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On lossless negative imaginary property for the characterization of LCL filters ⋆

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Abstract: This paper presents a brief survey of some of the main results in the negative imaginary systems theory and its applications. In particular, the paper concentrates on the application of lossless negative imaginary property for the characterization of the LCL filter, widely used for grid-connected inverters. The characterization is established when the (all possible) LCL filter transfer functions satisfy the lossless negative imaginary conditions. The results are compared with the frequency response of the LCL filter transfer functions.

Keywords: lossless negative imaginary system, LCL filter, grid converter

1. INTRODUCTION

The positive real (PR) systems theory is one of the main concepts of systems and control theory, and in particular of passivity theory. The concept dates back to the early 1930s, for a summary of the historic and recent contributions in this area, we refer the readers to expository article Petersen and Lanzon (2010) and monograph Brogliato et al. (2007); given the extensive amount of contributions. A fruitful novel development has been the introduction of the notion of negative imaginary (NI) systems, see Petersen and Lanzon (2010); Mabrok et al. (2014) and the references cited therein.

Given the transfer function or state-space realization, relevant characteristics or properties of (physical) systems are presented using both system theories. For instance, some basic relationships between positive real, passive, and negative imaginary systems are the following:

- According to the Theorem 2.8 of Brogliato et al. (2007), a passive system will have a transfer function $h(s)$ which satisfies
$$|\angle H(j\omega)| \leq 90^\circ \quad \text{for all} \quad \omega \in [-\infty, +\infty]$$
- The Nyquist plot of $H(j\omega)$ lies entirely in the (closed) right half complex plane. In other words, the phase of a positive real transfer function lies in $[-\frac{\pi}{2}, \frac{\pi}{2}]$ rad, and that of an strictly positive real transfer function lies in $(-\frac{\pi}{2}, \frac{\pi}{2})$ rad.
- The phase of negative imaginary systems satisfies $\angle H(s) \in [-\pi, 0]$ rad. This is why some transfer functions like $\frac{1}{s}$ can be both NI and PR, i.e., its phase belongs to $[-\frac{\pi}{2}, 0]$ rad.

Additionally, the NI systems theory can deal with systems of relative degrees zero, one or two, which complements the PR systems theory that copes with systems of relative degrees zero or one, Brogliato et al. (2007).

An important subclass of the NI systems is the lossless negative imaginary systems, where for the continuous-time proper lossless negative imaginary systems, the definition was first proposed in Xiong et al. (2012) by restricting no poles at the origin and infinity, and a minimal state-space characterization of such systems was also developed in Xiong et al. (2012). After, the realization of continuous-time lossless negative imaginary systems was studied in Roa and Rapisarda (2012), where an algebraic approach was given. Finally, Lui and Xiong (2016) extended the definition of continuous-time lossless negative imaginary systems to non-proper case by allowing poles at the origin and infinity.

The NI systems theory is being used in a range of applications including modelling and (robust) control of undamped or lightly damped mechanical flexible structures, Petersen and Lanzon (2010); Mabrok et al. (2014). In electrical systems, similar RLC electrical circuit interpretations of the negative imaginary stability results were presented in Petersen (2015), where an undamped or lightly damped (lossless) RLC circuit exhibits an inherent resonance behaviour. Indeed, control challenges of LCL-type grid-connected inverter arise from the resonance problem, Xinbo et at. (2018).

The negative imaginary systems (NIS) theory is used in the control design of grid converters, particularity the proportional resonant controller in Sarkar et al. (2019) and Haque et al. (2020, 2022) where proposed as a second-order controller to control the load voltage of a single- and three-phase islanded micro-grid (MG) systems. In these MG systems, voltage source converters with LCL filters are required to convert the dc voltage to ac voltage to run the load. Whereas in Badal et al. (2020) a control system is carried out for the active damping of a grid-

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connected inverter with an LCL filter by means of a resonant controller. However, in the above references the lossless negative imaginary property of the LC- and LCL-filters was not studied. The main result of this paper is the particularization of the results in Xiong et al. (2012); Liu et al. (2020) to the case where transfer functions of the LCL filter meet the lossless negative imaginary condition.

