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An LMI-Based Unknown Input Observer for Nonlinear Systems ⋆

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Abstract: This paper presents a novel unknown input observer for nonlinear systems which, based on as much time derivatives of the output as its relative degree plus one, is able to asymptotically reconstruct both the state and the unknown input for a class of nonlinear systems. Design conditions are expressed as linear matrix inequalities thanks to a convex representation of the nonlinear terms involved in the error system which, in turn, is obtained from a recently appeared factorization. Examples are provided to highlight the advantages of the proposal.

Keywords: Nonlinear Observer, Linear Matrix Inequality, Convex Embedding, Direct Lyapunov Method.

1. INTRODUCTION

The development of unknown input observers (UIOs) is a very active area due to its close relationship to fault estimation and fault-tolerant control (Alwi et al., 2012, Section 4.7), (Xu et al., 2019; Hosseini et al., 2020); indeed, fault diagnosis and isolation (Pertew et al., 2007; Quintana et al., 2019) and fault estimation (Tan and Edwards, 2002; Ichalal et al., 2014) can be viewed as determining an unknown input that may come from actuators or sensors (Zhang et al., 2002; Bedioui et al., 2019).

Design of UIOs is clearly a more challenging task than designing state observers: the latter relies upon measurements of the input and output while having full knowledge of the state dynamics; the former, on the other hand, has no access to the input signal and intends to reconstruct it without a model of its dynamics, i.e., no explicit equation governing the input behaviour is given. This is why some researchers have provided partial solutions based on further assumptions, e.g., an explicit equation governing the input dynamics or a finite number of non-zero derivatives of the input (assuming it to be polynomial as in Chua et al. (2020)). In most cases, the unknown input is estimated by solving a set of algebraic equations, thus circumventing the problem of lacking dynamic equations for it Ichalal and Mammar (2019).

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Estimating an unknown input faces deeper problems than those just mentioned, namely, the lack of full understanding of the meaning of classical concepts such as observability and detectability (both for linear (Bhattacharyya, 1978; Hautus, 1983) and nonlinear systems Pertew et al. (2005); Hammouri and Tmar (2010)), and the possibility of reproducing a given output with an observer that asymptotically has the same dynamics of the system while having different input and unobserved state signals. Indeed, in Moreno et al. (2014), the latter situation is referred to as *indistinguishability* which, up to our knowledge, has not been overcome.

Problem statement: Most proposals for UIOs are based on nominal linear systems; attempts to design them by fully exploiting nonlinear terms have been made by mimicking linear parameter varying (LPV) solutions (Ichalal and Mammar, 2015; Marx et al., 2019), which is a common practice within the Takagi-Sugeno (TS) framework (Marx et al., 2007; Orjuela et al., 2009; Rotondo et al., 2016).

Methodology: LPV and TS treatment of nonlinear systems, when appropriately made via exact convex rewriting (Bernal et al., 2022), have the advantage of leading to design conditions in the form of linear matrix inequalities (Boyd et al., 1994), which are efficiently solved via commercially available software (Gahinet et al., 1995); a very recent example of the latter is (Coutinho et al., 2022), where nonlinear systems, in contrast with LPV and TS approaches, are not approximate.

Contribution: This work follows the latter path for single-input single-output (SISO) distinguishable systems, while proposing an observer of the input dynamics based on its relationship with the $(\rho+1)$th-order time derivative of the output, where ρ is the relative degree of the latter w.r.t the input. An extended observer error system is constructed thanks to the factorization in Quintana et al. (2021) while convex treatment of nonlinearities allows LMI design conditions to be obtained.

Organization: Preliminaries on convex modelling, construction of error systems, and Levant's robuts differentiator, are shown in Section 2. The novel UIO is developed in Section 3; its LMI design conditions are deduced by means of convex properties and the direct Lyapunov method; its implementability requires the use of a finite-time convergent differentiator in the absence of noises. Two examples are presented in Section 4: a 1-input-2-output 4th-order single-link flexible joint robot manipulator and the Van Der Pol oscillator, both of which are subject to non-vanishing inputs and do not hold the decoupling condition between input and output (also known as unitary unknown input relative degree) that is usually required in former methodologies.

2. PRELIMINARIES

Convex rewriting of expressions, factorization of error signals and Levant's robust differentiators are the different elements on which rely our contribution; they are briefly presented in this section.

2.1 Convex rewriting of nonlinear expressions

Let $z(\cdot):\mathbb{R}^n\to\mathbb{R}$ be a well-defined function $\forall x\in\mathcal{C}\subset\mathbb{R}^n$; there exists z^0 and z^1 such that $z^0=\inf_{x\in\mathcal{C}}z(x)$ and $z^1=\sup_{x\in\mathcal{C}}z(x)$, that is, $z(x)\in[z^0,z^1]$. Defining

$$w_0(x)\equiv\frac{z^1-z(x)}{z^1-z^0},\ w_1(x)\equiv 1-w_0(x),$$

it can be verified that

$$z(x)=w_0(x)z^0+w_1(x)z^1=\sum_{i=0}^{1}w_i(x)z^i, \qquad (1)$$

is an identity that holds $\forall x\in\mathbb{R}^n$ while $w_0(x)\in[0,1]$ and $w_1(x)\in[0,1]$ are only guaranteed $\forall x\in\mathcal{C}$. Notice that the right-hand side of (1) is a convex sum of constant terms.

The previous concept can be extended to a collection of nonlinear expressions $z_1(x)$, $z_2(x)$, …, $z_r(x)$ as long as they are defined $\forall x\in\mathcal{C}\subset\mathbb{R}^n$. To do this, let us employ the following notation: let $z_i^0=\inf_{x\in\mathcal{C}}z_i(x)$, $z_i^1=\sup_{x\in\mathcal{C}}z_i(x)$, $i\in\{1,2,\dots,r\}$, $\forall x\in\mathcal{C}$; then, consider the expression $f(z_1,z_2,\dots,z_r)$ –it might be a scalar, vector, or matrix– so it can be rewritten as a convex sum of its bounds:

$$f(z_1,z_2,\dots,z_r)=\sum_{\mathbf{i}\in\mathbb{B}^r}\mathbf{w_i}(x)f_{\mathbf{i}},$$

