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Observer-Based Control with Hardware-in-the-Loop for Stabilization of a Driverless Two-Wheeled Vehicle

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Abstract: This paper presents the design of a control system whose objective is to stabilize a riderless bicycle. This is achieved from two torques applied to the angular positions of the vehicle. Also, the implementation of hardware in the loop is considered because only one of the states of the mathematical model is measured. The design of the control system addresses three important features: considering part of the real system (front frame or handlebars), a state observer and tacking into account these two parts, the solution of the roll angle variable ϕ is addressed with the corresponding differential equation (or hardware in the loop).

Keywords: LTI System, LPV System, Observer, Hardware in the loop, Differential Equation.

1. INTRODUCTION

In the discipline of automatic control, the dynamic behavior of a two-wheeled vehicle is one of the most interesting case studies. The nature of this physical system is defined as unstable and dependent on its translational (or motion) velocity. Previous research defines its behavior through a mathematical model resulting from an analysis considering physical and motion laws.

The mathematical model published in (Whipple, 1899) is developed from physical laws and differential equations of motion, applying the D'Alembert principle. This article presents the mathematical model of a two-wheeled vehicle, defining in its dynamics the translational velocity as a variable parameter, which turns the system into an LPV (Linear System of Variable Parameters) type.

Previous researches contemplate the objective of stabilizing the vehicle in its vertical position by means of different methodologies for the design of control systems, applied physically or validating the results through simulations. In (Wang et al., 2019) the design of a LQR controller focused on the manipulation of the steering of a vehicle and considering an actuator in the front wheel is presented. Cerone et al. (2010) designed a control system applied to a real vehicle, considering variable gains to stabilize it with different values of translational velocity.

The article by Brizuela-Mendoza et al. (2016) presents the design of a control scheme using the Fault Tolerant Control approach with the objective of stabilizing the vehicle even in the presence of faults. Ríos Ruiz (2016) designs control systems addressing Quasi-LPV models and validated by means of simulations. In (Baquero Suárez et al., 2017) the results of an active disturbance rejection control are reported, whose results were validated by means of a co-simulation between MATLAB and ADAMS software. In (Bravo and Rengifo, 2020) presents the design of two control schemes based on the kinematic model of a robotic bicycle, aiming to stabilize it in vertical position: the first controller focuses on the direction of the handlebars and the second one on smoothing the output values.

On the other hand, hardware-in-the-loop implementation is key to research development, as it allows for efficient and safe testing through a system equipped with electronic devices, whose data provide insights into system behavior.

In this paper, the case study of a riderless bicycle is addressed, contemplating the design of a control system capable of stabilizing it in its vertical position by means of two torques applied to the angular positions defined in its mathematical model.

The main contribution of this paper is the implementation of hardware in the loop and the coupling of this methodology in the control system designed and implemented to the instrumented two-wheeled vehicle. The approxi-

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mation of the particular mathematical model of the instrumented bicycle is considered (section 2), where the difference between other works oriented to the study of this system is also described: the complete mathematical model is used with emphasis on the design of the control law $u(t)$ since previous researches only consider the torque associated with the handlebar angle (T_δ).
The design of a state observer is also considered, obtaining the missing information associated with the model. Also, the solution of the differential equation associated with the roll angle of the bicycle, whose results are considered as virtual measurements (section 3).
The coupling between the real system, the observer, and the ϕ differential equation is defined as one of the main contributions of this work. Since the design of the control laws considers in its design measurements, estimated states and computed variables (section 4).
Similarly, it is important to clarify that the observer and the differential equation of ϕ are designed from the information obtained by the instrumentation implemented in the selected bicycle (section 5) and the resultant of both.

2. MATHEMATICAL MODEL OF A BICYCLE

The analysis and theoretical development of the mathematical model of the two-wheeled vehicle are based on Schwab et al. (2005) and Meijaard et al. (2007). Their procedure considers the vehicle divided into four subsystems.