The organization of the paper is as follows. Section II briefly recalls some basic concepts and results on negative imaginary transfer and lossless negative imaginary transfer functions. In Section III, Lossless Negative Imaginary Property of the LCL filter is established when its transfer functions satisfy the lossless negative imaginary condition. Validation through the frequency response is presented in Section IV. Finally, Section V concludes the paper.

2. PRELIMINARIES

In this section, for the sake of completeness we briefly present some concepts on (lossless) negative imaginary transfer functions, which will be used to develop the main results of this paper.

2.1 Negative imaginary systems with poles at the origin

We begin with a *generalized definition* of the negative imaginary property.

Definition 1. (Mabrok et al. (2014)) A square transfer function matrix $G(s)$ is *negative imaginary* (NI) if the following conditions are satisfied:

(1) $G(s)$ has no pole in $\mathrm{Re}[s] > 0$.
(2) For all $\omega > 0$ such that $j\omega$ is not a pole of $G(s)$, $j(G(j\omega) - G(j\omega)^*) \geq 0$.
(3) If $s = j\omega_0$ with $\omega_0 > 0$ is a pole of $G(s)$, then it is a simple pole and the residue matrix $K = \lim_{s \to j\omega_0}(s - j\omega_0)jG(s)$ is Hermitian and positive semidefinite.
(4) If $s = 0$ is a pole of $G(s)$, then $\lim_{s\to 0} s^k G(s) = 0$ for all $k \geq 3$ and $\lim_{s\to 0} s^2 G(s)$ is Hermitian and positive semidefinite.

Then, we define the following matrices which will be used in the stability conditions to be presented:

$$G_2 = \lim_{s\to 0} s^2 G(s), \quad G_1 = \lim_{s\to 0} s\left(G(s) - \frac{G_2(s)}{s^2}\right),$$
$$G_0 = \lim_{s\to 0}\left(G(s) - \frac{G_2(s)}{s^2} - \frac{G_1(s)}{s}\right) \tag{1}$$

where

- Transfer function matrices $G(s)$ with only single poles at the origin have $G_2 = 0$
- Transfer function matrices with only double poles at the origin have $G_1 = 0$

In Mabrok et al. (2014) is remarked that these matrices are the first three coefficients in the Laurent series expansion of the transfer function $G(s)$ around the zero. Note that the dc gain Condition (1) cannot be defined for an NI system with transfer function matrix $G(s)$ unless $G_2 = G_1 = 0$, which reduces to the case where the dynamical system has no free body motion. From Condition (4) in Definition 1, the matrix G_2 is required to be Hermitian and positive semidefinite.

Definition 2. Let $R : \mathbb{C} \longrightarrow \mathbb{C}^{m\times m}$ real, rational, proper transfer function matrix. Then, $R(s) \in \mathbb{R}^{m\times m}$ is said to be *strictly negative imaginary* (SNI) if

(1) $R(s)$ has no pole in $\mathrm{Re}[s] \geq 0$
(2) $j[(R(j\omega) - R(j\omega)^*] \geq 0$ for all $\omega \in (0, \infty)$.

For SISO systems, the NI property ensures that the positive branch of the Nyquist plot lies below the real axis.

2.2 Lossless negative imaginary transfer functions

Definition 3. A real-rational proper transfer function matrix $R(s) \in \mathbb{R}^{m\times m}$ is *lossless negative imaginary* if

(1) $R(s)$ is negative imaginary;
(2) $j[(R(j\omega) - R^*(j\omega)] = 0$ for all $\omega \in (0, \infty)$ except values of ω where $j\omega$ is a pole of $R(s)$

Remark 1. It can be seen from Definition 3 that the lossless negative imaginary property of a transfer function is simply defined by replacing the " $\geq$ " sign with the " $=$ " sign in the Condition (2) of Definition 1.