$$w_0^i(x)\equiv\frac{z_i^1-z_i(x)}{z_i^1-z_i^0},\ w_1^i(x)\equiv 1-w_0^i(x),$$

where $\mathbb{B}=\{0,1\}$, $\mathbf{i}=(i_1,i_2,\dots,i_r)$, $\mathbf{w_i}(x)=w_{i_1}^1(x)$ $w_{i_2}^2(x)\cdots w_{i_r}^r(x)$, and $f_{\mathbf{i}}=f(z_1,z_2,\dots,z_r)|_{\mathbf{w_i}(x)=\mathbf{1}}$. To illustrate the methodology, let us consider

$$f(z_1,z_2)=\begin{bmatrix}z_1(x)+2 & z_1(x)z_2(x)\\ -5z_2(x) & -1\end{bmatrix}$$
$$=\sum_{i_1=0}^{1}\sum_{i_2=0}^{1}w_{i_1}^1(x)w_{i_2}^2(x)\begin{bmatrix}z_1^{i_1}+2 & z_1^{i_1}z_2^{i_2}\\ -5z_2^{i_2} & -1\end{bmatrix}=\sum_{\mathbf{i}\in\mathbb{B}^2}\mathbf{w_i}(x)f_{\mathbf{i}}$$

where $\mathbf{w_i}(x)=w_{i_1}^1(x)w_{i_2}^2(x)$, $w_0^i(x)=(z_i^1-z_i(x))/(z_i^1-z_i^0)$, $w_1^i(x)=1-w_0^i(x)$, $f_{\mathbf{i}}=\begin{bmatrix}z_1^{i_1}+2 & z_1^{i_1}z_2^{i_2}\\ -5z_2^{i_2} & -1\end{bmatrix}$, $\mathbf{i}=(i_1,i_2)$.

Since this work is concerned with estimation, distinguishing measurable and unmeasurable signals is critical; thus, when required, measurable nonlinear expressions will be cast as convex sums using the previous definitions while unmeasurable ones will adopt the following notation: given unmeasurable expressions $\zeta_1(x)$, $\zeta_2(x)$, …, $\zeta_s(x)$ that are well-defined $\forall x\in\mathcal{C}\subset\mathbb{R}^n$, define $\zeta_j^0\equiv\inf_{x\in\mathcal{C}}\zeta_j(x)$, $\zeta_j^1\equiv\sup_{x\in\mathcal{C}}\zeta_j(x)$, $j\in\{1,2,\dots,s\}$, $\forall x\in\mathcal{C}$; thus, an expression $f(\zeta_1,\zeta_2,\dots,\zeta_s)$ can be rewritten as:

$$f(\zeta_1,\zeta_2,\dots,\zeta_s)=\sum_{\mathbf{j}\in\mathbb{B}^s}\boldsymbol{\omega}_{\mathbf{j}}(x)f_{\mathbf{j}},$$

$$\omega_0^j(x)\equiv\frac{\zeta_j^1-\zeta_j(x)}{\zeta_j^1-\zeta_j^0},\ \omega_1^j(x)\equiv 1-\omega_0^j(x),$$

where $\mathbb{B}=\{0,1\}$, $\mathbf{j}=(j_1,j_2,\dots,j_s)$, $\boldsymbol{\omega}_{\mathbf{j}}(x)=\omega_{j_1}^1(x)$ $\omega_{j_2}^2(x)\cdots\omega_{j_s}^s(x)$, and $f_{\mathbf{j}}=f(\zeta_1,\zeta_2,\dots,\zeta_s)|_{\boldsymbol{\omega}_{\mathbf{j}}(x)=\mathbf{1}}$.

2.2 Factorization of error signals

Expressions of the form $f(\hat{x})-f(x)$ usually arise in the context of observer design; Lyapunov analysis employed in such cases is based on the estimation error $e=\hat{x}-x$; therefore, factorizing such signal from the aforementioned differences is useful, i.e., finding $F(x,\hat{x})$ such that $f(\hat{x})-f(x)=F(x,\hat{x})(\hat{x}-x)$. An explicit solution to this problem is given in Quintana et al. (2021) for expressions holding the differential mean value theorem. Instead of introducing the somewhat cumbersome notation in the referred work, we will show the main ideas by means of suitable examples.

Polynomial expressions are easily handled by means of addition and substraction of proper terms that successively lower the polynomial degree, thus allowing factorization of the error signal in an infinite number of ways; for instance, given $e_i=\hat{x}_i-x_i$, $i\in\{1,2,3\}$, $e=[e_1\ e_2\ e_3]^T$, we have:

$$\begin{aligned}
&\hat{x}_1\hat{x}_2\hat{x}_3-x_1x_2x_3=\hat{x}_1\hat{x}_2\hat{x}_3-\hat{x}_1x_2x_3+\hat{x}_1x_2x_3-x_1x_2x_3\\
&=\hat{x}_1(\hat{x}_2\hat{x}_3-x_2x_3)+x_2x_3(\hat{x}_1-x_1)\\
&=\hat{x}_1(\hat{x}_2\hat{x}_3-\hat{x}_2x_3+\hat{x}_2x_3-x_2x_3)+x_2x_3e_1\\
&=\hat{x}_1\left(\hat{x}_2\left(\hat{x}_3-x_3\right)+x_3\left(\hat{x}_2-x_2\right)\right)+x_2x_3e_1\\
&=\hat{x}_1\hat{x}_2e_3+\hat{x}_1x_3e_2+x_2x_3e_1=[x_2x_3\ \hat{x}_1x_3\ \hat{x}_1\hat{x}_2]e.
\end{aligned}$$

Another option is

$$\hat{x}_1\hat{x}_2\hat{x}_3 - x_1x_2x_3 = [x_2x_3 \ \hat{x}_1\hat{x}_3 \ \hat{x}_1x_2]\, e.$$

Non-polynomial expressions are treated in a similar way by means of its Taylor series up to any degree of accuracy.

2.3 Levant's robust differentiator

The Levant's robust differentiator for a function $f(t)$ has the following structure (Levant, 2003):

$$\begin{aligned}
\dot{v}_0 &= -\lambda_0 \left|v_0 - f(t)\right|^{\frac{s}{s+1}} \operatorname{sign}\left(v_0 - f(t)\right) + v_1 \\
\dot{v}_1 &= -\lambda_1 \left|v_1 - v_0\right|^{\frac{s-1}{s}} \operatorname{sign}\left(v_1 - v_0\right) + v_2 \\
&\ \ \vdots \\
\dot{v}_{s-1} &= -\lambda_{s-1} \left|v_{s-1} - v_{s-2}\right|^{\frac{1}{2}} \operatorname{sign}\left(v_{s-1} - v_{s-2}\right) + v_s \\
\dot{v}_s &= -\lambda_s \operatorname{sign}\left(v_s - v_{s-1}\right),
\end{aligned} \tag{2}$$

where parameters $\lambda_i > 0$, $i \in \{0, 1, \ldots, s\}$, are chosen according to the Lipschitz constants of the successive time derivatives of the function under consideration. In the absence of input noises, after a finite time of a transient process, $v_i = f^{(i)}(t)$, $i \in \{0, 1, \ldots, s\}$. The precision of the estimation as well as the convergence time can be improved by increasing the order s of the differentiator beyond the maximum order sought.

