3 & 0 \\ 0 & -I_{q_1} \end{bmatrix}$, $\mathbb{B} = \begin{bmatrix} T_1 G - Z_1 T_2 G \\ 0 \end{bmatrix}$, $\mathbb{C}_i = \begin{bmatrix} T_1 D_i - Z_1 T_2 D_i \\ 0 \end{bmatrix}$ and $\mathbb{Y}_i = \begin{bmatrix} Y_{1i} & H_i \\ Y_{2i} & L_{i,} \end{bmatrix}$,
and from (21) we have

$$e(t) = \mathbb{P} \varphi(t), \tag{45}$$

where $\mathbb{P} = [P_1 \ 0]$ and $Y_3 = 0$ for simplicity, without loss of generality. Considering equation (10), $\tilde{f}_a(t) = f_a(t-\tau) - f_a(t)$, $e_f(t-\tau) = \hat{f}_a(t-\tau) - f_a(t-\tau)$ and the definition of $e_f(t)$, we have

$$\begin{aligned} e_f(t) &= \hat{f}_a(t-\tau) + \Phi_{fa} C(\hat{x}(t) - x(t)) - f_a(t) \\ e_f(t) &= e_f(t-\tau) + \Phi_{fa} C e(t) + \tilde{f}_a(t) \end{aligned} \tag{46}$$

From Assumption 3 it is assumed that $\tilde{f}_a(t)$ can be made zero, so then, estimation error (46) can be expressed as

$$e_f(t) = e_f(t-\tau) + \mathbb{K} \varphi(t). \tag{47}$$

where $\mathbb{K} = \begin{bmatrix} \Phi_{fa} C P_1 & 0 \end{bmatrix}$.
The observer design is obtained from the determination of matrices Z_1 and $\mathbb{Y}_i$ such that system (44) is asymptotically stable.

4.2 ***Stability analysis of the observer***

This section is devoted to the stability analysis of equation (44).

Theorem 1. Under Assumption 1, there exist two parameter matrices Z_1 and $\mathbb{Y}_i$ such that system (44) is asymptotically stable if there exists a positive definite matrix $X = \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix} > 0$ such that the following LMIs are satisfied

$$\mathcal{C}^{T\perp} \begin{bmatrix} \Pi_{1i} + \Pi_2 & \Pi_3 \\ (*) & -\gamma I_{nf} \end{bmatrix} \mathcal{C}^{T\perp T} < 0, \tag{48}$$

$$-\gamma < 0, \tag{49}$$

$$\begin{bmatrix} -\eta I & \mathbb{B}^T X + \mathbb{K} \\ (*) & -I \end{bmatrix} < 0 \tag{50}$$

where

$$\begin{aligned} &\mathcal{C}^{T\perp} = \begin{bmatrix} \mathbb{A}_2^{T\perp} & 0 \\ 0 & I \end{bmatrix}, \Pi_2 = \gamma \lambda^2 \begin{bmatrix} P_1{}^T F_L^T F_L P_1 & 0 \\ 0 & 0, \end{bmatrix}, \Pi_3 = \begin{bmatrix} X_1 T_1 D_i - \\ X_z T_2 D_i \\ 0 \end{bmatrix} \\ &\Pi_{1i} = \begin{bmatrix} X_1 N_{1i} + N_{1i}^T X_1 - X_z N_{2i} - N_{2i}^T X_z^T & 0 \\ 0 & 0 \end{bmatrix}, \end{aligned} \tag{51}$$

in this case $\eta > 0$, matrix $Z_1 = X_1^{-1} X_z$ and parameter matrix $\mathit{\Phi}_i^T$ is obtained as follows

$$\mathit{\Phi}_i^T = \mathcal{B}_r{}^+ \mathcal{K}_i \mathcal{C}_l{}^+ + \mathcal{Z} - \mathcal{B}_r{}^+ \mathcal{B}_r \mathcal{Z} \mathcal{C}_l \mathcal{C}_l{}^+, \tag{52}$$

where

$$\mathbb{Y}_i = (\mathit{\Phi}_i X_1^{-1})^T, \tag{53}$$

$$\mathcal{K}_i = -\mathcal{R}^{-1} \mathcal{B}_l{}^T \mathcal{V}_i \mathcal{C}_r{}^T (\mathcal{C}_r \mathcal{V}_i \mathcal{C}_r{}^T)^{-1} + \mathcal{S}_i^{\frac{1}{2}} \mathcal{L} (\mathcal{C}_r \mathcal{V}_i \mathcal{C}_r{}^T)^{-\frac{1}{2}}, \tag{54}$$