The following theorem, which was presented as a generalization of Lemma 2 in Xiong et al. (2012) by allowing poles at the origin and infinity, provides a necessary and sufficient condition in frequency domain for a system to be non-proper lossless negative imaginary.

Theorem 1. (Liu et al. (2020)). A square real-rational transfer function matrix $G(s)$ is lossless negative imaginary if and only if

(1) all poles of elements of $G(s)$ are purely imaginary;
(2) if $s = 0$ is a pole of $G(s)$, it is at most a double pole, $\lim_{s\to 0} s^2 G(s)$ is positive semidefinite Hermitian, and $\lim_{s\to 0} s^m G(s) = 0$ for all $m \geq 3$;
(3) if $s = j\omega_0$ with $\omega_0 > 0$ is a pole of $G(s)$, ω_0 is finite, it is at most a simple pole and the residue matrix $K = \lim_{s\to j\omega_0}(s - j\omega_0)jG(s)$ is positive semidefinite Hermitian;
(4) if $s = j\infty$ is a pole of $G(s)$, it is at most a double pole, $\lim_{\omega\to\infty} G(j\omega)$ is negative semidefinite Hermitian, and $\lim_{\omega\to\infty} \frac{G(j\omega)}{(j\omega)^2} = 0$ for all $m \geq 3$;
(5) $G(s) = G^\top(-s)$ for all s such that s is not a pole of any element of $G(s)$.

3. MAIN RESULT

Figure 1 shows the LCL-type filter, where L_1 is the inverter-side inductor, C_f is the filter capacitor, and L_2 is the grid-side inductor. By representing the inverter bridge output voltage with a voltage source v_i and v_g is the grid voltage.

3.1 Lossless LCL Filter

Grid-side current i_g as output signal. The transfer function from v_{in} to i_g can be derived as

$$G_{i_g}(s) = \frac{1}{C_f L_g L_1\, s}\frac{1}{s^2 + \omega_r^2} \tag{2}$$

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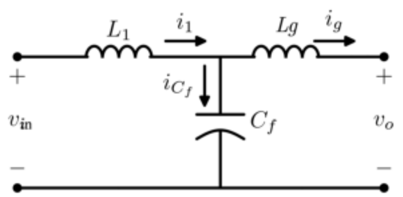

Fig. 1. Lossless LCL Filter

where $\omega_r = \sqrt{\frac{L_1+L_g}{C_f L_1 L_g}}$ is the resonance angular frequency and the resonance frequency is $f_r = \frac{\omega_r}{2\pi}$. The transfer function presents one pole at the origin and two pure imaginary poles at $\pm j\omega_r$. Moreover, it has no finite zeros. From Definition 1, clearly (2) has no pole in $\text{Re}[s] > 0$. For the Condition (2), we obtain

$$j[G_{i_g}(j\omega) - G_{i_g}(-j\omega)] = \frac{2}{\omega_r^2 - \omega^2} \geq 0 \;\Rightarrow\; \omega_r > \omega \quad (3)$$

which is not satisfied for all $\omega > 0$, only below ω_r. The residue for $s = j\omega_r$, with $\omega_r > 0$, is given by

$$\text{Res}_{s=j\omega_r} G_{i_g}(s) = \lim_{s\to j\omega_r} (s - j\omega_r) j G_{i_g}(s) = \frac{-j}{2C_f L_g L_1 \omega_r},$$

which is not positive semidefinite Hermitian. Finally, the first three coefficients in the Laurent series expansion of $G_{i_g}(s)$ around the zero are:

$$G_2 = 0,$$
$$G_1 = \frac{1}{L_1 L_g C \omega_r^2},$$
$$G_0 = 0.$$

Since, $G_{i_g}(s)$ has only single poles at the origin then $G_2 = 0$. Moreover, the dc gain Condition (1) cannot be defined since $G_1 \neq 0$.