3. MAIN RESULTS

Consider a SISO nonlinear system of the form

$$\dot{x}(t) = f(x) + g(x)u(t), \ y(t) = h(x), \tag{3}$$

where $f(\cdot) : \mathbb{R}^n \mapsto \mathbb{R}^n$, $g(\cdot) : \mathbb{R}^n \mapsto \mathbb{R}^n$, and $h(\cdot) : \mathbb{R}^n \mapsto \mathbb{R}$ are sufficiently smooth vector fields in a domain of interest $\mathcal{C}_x \subset \mathbb{R}^n$, $x \in \mathbb{R}^n$ is the state, $u \in \mathcal{C}_u \subset \mathbb{R}$ the input, and $y \in \mathbb{R}$ the system output. Both the state x and the input u are assumed unknown.

Consider system (3) has relative degree ρ; this means (Khalil, 2014):

$$y^{(\rho)}(t) = L_f^{\rho}h(x) + L_gL_f^{\rho-1}h(x)u(t), \tag{4}$$

while $L_gL_f^{i-1}h(x) = 0$ for $i \in \{1, 2, \ldots, \rho-1\}$, $L_gL_f^{\rho-1}h(x) \neq 0$, where $L_f(\cdot)$ and $L_g(\cdot)$ denote the Lie derivatives of the arguments with respect to the referred vector fields.

From (4) we have that (omitting arguments when convenient):

$$\begin{aligned}
y^{(\rho+1)} &= \frac{d}{dt}\left(L_f^{\rho}h(x) + L_gL_f^{\rho-1}h(x)u(t)\right) \\
&= \frac{\partial}{\partial x}\left(L_f^{\rho}h(x)\right)\dot{x} + \left(\frac{\partial}{\partial x}\left(L_gL_f^{\rho-1}h(x)\right)\dot{x}\right)u + L_gL_f^{\rho-1}h(x)\dot{u} \\
&= \frac{\partial}{\partial x}\left(L_f^{\rho}h\right)(f+gu) + \left(\frac{\partial}{\partial x}\left(L_gL_f^{\rho-1}h\right)(f+gu)\right)u + L_gL_f^{\rho-1}h\dot{u} \\
&= L_f^{\rho+1}h + \left(L_gL_f^{\rho}h\right)u + \left(L_fL_gL_f^{\rho-1}h\right)u + \left(L_g^2L_f^{\rho-1}h\right)u^2 + L_gL_f^{\rho-1}h\dot{u},
\end{aligned}$$

which allows solving for $\dot{u}$ as follows:

$$\begin{aligned}
\dot{u}(t) &= \frac{y^{(\rho+1)} - L_f^{\rho+1}h - \left(L_gL_f^{\rho}h + L_fL_gL_f^{\rho-1}h\right)u - \left(L_g^2L_f^{\rho-1}h\right)u^2}{L_gL_f^{\rho-1}h} \\
&\equiv q\left(y^{(\rho+1)}, x, u\right).
\end{aligned} \tag{5}$$

As customary in the UIO design problem, x and u are unknown, but the structure of the vector fields f, g, h, and q is known; thus, an observer of the form

$$\begin{bmatrix}\dot{\hat{x}} \\ \dot{\hat{u}}\end{bmatrix} = \begin{bmatrix} f(\hat{x}) + g(\hat{x})\hat{u} \\ q\left(y^{(\rho+1)}, \hat{x}, \hat{u}\right)\end{bmatrix} + \begin{bmatrix} L_1(y,\hat{x},\hat{u}) \\ L_2(y,\hat{x},\hat{u})\end{bmatrix}(\hat{y} - y), \ \hat{y} = h(\hat{x}), \tag{6}$$

can be proposed, where $\hat{x} \in \mathbb{R}^n$ is the observer state, $\hat{u} \in \mathbb{R}$ is the observer input, $\hat{y} \in \mathbb{R}$ is the observer output, and $L_1(y, \hat{x}, \hat{u}) \in \mathbb{R}^n$, $L_2(y, \hat{x}, \hat{u}) \in \mathbb{R}$ are possibly nonlinear observer gains to be found.

Defining the observer errors $e_x \equiv \hat{x} - x$ and $e_u \equiv \hat{u} - u$ and using the factorization in Quintana et al. (2021), briefly described in the previous section, we have that the error dynamics are:

$$\begin{aligned}
\begin{bmatrix}\dot{e}_x \\ \dot{e}_u\end{bmatrix} &= \begin{bmatrix} f(\hat{x}) + g(\hat{x})\hat{u} - f(x) - g(x)u \\ q\left(y^{(\rho+1)}, \hat{x}, \hat{u}\right) - q\left(y^{(\rho+1)}, x, u\right)\end{bmatrix} + \begin{bmatrix} L_1(y,\hat{x},\hat{u}) \\ L_2(y,\hat{x},\hat{u})\end{bmatrix}(h(\hat{x}) - h(x)) \\
&= \begin{bmatrix} F_1(x, \hat{x}, u, \hat{u}) & F_2(x, \hat{x}, u, \hat{u}) \\ Q_1(y^{(\rho+1)}, x, \hat{x}, u, \hat{u}) & Q_2(y^{(\rho+1)}, x, \hat{x}, u, \hat{u})\end{bmatrix}\begin{bmatrix} e_x \\ e_u\end{bmatrix} \\
&\quad + \begin{bmatrix} L_1(y, \hat{x}, \hat{u}) \\ L_2(y, \hat{x}, \hat{u})\end{bmatrix}[H(x, \hat{x}) \ 0]\begin{bmatrix} e_x \\ e_u\end{bmatrix},
\end{aligned} \tag{7}$$

where $F_1(x,\hat{x}, u,\hat{u}) \in \mathbb{R}^{n\times n}$, $F_2(x,\hat{x}, u,\hat{u}) \in \mathbb{R}^{n\times 1}$, $H(x, \hat{x}) \in \mathbb{R}^{1\times n}$, $Q_1(y^{(\rho+1)},x,\hat{x},u,\hat{u}) \in \mathbb{R}^{1\times n}$, and $Q_2(y^{(\rho+1)},x,\hat{x},u,\hat{u}) \in \mathbb{R}^{1\times 1}$, must satisfy $F_1(x,\hat{x}, u,\hat{u})e_x + F_2(x,\hat{x},u,\hat{u})e_u = f(\hat{x}) + g(\hat{x})\hat{u} - f(x) - g(x)u$, $H(x, \hat{x})e_x = h(\hat{x}) - h(x)$, and $Q_1(y^{(\rho+1)},x,\hat{x},u,\hat{u})e_x + Q_2(y^{(\rho+1)},x, \hat{x},u,\hat{u})e_u = q\left(y^{(\rho+1)},\hat{x},\hat{u}\right) - q\left(y^{(\rho+1)},x,u\right)$.