$$\mathcal{S}_i = \mathcal{R}^{-1} - \mathcal{R}^{-1} \mathcal{B}_l^T [\mathcal{V}_i - \mathcal{V}_i \mathcal{C}_r^T (\mathcal{C}_r \mathcal{V}_i \mathcal{C}_r{}^T)^{-1} \mathcal{C}_r \mathcal{V}_i] \mathcal{B}_l \mathcal{R}^{-1}, \tag{55}$$

$$\mathcal{V}_i = (\mathcal{B}_r \mathcal{R}^{-1} \mathcal{B}_l{}^T - \mathcal{D}_i)^{-1} > 0, \tag{56}$$

where matrices $\mathcal{Z}$, $\mathcal{L}$, $\mathcal{R}$ are arbitrary matrices such that $||\mathcal{L}|| < 1$ and $\mathcal{R} > 0$, with

$$\mathcal{D}_i = \begin{bmatrix} \Pi_{1i} + \Pi_2 & \Pi_3 \\ (*) & -\gamma I_{nf} \end{bmatrix},$$

$\mathcal{B} = \begin{bmatrix} -I \\ 0 \end{bmatrix}$ and $\mathcal{C} = \begin{bmatrix} \mathbb{A}_2 & 0 \end{bmatrix}$, such that there exist matrices $\mathcal{B}_l$, $\mathcal{B}_r$, $\mathcal{C}_l$ and $\mathcal{C}_r$, are such that $\mathcal{B} = \mathcal{B}_l \mathcal{B}_r$ and $\mathcal{C} = \mathcal{C}_l \mathcal{C}_r$, respectively.

Proof. Consider the following Lyapunov candidate function

$$V(\varphi(t)) = \varphi^T(t) X \varphi(t) + \int_{t-\tau}^{t} e_f^T(t) e_f(t) dt \tag{57}$$

such that

$$X = \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix} > 0, \quad X_1 = X_1^T. \tag{58}$$

The derivative of (57) along the trajectory of (44) gives

$$\begin{aligned} \dot{V}(\varphi(t)) \leq \sum_{i=1}^{k} \mu_i(\varrho(t)) \Big[&\varphi^T(t) X \dot{\varphi}(t) + \\ &\dot{\varphi}^T(t) X \varphi(t) + e_f^T(t) e_f(t) - e_f^T(t-\tau) e_f(t-\tau) \Big], \end{aligned} \tag{59}$$

replacing (44) and considering (47) in equation (59), we have

$$\begin{aligned} \dot{V}(\varphi(t)) \leq \sum_{i=1}^{k} \mu_i(\varrho(t)) \Big[&\varphi^T(t) (\bar{\mathbb{A}}_i^T X + X^T \bar{\mathbb{A}}_i) \varphi(t) + \\ &\varphi^T(t) X^T \mathbb{C}_i \Delta_f(t) + \Delta_f^T(t) \mathbb{C}_i^T X \varphi(t) + \\ &2\varphi^T(t) [X\mathbb{B} + \mathbb{K}^T] e_f(t-\tau) + \varphi^T(t) [2X\mathbb{B} + \mathbb{K}^T] \mathbb{K} \varphi(t) \Big], \end{aligned} \tag{60}$$

taking into account the following restriction

$$\mathbb{B}^T X = -\mathbb{K} \tag{61}$$

we can obtain the following equivalence

$$\varphi^T(t) [2X\mathbb{B} + \mathbb{K}^T] \mathbb{K} \varphi(t) = -\varphi(t) \mathbb{K}^T \mathbb{K} \varphi(t) < 0 \tag{62}$$

It is important to note that Equation (62) is formed by a quadratic term and by containing the negative sign, the condition of being defined negative will be fulfilled as long as $\varphi(t) \neq 0$.
Now, Equation (60) can be written as

$$\begin{aligned} \dot{V}(\varphi(t)) \leq \sum_{i=1}^{k} \mu_i(\varrho(t)) \Big[&\varphi^T(t) (\bar{\mathbb{A}}_i^T X + X^T \bar{\mathbb{A}}_i) \varphi(t) + \\ &\varphi^T(t) X^T \mathbb{C}_i \Delta_f(t) + \Delta_f^T(t) \mathbb{C}_i^T X \varphi(t) \Big], \end{aligned} \tag{63}$$