Inverter-side current i_1 as output signal. The transfer function from v_{in} to i_1 can be derived as

$$G_{i_1}(s) = \frac{C_f L_g s^2 + 1}{C_f L_g L_i s^3 + (L_g + L_i)s}, \quad (4)$$

$$= \frac{1}{L_i\, s} \frac{s^2 + \omega_f^2}{s^2 + \omega_r^2}, \quad (5)$$

where $\omega_f = \sqrt{\frac{1}{C_f L_g}}$ is the anti-resonance angular frequency. The transfer function $G_{i_1}(s)$ has two finite zeros at $\pm j\omega_f$. From Definition 1, clearly (5) has no pole in $\text{Re}[s] > 0$. The condition $j[G_{i_1}(j\omega) - G_{i_1}(-j\omega)]$ is given by

$$j[G_{i_1}(j\omega) - G_{i_1}(-j\omega)] = \frac{2}{L_1\omega} \left(\frac{\omega_f^2 - \omega^2}{\omega_r^2 - \omega^2} \right) \geq 0 \quad (6)$$

or

$$j[G_{i_1}(j\omega) - G_{i_1}(-j\omega)] = \frac{2}{L_1\omega} \left(\frac{\omega^2 - \omega_f^2}{\omega^2 - \omega_r^2} \right) \geq 0 \quad (7)$$

which are satisfied for all positive $\omega \notin (\omega_f, \omega_r]$ since from the angular frequency definitions

$$\omega_r^2 = \frac{L_1 + L_g}{C_f L_1 L_g} = \omega_f^2 + \frac{1}{C_f L_1} \quad \Rightarrow \quad \omega_r^2 > \omega_f^2, \quad (8)$$

which equals

$$\frac{\omega_f^2 - \omega^2}{\omega_r^2 - \omega^2} \leq 1, \quad \forall\, \omega > 0, \quad (9)$$

except for $\omega = \omega_r$. If $\omega > \omega_r$, then $\omega > \omega_f$ and hence (6) and (7) hold. If $\omega_f > \omega$, then $\omega_r > \omega$ and hence (6) and (7) are satisfied. But, when $\omega_f < \omega \leq \omega_r$, Condition (2) in Definition 1 is not satisfied.
The residue for $s = j\omega_r$, with $\omega_r > 0$,

$$\text{Res}_{s=j\omega_r} G_{i_1}(s) = \lim_{s\to j\omega_r} (s - j\omega_r) j G_{i_1}(s) = \frac{j\,(\omega_r^2 - \omega_f^2)}{2L_1\omega_r} \geq 0$$

is positive semidefinite Hermitian provided that $\omega_r > \omega_f$. The quantities G_0, G_1 and G_2 defined in (1) are given as follows:

$$G_2 = 0,$$
$$G_1 = \frac{1}{L_1 + L_g},$$
$$G_0 = 0.$$

Since, $G_{i_1}(s)$ has only one pole at the origin then $G_2 = 0$ and the dc gain Condition (1) cannot be defined because $G_1 \neq 0$.

Filter capacitor current i_{C_f} as output signal. The transfer function from v_{in} to i_{C_f} can be derived as

$$G_{i_{C_f}}(s) = \frac{1}{L_i\, s} \frac{s^2}{s^2 + \omega_r^2} \quad (10)$$

where there is a pole zero cancellation. The transfer function presents one pole at the origin and two pure imaginary poles at $\pm j\omega_r$.
For the Condition (2), we obtain

$$j[G_{i_{C_f}}(j\omega) - G_{i_{C_f}}(-j\omega)] = \frac{2}{\omega^2 - \omega_r^2} \geq 0 \;\Rightarrow\; \omega > \omega_r \quad (11)$$