Convex treatment of nonlinear expressions $F_1(\cdot)$, $F_2(\cdot)$, $Q_1(\cdot)$, $Q_2(\cdot)$, and $H(\cdot)$, comes now at hand to find possibly nonlinear gains $L_1(\cdot)$ and $L_2(\cdot)$ capable of guaranteeing $\lim_{t\to\infty} e(t)$, where $e(t) = [e_x^T(t) \ \ e_u(t)]^T$; shorthand notation $\mathrm{He}(M) = M + M^T$ will be employed:

Theorem 1. The origin of the nonlinear error system (7) is asymptotically stable if there exists matrices $P_1 \in \mathbb{R}^{n\times n}$, $P_2 \in \mathbb{R}^{n\times 1}$, $P_3 \in \mathbb{R}$, $N_1^{\mathbf{k}} \in \mathbb{R}^{n\times 1}$, $N_2^{\mathbf{k}} \in \mathbb{R}$, $\mathbf{k} \in \mathbb{B}^r$, such that the following LMIs hold $\forall\ \mathbf{i}, \mathbf{k} \in \mathbb{B}^r$ and $\forall\ \mathbf{j} \in \mathbb{B}^s$:

$$\begin{bmatrix} P_1 & P_2 \\ P_2^T & P_3\end{bmatrix} > 0, \ \mathrm{He}\left(\begin{bmatrix} P_1 & P_2 \\ P_2^T & P_3\end{bmatrix}\begin{bmatrix} F_1^{\mathbf{ij}} & F_2^{\mathbf{ij}} \\ Q_1^{\mathbf{ij}} & Q_2^{\mathbf{ij}}\end{bmatrix} + \begin{bmatrix} N_1^{\mathbf{k}} \\ N_2^{\mathbf{k}}\end{bmatrix}\left[H^{\mathbf{ij}} \ \ 0\right]\right) < 0, \tag{8}$$

provided that for $l \in \{1, 2\}$

$$\begin{aligned}
F_l(x,\hat{x}, u,\hat{u}) &= \sum_{\mathbf{i}\in\mathbb{B}^r}\sum_{\mathbf{j}\in\mathbb{B}^s} \mathbf{w_i}(y,\hat{x},\hat{u},y^{(\rho+1)})\boldsymbol{\omega}_{\mathbf{j}}(x,\hat{x}, u,\hat{u})F_l^{\mathbf{ij}}, \\
Q_l(y^{(\rho+1)},x,\hat{x},u,\hat{u}) &= \sum_{\mathbf{i}\in\mathbb{B}^r}\sum_{\mathbf{j}\in\mathbb{B}^s} \mathbf{w_i}(y,\hat{x},\hat{u},y^{(\rho+1)})\boldsymbol{\omega}_{\mathbf{j}}(x,\hat{x}, u,\hat{u})Q_l^{\mathbf{ij}}, \\
H(x,\hat{x}) &= \sum_{\mathbf{i}\in\mathbb{B}^r}\sum_{\mathbf{j}\in\mathbb{B}^s} \mathbf{w_i}(y,\hat{x},\hat{u},y^{(\rho+1)})\boldsymbol{\omega}_{\mathbf{j}}(x,\hat{x}, u,\hat{u})H^{\mathbf{ij}}, \\
L_l(y,\hat{x},\hat{u}) &= \sum_{\mathbf{k}\in\mathbb{B}^r} \mathbf{w_k}(y,\hat{x},\hat{u},y^{(\rho+1)})L_l^{\mathbf{k}},
\end{aligned}$$

where $\forall\ x,\ u,\ \hat{x},\ \hat{u}$, we have

$$\mathbf{w_i}(y,\hat{x},\hat{u},y^{(\rho+1)}) = w^1_{i_1}(y,\hat{x},\hat{u},y^{(\rho+1)}) \cdots w^r_{i_r}(y,\hat{x},\hat{u},y^{(\rho+1)})$$
$$\boldsymbol{\omega}_\mathbf{j}(x,\hat{x},u,\hat{u}) = \omega^1_{j_1}(x,\hat{x},u,\hat{u})\omega^2_{j_2}(x,\hat{x},u,\hat{u}) \cdots \omega^s_{j_s}(x,\hat{x},u,\hat{u}),$$

and $\forall x, \hat{x} \in \mathcal{C}_x$, $\forall u, \hat{u} \in \mathcal{C}_u$ we have

$$0 \le w^i_0(\cdot) \le 1,\ 0 \le w^i_1(\cdot) \le 1,\ i \in \{1, 2, \dots, r\},$$
$$0 \le \omega^j_0(\cdot) \le 1,\ 0 \le \omega^j_1(\cdot) \le 1,\ j \in \{1, 2, \dots, s\},$$

and $\forall\ \mathbf{k} \in \mathbb{B}^r$ we have

$$\begin{bmatrix} L^\mathbf{k}_1 \\ L^\mathbf{k}_2 \end{bmatrix} = \begin{bmatrix} P_1 & P_2 \\ P^T_2 & P_3 \end{bmatrix}^{-1} \begin{bmatrix} N^\mathbf{k}_1 \\ N^\mathbf{k}_2 \end{bmatrix}.$$

Moreover, any trajectory within any level set

$$\Omega_k = \left\{ \begin{bmatrix} e_x \\ e_u \end{bmatrix}^T \begin{bmatrix} P_1 & P_2 \\ P^T_2 & P_3 \end{bmatrix} \begin{bmatrix} e_x \\ e_u \end{bmatrix} \le k \right\} \subset \mathcal{C}_e \subset \mathbb{R}^{n+1},$$

with $k > 0$ and $\mathcal{C}_e$ being the region induced in e by $\mathcal{C}_x$ and $\mathcal{C}_u$, goes asymptotically to 0.