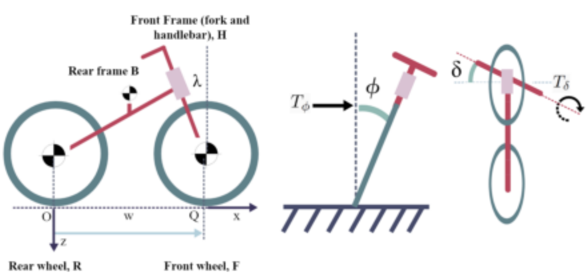

Fig. 1. Schematics of the two-wheeled vehicle (From left to right: lateral view, front view, top view).

Fig. 1 shows the location of the subsystems and the schematics of the vehicle structure associated with its mathematical model: ϕ is the roll angle, which represents the angle between the general frame and the vertical, δ is the handle angle, which represents the angle between the front frame and the perpendicular of the general frame, T_ϕ and T_δ are defined as the generalized input forces applied to the general and front frames, respectively. Finally, the position and direction of the coordinate system according to the theory of the mathematical model of the vehicle analyzed in Meijaard et al. (2007) are shown. Each of the variables mentioned above will be defined and used later.
Considering all the above, the dynamic behavior of the vehicle is analyzed through two second-order differential equations derived from the physical laws of motion. One equation is associated to the vehicle's inclination, while the other addresses its direction, taking into account translational velocity and gravity. These equations are then dynamically combined in the following form:

$$M\ddot{q} + vC_1\dot{q} + [gK_0 + v^2K_2]q = f \tag{1}$$

where $q = [\phi\ \delta]^\top$ is the vector of angular positions and $f = [T_\phi\ T_\delta]^\top$ is the vector of generalized input forces. T_ϕ is the torque associated with the roll angle and T_δ is the torque associated with the handlebar angle. v represents the translational velocity and g represents gravity.
Matrices M and C_1 encompass parameters associated with masses and dampings, while K_0 and K_2 represent matrices containing stiffness parameters. Each coefficient within these matrices is derived from mathematical operations based on the physical and motion laws governing the vehicle, along with parameters and dimensions sourced from the CAD model created in SolidWorks.
In table A.1 the dimensions and parameters of the selected two-wheeled vehicle are presented. Based on this, the matrix coefficients of M, C_1, K_0 and K_2 are defined with the following values:

$$\begin{gathered} M = \begin{bmatrix} 1.0239 & 0.1602 \\ 0.1602 & 0.4594 \end{bmatrix},\ K_0 = \begin{bmatrix} -3.6400 & -0.7059 \\ -0.7059 & -0.2414 \end{bmatrix} \\ K_2 = \begin{bmatrix} 0 & 5.9226 \\ 0 & 1.1096 \end{bmatrix},\ C_1 = \begin{bmatrix} 0 & 0.5302 \\ -0.3796 & 0.8999 \end{bmatrix} \end{gathered} \tag{2}$$

The mathematical model of the two-wheeled vehicle defined in Eq. (1) can be represented in state-space form by considering the following state variables: $\phi(t)$: *Roll angle*, $\delta(t)$: *Handlebar angle*, $\dot{\phi}(t)$: *Angular velocity of roll*, $\dot{\delta}(t)$: *Angular velocity of the handlebar*.
Then $x(t) = \left[\phi\ \delta\ \dot{\phi}\ \dot{\delta}\right]^\top$, the mathematical model is:

$$\begin{aligned} \dot{x}(t) &= A(v)x(t) + Bu(t) \\ y &= Cx(t) \end{aligned} \tag{3}$$

with:

$$A(v) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 3.5058 & 0.6422 - 5.7184v^2 & -0.1368v & -0.2235v \\ 0.3138 & 0.3015 - 0.4209v^2 & 0.8739v & -1.8809v \end{bmatrix},$$
$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1.0330 & -0.3603 \\ -0.3603 & 2.3023 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{4}$$