which is not satisfied for all $\omega_r > \omega > 0$. The residue for $s = j\omega_r$, with $\omega_r > 0$, is given by

$$\text{Res}_{s=j\omega_r} G_{i_{C_f}}(s) = \lim_{s\to j\omega_r} (s - j\omega_r) j G_{i_{C_f}}(s) = \frac{j\omega_r}{2L_1} > 0,$$

which is positive semidefinite Hermitian. And, after carrying out the pole zero cancellation, we have

$$G_2 = 0,$$
$$G_1 = 0,$$
$$G_0 = 0.$$

Since, $G_{v_C}(s)$ has one pole at the origin then $G_2 = G_1 = G_0 = 0$. In fact, it is a negative imaginary system as long as $\omega > \omega_r$, according to Definition 3 .

Grid-side inductor voltage v_{L_g} as output signal. The transfer function from v_{in} to v_{L_g} can be derived as

$$G_{v_{L_g}}(s) = \frac{1}{C_f L_1} \frac{1}{s^2 + \omega_r^2} \quad (12)$$

where the transfer function presents two pure imaginary poles at $\pm j\omega_r$ and has no finite zeros. From Definition 1, clearly (12) has no pole in $\text{Re}[s] > 0$. For the Condition (2), we obtain

$$j[G_{v_{L_g}}(j\omega) - G_{v_{L_g}}(-j\omega)] = \frac{j}{L_1 C} \left(\frac{1}{\omega_r^2 - \omega^2} - \frac{1}{\omega_r^2 - \omega^2} \right)$$
$$= 0,$$

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$\forall\,\omega > 0$, except ω_r, therefore, Condition (3) in Definition 1 is satisfied. The residue for $s = j\omega_r$, with $\omega_r > 0$, is given by

$$\mathrm{Res}_{s=j\omega_r} G_{i_g}(s) = \lim_{s\to j\omega_r} (s - j\omega_r) j G_{i_g}(s) = \frac{-j}{2C_f L_g L_1 \omega_r},$$

which is not positive semidefinite Hermitian.
Since, $G_{v_{L_g}}(s)$ has no poles at the origin then $G_2 = G_1 = 0$, with dc gain $G_0 = \frac{1}{L_1 C \omega_r^2}$. However, it is not a lossless negative imaginary system, according to Definition 3.

Inverter-side inductor voltage v_{L_1} as output signal. The transfer function from v_{in} to v_{L_1} can be derived as

$$G_{v_{L_1}}(s) = \frac{s^2 + \omega_f^2}{s^2 + \omega_r^2}, \tag{13}$$

with ω_r and ω_f defined for (2) and (5), respectively. The transfer function $G_{v_{i_1}}(s)$ has two zeros at $\pm j\omega_f$. From Definition 1, clearly (5) has no pole in $\mathrm{Re}[s] > 0$. The condition $j[G_{v_{L_1}}(j\omega) - G_{v_{L_1}}(-j\omega)]$ is given by

$$j[G_{v_{L_1}}(j\omega) - G_{v_{L_1}}(-j\omega)] = j\left(\frac{\omega_f^2 - \omega^2}{\omega_r^2 - \omega^2} - \frac{\omega_f^2 - \omega^2}{\omega_r^2 - \omega^2}\right) = 0 \tag{14}$$

$\forall\,\omega > 0$, except ω_r, therefore, Condition (3) in Definition 1 is satisfied. The residue for $s = j\omega_r$, with $\omega_r > 0$, is

$$\mathrm{Res}_{s=j\omega_r} G_{v_{L_1}}(s) = \lim_{s\to j\omega_r} (s - j\omega_r) j G_{v_{L_1}}(s) = \frac{\omega_f^2 - \omega_r^2}{2\omega_r} > 0, \tag{15}$$

which is positive semidefinite Hermitian only when $\omega_f > \omega_r$. Since, $G_{v_{L_1}}(s)$ has no poles at the origin then $G_2 = G_1 = 0$, with dc gain $G_0 = \frac{\omega_f}{\omega_r}$. In fact, it is a lossless negative imaginary system as long as $\omega_f > \omega_r$, according to Definition 3.