Proof. The first LMI expression in (8) guarantees

$$V(e) = \begin{bmatrix} e_x \\ e_u \end{bmatrix}^T \begin{bmatrix} P_1 & P_2 \\ P^T_2 & P_3 \end{bmatrix} \begin{bmatrix} e_x \\ e_u \end{bmatrix}$$

is a Lyapunov function candidate for the error dynamics in (7). The second LMI expression allows writing (omitting arguments when convenient)

$$\begin{aligned}
&\begin{bmatrix} e_x \\ e_u \end{bmatrix}^T \Bigg(\sum_{\mathbf{i},\mathbf{k}\in\mathbb{B}^r} \sum_{\mathbf{j}\in\mathbb{B}^s} \mathbf{w_i}(y,\hat{x},\hat{u},y^{(\rho+1)}) \boldsymbol{\omega}_\mathbf{j}(x,\hat{x},u,\hat{u}) \mathbf{w_k}(y,\hat{x},\hat{u},y^{(\rho+1)}) \\
&\quad \times \mathrm{He}\left(\begin{bmatrix} P_1 & P_2 \\ P^T_2 & P_3 \end{bmatrix} \begin{bmatrix} F^\mathbf{ij}_1 & F^\mathbf{ij}_2 \\ Q^\mathbf{ij}_1 & Q^\mathbf{ij}_2 \end{bmatrix} + \begin{bmatrix} N^\mathbf{k}_1 \\ N^\mathbf{k}_2 \end{bmatrix} \begin{bmatrix} H^\mathbf{ij} & 0 \end{bmatrix} \right) \Bigg) \begin{bmatrix} e_x \\ e_u \end{bmatrix} \\
&= \begin{bmatrix} e_x \\ e_u \end{bmatrix}^T \Bigg(\mathrm{He}\Bigg(\begin{bmatrix} P_1 & P_2 \\ P^T_2 & P_3 \end{bmatrix} \begin{bmatrix} \sum_{\mathbf{i}\in\mathbb{B}^r}\sum_{\mathbf{j}\in\mathbb{B}^s} \mathbf{w_i}\boldsymbol{\omega}_\mathbf{j} F^\mathbf{ij}_1 & \sum_{\mathbf{i}\in\mathbb{B}^r}\sum_{\mathbf{j}\in\mathbb{B}^s} \mathbf{w_i}\boldsymbol{\omega}_\mathbf{j} F^\mathbf{ij}_2 \\ \sum_{\mathbf{i}\in\mathbb{B}^r}\sum_{\mathbf{j}\in\mathbb{B}^s} \mathbf{w_i}\boldsymbol{\omega}_\mathbf{j} Q^\mathbf{ij}_1 & \sum_{\mathbf{i}\in\mathbb{B}^r}\sum_{\mathbf{j}\in\mathbb{B}^s} \mathbf{w_i}\boldsymbol{\omega}_\mathbf{j} Q^\mathbf{ij}_2 \end{bmatrix} \\
&\quad + \begin{bmatrix} P_1 & P_2 \\ P^T_2 & P_3 \end{bmatrix} \begin{bmatrix} \sum_{\mathbf{k}\in\mathbb{B}^r} \mathbf{w_k} L^\mathbf{k}_1 \\ \sum_{\mathbf{k}\in\mathbb{B}^r} \mathbf{w_k} L^\mathbf{k}_2 \end{bmatrix} \begin{bmatrix} \sum_{\mathbf{i}\in\mathbb{B}^r}\sum_{\mathbf{j}\in\mathbb{B}^s} \mathbf{w_i}\boldsymbol{\omega}_\mathbf{j} H^\mathbf{ij} & 0 \end{bmatrix} \Bigg) \Bigg) \begin{bmatrix} e_x \\ e_u \end{bmatrix} \\
&= \begin{bmatrix} e_x \\ e_u \end{bmatrix}^T \Bigg(\mathrm{He}\Bigg(\begin{bmatrix} P_1 & P_2 \\ P^T_2 & P_3 \end{bmatrix} \begin{bmatrix} F_1(x,\hat{x},u,\hat{u}) & F_2(x,\hat{x},u,\hat{u}) \\ Q_1(y^{(\rho+1)},x,\hat{x},u,\hat{u}) & Q_2(y^{(\rho+1)},x,\hat{x},u,\hat{u}) \end{bmatrix} \\
&\quad + \begin{bmatrix} P_1 & P_2 \\ P^T_2 & P_3 \end{bmatrix} \begin{bmatrix} L_1(y,\hat{x},\hat{u}) \\ L_2(y,\hat{x},\hat{u}) \end{bmatrix} [H(x,\hat{x})\ \ 0] \Bigg) \Bigg) \begin{bmatrix} e_x \\ e_u \end{bmatrix} = \dot{V}(e) < 0.
\end{aligned}$$

Thus, $V(e)$ is a valid Lyapunov function associated to the nonlinear error system (7), which establishes asymptotic stability of the origin. Now, positive-definiteness of $V(e)$ and $-\dot{V}(e)$ as well as its inclusion in the convex sums above depend on functions $\mathbf{w_i}(\cdot)$, $\mathbf{w_k}(\cdot)$, $\boldsymbol{\omega}_\mathbf{j}(\cdot)$, holding $0 \le \mathbf{w_i}(\cdot) \le 1$, $0 \le \mathbf{w_k}(\cdot) \le 1$, and $0 \le \boldsymbol{\omega}_\mathbf{j}(\cdot) \le 1$, as well as $\sum_{\mathbf{i}\in\mathbb{B}^r} \mathbf{w_i}(\cdot) = \sum_{\mathbf{k}\in\mathbb{B}^r} \mathbf{w_k}(\cdot) = \sum_{\mathbf{j}\in\mathbb{B}^s} \boldsymbol{\omega}_\mathbf{j}(\cdot) = 1$, which is ensured only in region $\mathcal{C}_e$ induced by x and u belonging to regions $\mathcal{C}_x$ and $\mathcal{C}_u$, respectively. Thus, trajectories belonging to any level set Ω_k as defined above go asymptotically to 0, which concludes the proof. □

Remark 2. Although conditions in Theorem 1 were developed for SISO systems, they can be easily generalized for multiple outputs by using all of them for observer feedback and one of them to propose input dynamics. See example 1.