By using Lemma 2 we can obtain the following inequality

$$\begin{aligned} &\Delta_f^T(t) \mathbb{C}_i^T X \varphi(t) + \varphi^T(t) X^T \mathbb{C}_i \Delta_f(t) \leq \\ &\gamma \Delta_f^T(t) \Delta_f(t) + \frac{1}{\gamma} \varphi^T(t) X^T \mathbb{C}_i \mathbb{C}_i^T X \varphi(t), \end{aligned} \tag{64}$$

and from the Lipschitz condition, we have

$$\Delta_f^T \Delta_f(t) \leq \gamma \lambda^2 \varphi^T(t) \mathbb{P}^T F_L^T F_L \mathbb{P} \varphi(t), \tag{65}$$

by inserting (64) and (65) into (63), we obtain

$$\begin{aligned} \dot{V}(\varphi(t)) \leq \sum_{i=1}^{k} \mu_i(\varrho(t)) \Big[&\varphi^T(t) \Big(\bar{\mathbb{A}}_i^T X + X^T \bar{\mathbb{A}}_i + \\ &\frac{1}{\gamma} X^T \mathbb{C}_i \mathbb{C}_i^T X + \gamma \lambda^2 \mathbb{P}^T F_L^T F_L \mathbb{P} \Big) \varphi(t) \Big]. \end{aligned} \tag{66}$$

Now, if the following LMI is satified then $\dot{V}(\varphi(t)) < 0$.

$$\bar{\mathbb{A}}_i^T X + X^T \bar{\mathbb{A}}_i + \frac{1}{\gamma} X^T \mathbb{C}_i \mathbb{C}_i^T X + \gamma \lambda^2 \mathbb{P}^T F_L^T F_L \mathbb{P} < 0, \tag{67}$$

By using the Schur complement and replacing matrix $\bar{\mathbb{A}}_i$ in inequality (67), we obtain

$$\left[\begin{array}{c|c} X(\mathbb{A}_1 - \mathbb{Y}\mathbb{A}_2) + (\mathbb{A}_1 - \mathbb{Y}\mathbb{A}_2)^T X + & \\ \gamma\lambda^2 \mathbb{P}^T F_L^T F_L \mathbb{P} & X\mathbb{C}_i \\ \hline (*) & -\gamma I_{nf} \end{array}\right] < 0, \tag{68}$$

Now, by inserting the values of matrices $\mathbb{A}_{1i}$, $\mathbb{P}$ and X we obtain

$$\begin{bmatrix} \Pi_{1i} + \Pi_2 - \mathbb{A}_2^T \Phi_i - \Phi_i^T \mathbb{A}_2 & \Pi_3 \\ (*) & -\gamma I_{nf} \end{bmatrix} < 0, \tag{69}$$

where $\Phi_i = \mathbb{Y}_i^T X$ and $X_z = X_1 Z_1$. Π_{1i}, Π_2 and Π_3 are defined in (51). Inequality (69) can also be rewritten as

$$\mathcal{B}\Phi_i^T \mathcal{C} + (\mathcal{B}\Phi_i^T \mathcal{C})^T + \mathcal{D}_i < 0, \tag{70}$$

where

$$\mathcal{D}_i = \begin{bmatrix} \Pi_{1i} + \Pi_2 & \Pi_3 \\ (*) & -\gamma I_{nf} \end{bmatrix}, \ \mathcal{B} = \begin{bmatrix} -I \\ 0 \end{bmatrix} \text{ and } \mathcal{C} = \begin{bmatrix} \mathbb{A}_2 & 0 \end{bmatrix}.$$

According to Lemma 1, inequality (70) is satisfied if and only if the following inequalities verified

$$\mathcal{C}^{T\perp} \mathcal{D}_i \mathcal{C}^{T\perp T} < 0, \tag{71}$$

$$\mathcal{B}^{\perp} \mathcal{D}_i \mathcal{B}^{\perp T} < 0, \tag{72}$$

the inequalities (71) and (72) are equivalent to (73) and (74) respectively

$$\mathcal{C}^{T\perp} \begin{bmatrix} \Pi_{1i} + \Pi_2 & \Pi_3 \\ (*) & -\gamma I_{nf} \end{bmatrix} \mathcal{C}^{T\perp T} < 0, \tag{73}$$

$$-\gamma I_{nf} < 0. \tag{74}$$

with $\mathcal{B}^{\perp} = [0 \ I]$ and $\mathcal{C}^{T\perp} = \begin{bmatrix} \mathbb{A}_2^{T\perp} & 0 \\ 0 & I \end{bmatrix}$. If condition (61) is satisfied, conditions (73) and (74) can be solved using a standard tool for linear matrix inequalities (LMIs). However, condition (61) is a matrix equality to solve it, it can be rewritten as (Jia et al., 2016)

$$(\mathbb{B}^T X + \mathbb{K})(\mathbb{B}^T X + \mathbb{K})^T < \eta^2 I \tag{75}$$

where η is a positive scalar. By using the Schur complement lemma (Boyd et al., 1994), equation (75) can be written as

$$\begin{bmatrix} -\eta^2 I & \mathbb{B}^T X + \mathbb{K} \\ (*) & -I \end{bmatrix} < 0 \tag{76}$$