Filter capacitor voltage v_{C_f} as output signal. The transfer function from v_{in} to v_{C_f} can be derived as

$$G_{v_{C_f}}(s) = \frac{1}{C_f L_g L_i\, s}\frac{L_g\, s}{s^2 + \omega_r^2} \tag{16}$$

where there is a pole zero cancellation. From Definition 1, clearly (16) has no pole in $\mathrm{Re}[s] > 0$. For the condition $j[G_{v_{C_f}}(j\omega) - G_{v_{C_f}}(-j\omega)]$, we have

$$j[G_{v_{C_f}}(j\omega) - G_{v_{C_f}}(-j\omega)] = \frac{j}{j\omega L_1 C}\left(\frac{j\omega}{\omega_r^2 - \omega^2} - \frac{j\omega}{\omega_r^2 - \omega^2}\right) = 0,$$

$\forall\,\omega > 0$, except ω_r, therefore, Condition (3) in Definition 1 is satisfied. The residue for $s = j\omega_r$, with $\omega_r > 0$, is

$$\mathrm{Res}_{s=j\omega_r} G_{v_{C_f}}(s) = \lim_{s\to j\omega_r} (s - j\omega_r) j G_{v_{C_f}}(s) = \frac{1}{2C_f L_1 \omega_r} > 0,$$

which is positive semidefinite Hermitian. And, after carrying out the pole zero cancellation, we have

$$G_2 = 0,$$
$$G_1 = 0,$$
$$G_0 = \frac{1}{L_1 C \omega_r^2}.$$

Since, $G_{v_{C_f}}(s)$ has no poles at the origin then $G_2 = G_1 = 0$. In fact, it is a lossless negative imaginary system, according to Definition 3.

4. SIMULATIONS

In this section, the frequency response of LCL filter (as negative imaginary system) is presented. The parameter values are: $L_1 = 1\,\mathrm{mH}$, $L_g = 2\,\mathrm{mH}$, and $C_f = 10\,\mu\mathrm{F}$. Moreover, the resonance frequency is $f_r = 50.342\,\mathrm{kHz}$ and the anti-resonant frequency is $f_f = 1.1254\,\mathrm{kHz}$, then $\omega_r > \omega_f$.

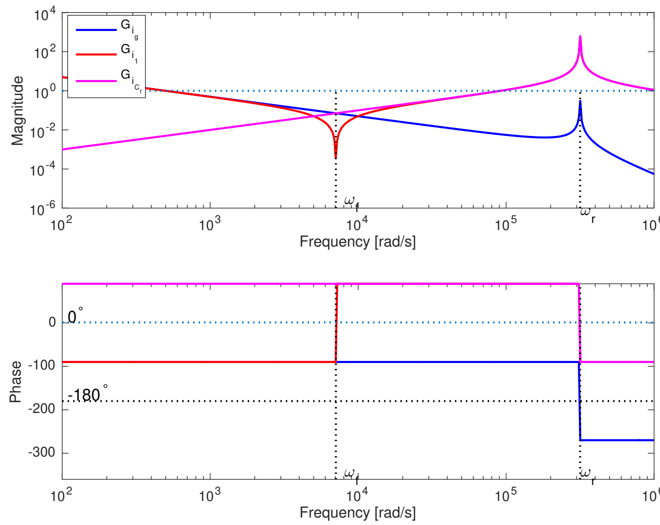

Fig. 2. Frequency responses of $G_{i_g}(s)$ (blue graph), $G_{i_1}(s)$ (red graph), and $G_{i_{C_f}}(s)$ (magenta graph), where ω_r and ω_f are shown as dotted lines.