4. EXAMPLES

Example 1: Consider the single-link flexible joint robot in Fan and Arcak (2003)

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{1}{J_m}(k_1(x_1-x_3)+k_2(x_1-x_3)^3) - \frac{B}{J_m}x_2 + \frac{K_\tau}{J_m}u \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= -\frac{1}{J_l}(k_1(x_1-x_3)+k_2(x_1-x_3)^3) - \frac{mgh}{Jl}\sin x_1
\end{aligned} \tag{9}$$

where x_1 and x_3 are the motor and link position, respectively; x_2 and x_3 are the motor and link velocities, respectively; $J_m = 3.7 \times 10^{-3}\mathrm{kgm}^2$ is the motor inertia, $J_l = 9.3 \times 10^{-3}\mathrm{kgm}^2$ is the link inertia, $2h = 3 \times 10^{-1}\mathrm{m}$ is the length of the link, $m = 0.21\mathrm{kg}$ is the mass of the link, $B = 4.6 \times 10^{-2}\mathrm{NmV}^{-1}$ is the viscous friction, $k_1 = k_2 = 1\mathrm{Nm\ rad}^{-1}$ are the torsional spring constant, and $K_\tau = 8 \times 10^{-2}\mathrm{NmV}^{-1}$ is the amplifier gain; the measurable output of the system $y = [x_1\ x_3]^T$, i.e., the motor and link position. Without loss of generality, let us assume u as the unknown input, this case contains as a particular one that presented in Wang et al. (2021). The relative degree of the output $y_1 = x_1$ w.r.t. u is $\rho = 2$, thus, taking the derivative $y^{(3)}_1$ and solving for $\dot{u}$, the input dynamics is

$$\begin{aligned}
\dot{u} &= \frac{k_1}{J_m}(x_2 - x_4) + \frac{B}{J_m}u + \frac{3k_2}{K_\tau}(x_3 - x_1)^2(x_2 - x_4) \\
&\quad - \frac{B}{J_m K_\tau}(Bx_2 - k_2(x_3 - x_1)^3 - k_1(x_3 - x_1)) + \frac{J_m}{K_\tau}y^{(3)}_1.
\end{aligned}$$

Now, mimicking the above system structure with the extended state $[x_1\ x_2\ x_3\ x_4\ u]^T$, the proposed observer (omitting arguments in the observer gain) is

$$\begin{aligned}
\dot{\hat{x}}_1 &= \hat{x}_2 + L_{11}(\hat{y} - y) \\
\dot{\hat{x}}_2 &= \frac{1}{J_m}(k_1(x_3-x_1)+k_2(x_3-x_1)^3) - \frac{B}{J_m}\hat{x}_2 + \frac{K_\tau}{J_m}\hat{u} + L_{12}e_y \\
\dot{\hat{x}}_3 &= \hat{x}_4 + L_{13}e_y \\
\dot{\hat{x}}_4 &= -\frac{1}{J_l}(k_1(x_3-x_1)+k_2(x_3-x_1)^3) - \frac{mgh}{J_l}\sin x_1 + L_{14}e_y \\
\dot{\hat{u}} &= \frac{k_1}{J_m}(\hat{x}_2 - \hat{x}_4) + \frac{B}{J_m}\hat{u} + \frac{J_m}{K_\tau}y^{(3)}_1 + \frac{3k_2}{K_\tau}(x_3-x_1)^2(\hat{x}_2 - \hat{x}_4) \\
&\quad - \frac{B}{J_m K_\tau}(B\hat{x}_2 - k_2(x_3-x_1)^3 - k_1(x_3-x_1)) + L_2 e_y.
\end{aligned}$$

which exploits the information available from the output; $\hat{x}_1$, $\hat{x}_2$, $\hat{x}_3$, $\hat{x}_4$, $\hat{u}$, $\hat{y}$ are the estimation of x_1, x_2, x_3, x_4, u, y, respectively; $L_1(\hat{x},y,\hat{u}) = [L^T_{11}(\cdot)\ L^T_{12}(\cdot)\ L^T_{13}(\cdot)\ L^T_{14}(\cdot)]^T$ and $L_2(\hat{x},y,\hat{u})$ are the observer gains to be designed such that $\lim_{t\to\infty} \hat{x} - x = 0$ and $\lim_{t\to\infty} \hat{u} - u = 0$; and $e_y = \hat{y} - y$ is the output error. Taking into account the factorization given in section 2.2, the nonlinear error system (7) is obtained with

$$F_1=\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & -\frac{B}{J_m} & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0\end{bmatrix}, F_2=\begin{bmatrix}0\\ \frac{K_\tau}{J_m}\\ 0\\ 0\end{bmatrix}, Q_2=\frac{B}{J_m}, H=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix},$$

$$Q_1(y)=\left[0 \;\; \tfrac{1}{K_\tau}(k_1+3k_2z_1-\tfrac{B^2}{J_m}) \;\; 0 \;\; -\tfrac{1}{K_\tau}(k_1+3k_2z_1)\right],$$

where $z_1=(x_3-x_1)^2$ is the sole nonlinearity in $Q_1(y)$. Taking the bounds $|x_3-x_1|\leq 2.8$ given in Yan and Edwards (2007), z_1 can be bounded as $z_1\in[0\;\; 7.84]$, thus enabling the convex modelling in section 2.1 with $w_0^1(z_1)=1-0.128z_1$ and $w_1^1(z_1)=0.128z_1$. The LMI conditions (8) are feasible. The observer gains L_1^k, L_2^k are

$$L_1^0=\begin{bmatrix}-2126 & 138\\ -66267 & -10244\\ 122 & -3109\\ 113 & -1669\end{bmatrix}, \; L_1^1=\begin{bmatrix}-2097 & 1331\\ -63583 & -10670\\ 126 & -3110\\ 114 & -1669\end{bmatrix},$$

$$L_2^0=[-162827\;\; 5809], \quad L_2^1=[-158051\;\; 5050],$$

$$P=\begin{bmatrix}409835 & 2122 & 2280 & 3784 & -3704\\ 2122 & 358 & -1353 & -95 & -172\\ 2280 & -1353 & 201319 & -119889 & 588\\ 3784 & -95 & -119889 & 224558 & 55\\ -3704 & -172 & 588 & 55 & 119\end{bmatrix}$$

For simulation purposes the unknown input was considered as

$$u(t)=\begin{cases}\begin{array}{c}0.5\sin t+0.2\sin 5t\\ +0.2\sin 10t+0.1\sin 20t,\end{array} & 3<t\leq 5\\ 0.01t, & 6<t\leq 10\\ -0.5, & 10<t\leq 15\\ \begin{array}{c}0.2(\sin t-0.2\sin 2t-0.25\cos 25t\\ +0.5\sin 10t+\cos 15t-1.5\sin 40t\\ -0.2\cos 30t-0.1\cos 60t),\end{array} & 20<t\leq 25,\\ 0, & \text{otherwise}\end{cases}$$