The design problem can be simplified by considering a scalar $\eta > 0$ and $\gamma > 0$ to solve the inequalities (48), (49), and (50), resulting in a positive definite matrix X. The parameter matrix $\mathbb{Y}_i$ can be obtained as (53), which completes the proof of the theorem. ■

□

5. APPLICATION TO HEAT EXCHANGER

This section is devoted to the performance analysis of the observer (7)-(10) applied to the heat exchanger model.

5.1 Model of the heat exchanger

To show the effectiveness of the observer, a heat exchanger with two countercurrent cells presented in Hidalgo Reyes (2008) is considered. The equations that represent the energy balance are given by

$$\begin{aligned}
\frac{dT_{1,c}}{dt} &= \frac{2v_c}{V_c}(T_{2,c} - T_{1,c}) + \frac{UA}{\rho_c C\rho_c V_c}\Delta T_1 \\
\frac{dT_{1,h}}{dt} &= \frac{2v_h}{V_h}(T_{0,h} - T_{1,h}) - \frac{UA}{\rho_h C\rho_h V_h}\Delta T_1 \\
\frac{dT_{2,c}}{dt} &= \frac{2v_c}{V_c}(T_{3,c} - T_{2,c}) + \frac{UA}{\rho_c C\rho_c V_c}\Delta T_2 \\
\frac{dT_{2,h}}{dt} &= \frac{2v_h}{V_h}(T_{1,h} - T_{2,h}) - \frac{UA}{\rho_h C\rho_h V_h}\Delta T_2 \\
\Delta T_1 &= \frac{[T_{1,h} - T_{2,c}] - [T_{0,h} - T_{1,c}]}{ln\left[\frac{[T_{1,h} - T_{2,c}]}{[T_{0,h} - T_{1,c}]}\right]} \\
\Delta T_2 &= \frac{[T_{2,h} - T_{3,c}] - [T_{1,h} - T_{2,c}]}{ln\left[\frac{[T_{2,h} - T_{3,c}]}{[T_{1,h} - T_{2,c}]}\right]} \\
Q_1 &= UA\Delta T_1 \\
Q_2 &= UA\Delta T_2
\end{aligned} \tag{77}$$

$T_{3,c}(t)$ and $T_{0,h}(t)$ represent the inlet temperatures for cold and hot water, respectively. $T_{1,c}(t)$ and $T_{2,h}(t)$ indicate the outlet temperatures for cold and hot water, respectively. The rate of heat flow across the solid-fluid interface is Q_1 and Q_2. V_c is the volume in external side $(134.99 \times 10^{-6} m^3)$, V_h is the volume in the inner side $(15.512 \times 10^{-6} m^3)$, v_c is the flow in the cold stream (6.399×10^{-6}cm^3/min), $C\rho_c$ is the specific heat of cold water (4181.5 J/Kg°C), $C\rho_h$ is the specific heat of hot water (4196.5 J/Kg°C), ρ_c is the density of cold water 996.781Kg/m^3, ρ_h is the density of hot water (971.150Kg/m^3), A is the heat transfer surface area (0.015387511m^2) and U is the global heat transfer coefficient (1400W/m^2).

5.2 NLPV system formulation

Following, the state space formulation for the heat exchanger is presented.