In Eq. (3), $u(t)$ is the vector containing the generalized input forces considered for the design of the control law that stabilizes the vehicle:

$$u(t) = [T_\phi\ T_\delta]^\top \tag{5}$$

By incorporating this input, this paper distinguishes itself from previous works, which only consider T_δ for stabilization. Therefore, the primary contribution lies in utilizing the complete mathematical model for the design of the control system.
As evidenced by the matrix $A(v)$, the stability of the vehicle depends on the translational velocity $v(t)$. Consequently, the system is linear parameter-varying. How-

ever, in this article, a constant translational velocity is considered as a first LTI approach to be implemented in the proposed hardware-in-the loop scheme.
According to (4) and the real instrumented two-wheeler described later, only the handlebar angle is measured, so the design of an observer is considered to estimate the rest of the states of the mathematical model.

3. CONTROL SYSTEM

The hardware implementation in the loop consists of the solution of the ϕ differential equation considering the information generated by the bicycle instrumentation and the observer.

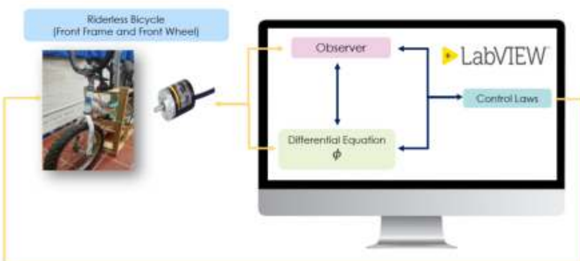

Fig. 2. General structure of the control system.

A general scheme of the control system coupling dynamics is shown in Fig. 2: the information obtained from the real system is used for the design of the observer and the solution of the ϕ differential equation. Likewise, both elements share information to generate the control laws contemplated in their design, while only the control law associated with the real system (or physical vehicle) is applied to it.
The control scheme presented in Fig. 3 describes in detail the coupling between a part of the real system (the front frame and the front wheel), a Luenberger observer and the hardware in the loop (i.e., the ϕ differential equation).

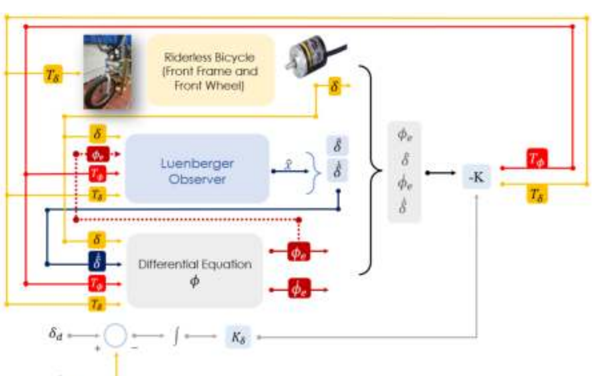

Fig. 3. The hardware in the loop scheme

3.1 Real system (Front frame and front wheel)

The real system (front frame and front wheel) delivers only a measured state through an angular position sensor: the handlebar angle δ.

3.2 State observer

The observer-based control system considers a Luenberger observer:

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x}) \tag{6}$$

Its design considers two measured states δ and ϕ_e (both angular positions), obtaining as a result the whole vector of estimated states:

Notation	*Parameter*
$\hat{\phi}$	Estimated roll angle
$\hat{\delta}$	Estimated handlebar angle
$\dot{\hat{\phi}}$	Estimated angular velocity of roll
$\dot{\hat{\delta}}$	Estimated angular velocity of the handlebar

3.3 Differential equation of ϕ

The mathematical model of the bicycle defines the ϕ differential equation as:

$$\begin{aligned}\ddot{\phi} = 3.5058 + (0.6422 - 5.7184v^2)\delta - 0.1368v\ \dot{\phi} \\ -0.2235v\ \dot{\delta} + 1.0330\ T_\phi - 0.3603\ T_\delta\end{aligned} \tag{7}$$