Figures 1-2 depicts the time evolution of the unmeasurable signals and their estimates; the observation task is effectively achieved. It is worth noticing that system (9) does not hold the relative degree condition $\text{rank}(CB)\neq\text{rank}(B)$, i.e., $\text{rank}(CB)=0$, which makes works as Yan and Edwards (2007); Marx et al. (2019); Ichalal and Guerra (2019) to be inapplicable; works as Chua et al. (2020); Tavasolipour et al. (2021) can be applied, however the estimation error is no guaranteed to be $\lim_{t\to\infty}e(t)=0$ because they use the $\mathcal{L}_2$-norm to attenuate the perturbation due to the uncoupled UI ($\text{rank}(CB)=0$); due to the non-low variation (non-vanishing derivative of the UI at the o-*th* order) nature of the UI, the example does not meet conditions required to employ the observers proposed in Guzman et al. (2021).

Example 2: Consider the forced van der Pol equation (Khalil, 2014, (A.13))

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}\frac{x_2}{\epsilon}\\ \epsilon(-x_1+x_2-\frac{1}{3}x_2^3+u)\end{bmatrix}, \quad y=x_1 \tag{10}$$

where $x=[x_1\; x_2]^T$ is the state vector, $u\in\mathbb{R}$ is the input, y is the measured output and $\epsilon=0.1$ is a parameter; let us to assumed u as the unknown input. The relative degree of the system is 2, thus, taking the derivative $y^{(3)}$ and solving for $\dot{u}$, the dynamic $\dot{u}=\epsilon(-u+x_1-x_2+x_2/\epsilon^2+4x_2^3/3-x_2^5/3+ux_2^2-x_1x_2^2)+y^{(3)}$ can be obtained. Considering the extended system with $[x_1\; x_2\; u]^T$ as the state, the unknown input observer (6) can be proposed with $g(\hat{x})=[0\;\; 1]^T$, $q(y^{(\rho+1)},\hat{x},\hat{u})=\epsilon(\hat{x}_1-\hat{u}-\hat{x}_2+\frac{\hat{x}_2}{\epsilon^2}+\frac{4\hat{x}_2^3}{3}-\frac{\hat{x}_2^5}{3}+\hat{u}\hat{x}_2^2-\hat{x}_1\hat{x}_2^2)+y^{(3)}$, and

$$f(\hat{x})=\begin{bmatrix}\hat{x}_2/\epsilon\\ \epsilon(-\hat{x}_1+\hat{x}_2-\frac{1}{3}\hat{x}_2^3)\end{bmatrix},$$

where $[\hat{x}_1\; \hat{x}_2\; \hat{u}]^T$ is the observer state; $L_1(\hat{x},y,\hat{u})\in\mathbb{R}^{2\times 1}$ and $L_2(\hat{x},y,\hat{u})\in\mathbb{R}$ are the nonlinear observer gains to be designed so that $\lim_{t\to\infty}\hat{x}-x=0$ and $\lim_{t\to\infty}\hat{u}-u=0$.

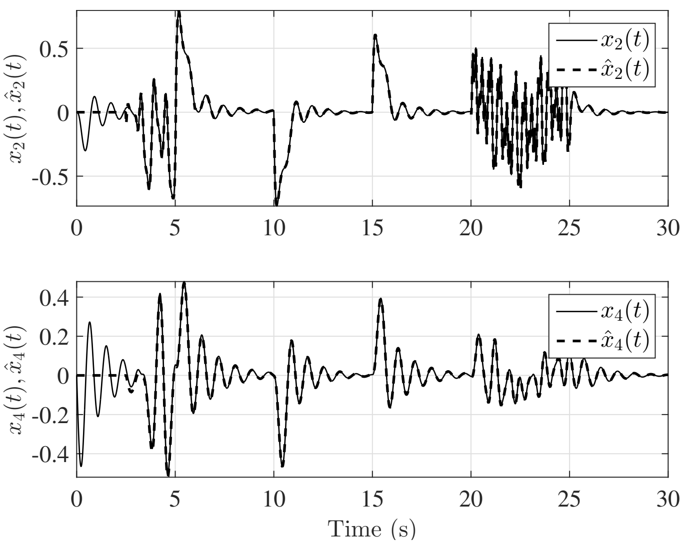

Fig. 1. Time evolution of the unmeasurable states x_2 and x_4 and their estimations $\hat{x}_2$ and $\hat{x}_4$ in Example 1.

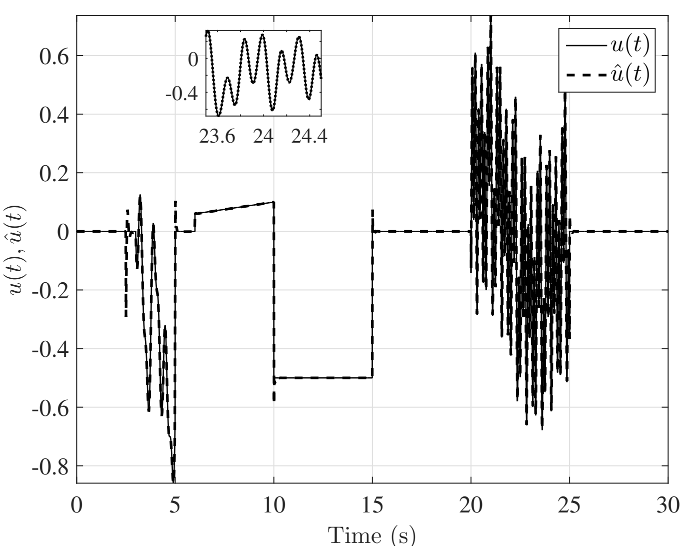

Fig. 2. Time evolution of the unknown input u and its estimation $\hat{u}$ in Example 1.