Figure 2 shows that the phase of all systems, with a current as output, do not satisfy $\angle G(s) \in [-\pi, 0]$ rad. However, for instance, the phase of G_{i_1} when $\omega_f < \omega \le \omega_r$, Condition (2) in Definition 1 is not satisfied and $\angle G_{i_1}(s) \notin [-\pi, 0]$, therefore it is not a negative imaginary system. Similar conclusion, for each system G_{i_g} and G_{i_c} can be derived from conditions (3) and (11) respectively.

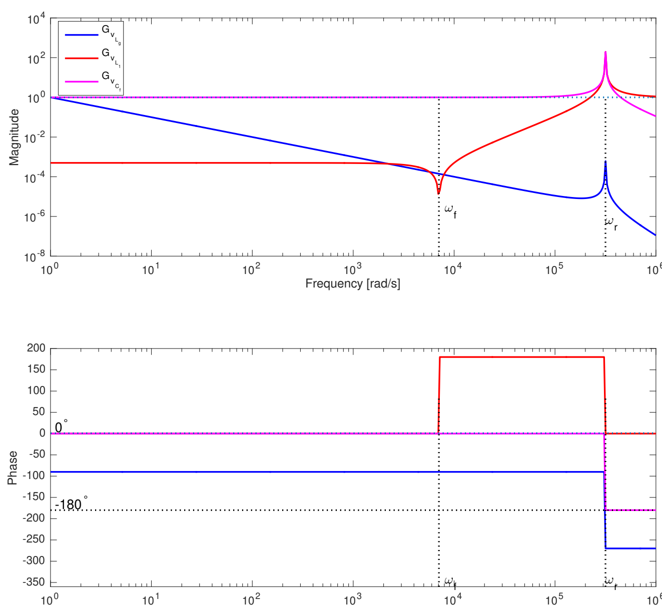

Fig. 3. Frequency responses of $G_{v_{L_g}}(s)$ (blue graph), $G_{v_{L_1}}(s)$ (red graph), and $G_{v_{C_f}}(s)$ (magenta graph), where ω_r and ω_f are shown as dotted lines.

Similarly, the phase of all systems, with a voltaje as output, are presenten in Fig. 3. Notice that only the system $G_{v_{C_f}}$ defines a lossless negative imaginary system since its phase satisfies $\angle G(s) \in [-\pi, 0]$ rad, see the magenta graph. For the system $G_{v_{L_1}}$ the Condition (15) requires $\omega_f > \omega_r$, but from (8) we have that $\omega_r > \omega_f$ for the LCL filter. Then, the system $G_{v_{L_1}}$ is not a NIS. Likewise, conclusion for the system $G_{v_{L_g}}$ can be derived.

5. CONCLUSIONS AND PERSPECTIVES

This paper has studied the lossless negative imaginary properties of LCL filter transfer functions, where these were established by means of each transfer function satisfies the (lossless) negative imaginary conditions. The voltaje or current of each passive element was used as output signal for the LCL filter transfer functions. In fact, as main conclusion, only when the filter capacitor voltage is used as output signal defines a lossless negative imaginary system.

Dynamic systems with lossless negative imaginary transfer functions have applications in control of lossless electrical circuits. From the results presented in this work, some issues that remain open, and are currently being explored, include for instance: (a) A control challenge of LCL-type grid-connected inverter arise from the resonance problem, then resonance damping methods of LCL Filter by means of the existing passive- and active-damping solutions are studied systematically by the NIS conditions, and (b) The positive feedback interconnection of an NI system $R(s)$ and a strictly negative imaginary (SNI) system $R_s(s)$ with $R(\infty)R_s(\infty) = 0$ and $R_s(\infty) \geq 0$, internal stability is achieved if and only if the dc loop gain of the interconnection is strictly less than unity; i.e., $\lambda_{\max}(R(0)R_s(0)) < 1$ (see Petersen and Lanzon (2010)). Then, regarding the controller design, it is interesting to extent the results presented here and to the references in this work.

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