The solution of the differential equation contemplates two states: δ and $\dot{\hat{\delta}}$ being a real state and an estimated state, having as a result:

Notation	*Parameter*
ϕ_e	Roll angle equation
$\dot{\phi}_e$	Angular velocity of roll equation

3.4 New vector of states

In order to highlight the dynamic coupling between the three elements (real system, observer and ϕ differential equation) described above, the new vector of states $\bar{x}$ used for the calculation of the control law is defined as: $\bar{x} = \left[\phi_e\ \hat{\delta}\ \dot{\phi}_e\ \dot{\hat{\delta}}\right]^\top$, where two of its states are generated from the observer and the rest by the differential equation of ϕ.
In general, both part of the real system, the observer and the differential equation ϕ communicate dynamically as follows. The real system delivers only δ, the state that feeds the observer (in addition to ϕ_e) for its design, and the differential equation ϕ. On the other hand, the observer generates the entire vector of estimated states, of which only those associated with the handlebars are considered. Finally, the solution of the differential equation of ϕ contemplates δ and $\dot{\hat{\delta}}$, thus generating ϕ_e and $\dot{\phi}_e$.

Observing Fig. 3, both control laws (T_ϕ and T_δ) are considered in the design of the observer and the differential equation, while only the generalized input force T_δ is applied to the real system.

4. DESIGN OF CONTROL LAWS

With all of the above and with the objective of stabilizing and regulating the position of the vehicle, both control

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laws are defined as follows (for simplicity, the argument (t) will be omitted in the remainder of this paper):

$$T_\phi = -(K_{11}\phi_e + K_{12}\hat{\delta} + K_{13}\dot{\hat{\phi}}_e + K_{14}\dot{\hat{\delta}}) + K_{15}\int(\delta_d - \delta)dt \quad (8)$$

$$T_\delta = -(K_{21}\phi_e + K_{22}\hat{\delta} + K_{23}\dot{\hat{\phi}}_e + K_{24}\dot{\hat{\delta}}) + K_{25}\int(\delta_d - \delta)dt \quad (9)$$

Where the elements in color red represent those that are generated and applied virtually, the elements in color blue correspond to those generated by the observer and those in color yellow to the real elements (same logic applied to the contents in Fig.(3)), thus concluding that in its dynamics a real part, an estimated part and the hardware in the loop are considered.
Simplifying to (8) and (9), it follows that

$$u = -K\bar{x} + K_\delta \int(\delta_d - \delta)dt \quad (10)$$

where K is the vector of controller gains, K_δ is the gain associated with the integral action applied to the handlebar angle and $\bar{x}$ is the new vector of states defined above. Likewise, $(\delta_d - \delta)$ represents the ***error*** between a desired angular position of the handlebar angle and the measured state of the handlebar, respectively.
K and K_δ are part of a single vector of gains calculated with the *place* command of MATLAB contemplating the poles $[-1\ -1.5\ -2\ -2.5\ -3]$ seeking to have no oscillations in its response and to ensure a convergence time such that the state reaches the programmed references, considering matrix A and B as augmented, and matrix C as follows:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 3.5058 & 0.6422 - 5.7184v^2 & -0.1368v & -0.2235v & 0 \\ 0.3138 & 0.3015 - 0.4209v^2 & 0.8739v & -1.8809v & 0 \\ 0 & -1 & 0 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1.0330 & -0.3603 \\ -0.3603 & 2.3023 \\ 0 & 0 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad (11)$$

Similarly, the observer structure defined in (6) considers a vector of gains L whose values are calculated through the *place* command of MATLAB contemplating the poles $[-12.1\ -12.2\ -12.3\ -12.4]$, which were selected in order to ensure an ideal convergence time to obtain a response dynamics in conjunction with the designed controller.

5. INSTRUMENTED BICYCLE

The real bicycle selected, and the control system designed only consider the front frame and the front wheel as a degree of freedom, so a structure was designed and built in order to assemble it and keep it suspended, avoiding that both wheels have contact with the ground and that the general frame moves (Important feature in system modeling).