Employing the factorization in section 2.2, the nonlinear error system (7) can be obtained with

$$F_1(x,\hat{x},u)=\begin{bmatrix}0 & \frac{1}{\epsilon}\\ -\epsilon & \epsilon(1-\frac{\zeta_3}{3})\end{bmatrix},\quad F_2(x,\hat{x},u)=\begin{bmatrix}0\\ \epsilon\end{bmatrix},$$

$$Q_1(y^{(\rho+1)},x,\hat{x},u,\hat{u})=\begin{bmatrix}\epsilon(1-z_2^2) & \begin{matrix}\epsilon(\frac{1}{\epsilon^2}-1+\zeta_1 z_2+\zeta_1\zeta_2\\ -z_1 z_2-z_1\zeta_2+\frac{4\zeta_3}{3}-\frac{\zeta_4}{3})\end{matrix}\end{bmatrix}$$

$$Q_2(y^{(\rho+1)},x,\hat{x},u,\hat{u})=\epsilon(z_2^2-1)$$

where $z_1 = x_1$, $z_2 = \hat{x}_2$, $\zeta_1 = u$, $\zeta_2 = x_2$, $\zeta_3 = \hat{x}_2^2 + \hat{x}_2 x_2 + x_2^2$, and $\zeta_4 = \hat{x}_2^4 + \hat{x}_2^3 x_2 + \hat{x}_2^2 x_2^2 + \hat{x}_2 x_2^3 + x_2^4$. Considering the design region $\mathcal{C}_x = \{x \in \mathbb{R}^2 : |x_1| \le 5, |x_2| \le 2.5\}$ and $\mathcal{C}_u = \{u \in \mathbb{R} : |u| \le 0.4\}$ and mimicking them for the input and observer states, the non-constant terms are bounded into the regions as $z_1 \in [-5,5]$, $z_2 \in [-2,2]$, $\zeta_1 =\in [-0.4, 0.4]$, $\zeta_2 \in [-2.5, 2.5]$, $\zeta_3 \in [0, 18.75]$, and $\zeta_4 \in [0, 195.3125]$. As before, the convex model can be obtained employing the methodology in section 1. Using the matrices obtained in the latter, the LMI conditions in (8) are feasible; matrix P and 2 of the 8 gains $L^{\mathbf{k}} = [L_1^{\mathbf{k}}\ L_2^{\mathbf{k}}]$, $\mathbf{k} \in \mathbb{B}^3$ are

$$L^{000}=\begin{bmatrix}-9.2412\\ -21.1889\\ -583.6710\end{bmatrix},\ L^{010}=\begin{bmatrix}-9.2924\\ -21.3303\\ -591.5769\end{bmatrix}$$

$$P=\begin{bmatrix}351.2866 & -96.9064 & -0.6029\\ -96.9064 & 434.8538 & -8.9663\\ -0.6029 & -8.9663 & 0.3154\end{bmatrix}.$$

The unknown input employed in the simulation is

$$u=\begin{cases}0.3\sin 5t, & 5<t\le 10\\ 0.3\sin 3t, & 5<t\le 8\\ 0.01t, & 8<t\le 15\\ -0.2, & 15<t\le 20\\ -0.2\sin 10t+0.1\cos 4t+0.1\cos 15t, & 20<t\le 25\\ 0, & \text{otherwise.}\end{cases}$$

The simulation results are depicted in Figure 3-4; it can be seen that the observation task is achieved successfully for the states and the UI by the sole information on the output. As for the example above, works as Yan and Edwards (2007); Marx et al. (2019); Ichalal and Guerra (2019) are inapplicable; works as Chua et al. (2020); Tavasolipour et al. (2021) can be applied using the $\mathcal{L}_2$-norm to attenuate the perturbation due to the uncoupled UI ($\text{rank}(CB) = 0$); the UI in this example does not meet the conditions required to employ the observer proposed in Guzman et al. (2021).

5. CONCLUSION

A novel unknown input observer for nonlinear systems has been presented, whose structure is based on as much time derivatives of the output as its relative degree plus one. It has been shown that an error system can be obtained via a proper factorization, which is amenable to a convex form that, combined with the direct Lyapunov method, enables design conditions in the form of linear matrix inequalities to be found. Detailed examples have been provided that show the advantages of the proposal over former methodologies.

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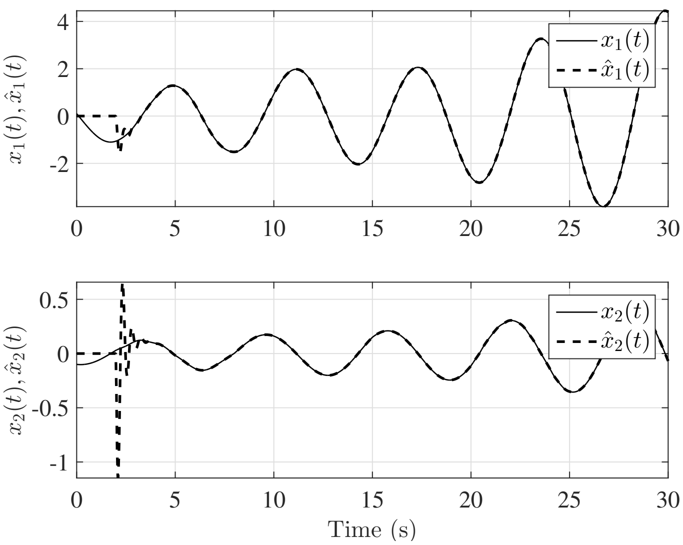

Fig. 3. Time evolution of the system states x_1 and x_2 and their estimations $\hat{x}_1$ and $\hat{x}_2$ in Example 2.

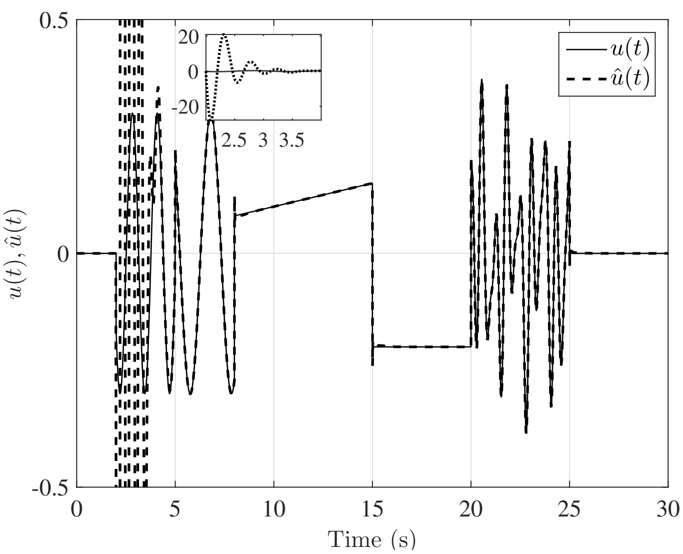

Fig. 4. Time evolution of the unknown input u and its estimation $\hat{u}$ in Example 2.

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