$$\underbrace{\begin{bmatrix} 1&0&0&0&0&0 \\ 0&1&0&0&0&0 \\ 0&0&1&0&0&0 \\ 0&0&0&1&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \end{bmatrix}}_{E} \underbrace{\begin{bmatrix} \dot{T}_{1,c}(t) \\ \dot{T}_{1,h}(t) \\ \dot{T}_{2,c}(t) \\ \dot{T}_{2,h}(t) \\ \dot{Q}_1(t) \\ \dot{Q}_2(t) \end{bmatrix}}_{\dot{x}(t)} = \underbrace{\begin{bmatrix} 0 & 0 \\ \frac{2v_h}{V_h} & 0 \\ 0 & \frac{2v_c}{V_c} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} T_{0,h} \\ T_{3,c} \end{bmatrix}}_{u(t)} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \frac{2v_c}{V_c} \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{G} f_a(t)$$

$$\underbrace{\begin{bmatrix} -\frac{2v_c}{V_c} & 0 & \frac{2v_c}{V_c} & 0 & \frac{1}{C\rho_c\rho_c V_c} & 0 \\ 0 & -\frac{2v_h(t)}{V_h} & 0 & 0 & -\frac{1}{C\rho_c\rho_c V_c} & 0 \\ 0 & 0 & -\frac{2v_c}{V_c} & 0 & 0 & \frac{1}{C\rho_c\rho_c V_c} \\ 0 & \frac{2v_h(t)}{V_h} & 0 & -\frac{2v_h(t)}{V_h} & 0 & -\frac{1}{C\rho_c\rho_c V_c} \\ 0 & 0 & 0 & 0 & -\frac{1}{UA} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{UA} \end{bmatrix}}_{A(v_h(t))} x(t) +$$

$$\underbrace{\begin{bmatrix} 0&0 \\ 0&0 \\ 0&0 \\ 0&0 \\ 1&0 \\ 0&1 \end{bmatrix}}_{D} \underbrace{\begin{bmatrix} \Delta T_1 \\ \Delta T_2 \end{bmatrix}}_{f(t,F_L\hat{x})} \text{ and } C = \begin{bmatrix} 0&0&1&0&0&0 \\ 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \end{bmatrix},$$

where the nonlinearity $f(t, F_L x)$ is considered as a Lipschitz function. The input signal is considered constant $u(t) = [77.9°C \ \ 21.2°C]$. The fault signal is defined as

$$f_a(t) = \begin{cases} 8°C & 80s \leq t \leq 120s \\ 0°C & \text{otherwise (s).} \end{cases} \quad (78)$$

The Lipschitz constant is selected as $\lambda = 100$ and $\tau = 0.00001$. The initial conditions for the nonlinear system are $x(0) = [10,\ 60,\ 15,\ 55]^T$, and for the observer (7) - (10) $\hat{x}(0) = [0,\ 0,\ 0,\ 0]^T$. The perfomance of the proposed observer is shown in Figures 1 - 2. The fault occurs at the time $t = 80s$. In a real context, the fault can be due to problems in the cold water inlet valve. From the simulation results shown in Figure 1, it can be observed that the proposed observer successfully estimates the simultaneous unmeasured states of the system, achieving a notable reduction in convergence time. The estimate fault manages to reduce the convergence time due to the learnig part as seen in Figure 2.

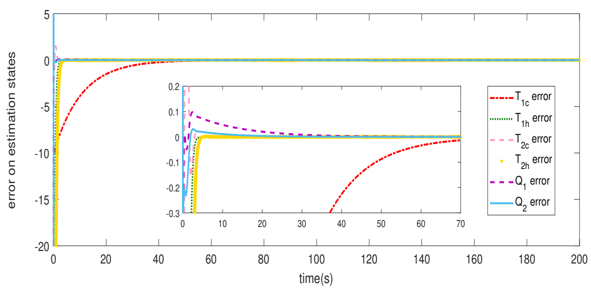

Figure 1. Convergence of the states estimation errors.

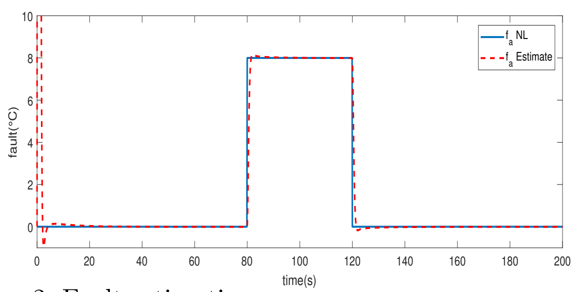

Figure 2. Fault estimation.

6. CONCLUSIONS

A GLO structure is synthesized to perform simultaneous estimation of state variables and actuator faults in D-NLPV systems. The advantages of the proposed observer are mainly: (i) This methodology can be applied to standard LTI formulations, considering them as particular cases; (ii) The use of the generalized structure improves robustness in estimation performances; (iii) The proposed GLO methodology is capable of reducing the convergence time for fault estimation due to the learning part. A heat exchanger in D-NLPV form was considered to show the performances of the observer designed.

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