Fig. 4. Main base for bicycle.

Fig. 4 presents the main base that is considered to keep the vehicle in its upright position, leaving the front frame as the only subsystem with movement.

5.1 Instrumentation mechanisms

The two-wheeled vehicle motion considerations described above lead to the design of two important mechanisms:

- *Mechanism associated with handlebar angle δ*: Its objective is to locate the sensor in charge of measuring the angular position of the front frame (handlebar angle) and the front wheel.
- *Mechanism associated with the generalized input force T_δ*: Associated with the same elements as the previous mechanism, this one contemplates an actuator, whose main task is to manipulate the front frame and the front wheel in order to steer and stabilize the vehicle (see Fig. 5).

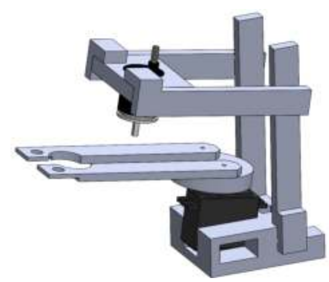

Fig. 5. Mechanisms designed in SolidWorks.

5.2 Electronic components

With all the above, the instrumentation of the vehicle only contemplates those elements associated to one of the states of the mathematical model, in this case the handlebar angle δ. Therefore, the elements contemplated for such instrumentation are the following:

- *OMRON E6A2 - CWZ3E Rotary ENCODER (INCREMENTAL)*: Sensor coupled with the mechanism associated with the handlebar angle to deliver the angular position of the subsystems (the front frame and the front wheel) in charge of steering the two-wheeled vehicle (see Fig. 6a).
- *Servomotor DS3225 MG*: The actuator coupled with the mechanism associated with the generalized input force applied to the handlebar angle allows the

movement or manipulation of the front frame and the front wheel, being that one of the pairs considered in the mathematical model (see Fig. 6b).

(a) Sensor in charge of measuring the angular position of the handlebars δ.

(b) Actuator to manipulate the front frame of the vehicle.

Fig. 6. Electronic components for instrumentation.

6. RESULTS

This section shows the results of the control system presented in Fig. 3 developed in LabVIEW software.
The control system design considers a constant translational velocity $v = 1.271\ m/s$. Its objective is to stabilize the vehicle at a desired angular position by means of an integral action applied to the handlebar angle.

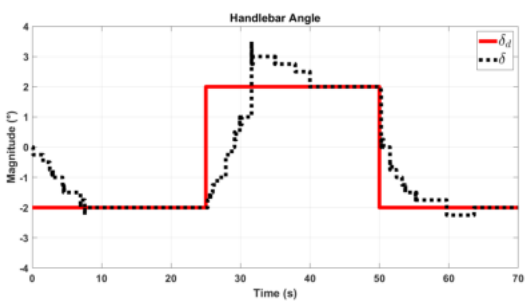

Fig. 7. Handlebar angle δ.

The experimental results obtained from the control system applied to the instrumented bicycle are shown in Fig. 7. The present test contemplates reference changes from $-2°$ to $2°$ programmed in the integral action applied to the handlebar angle δ, observing that delta perfectly reaches the desired angular position.

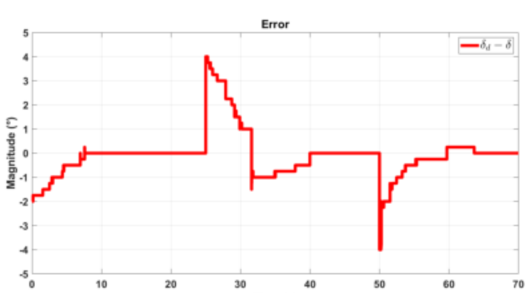

Fig. 8. Error between δ_d - δ.

In order to validate the above, in Fig. 8 the error between (δ_d - δ) is plotted, resulting that during certain periods of time the value is 0, concluding that delta takes the exact value at the desired angular position.

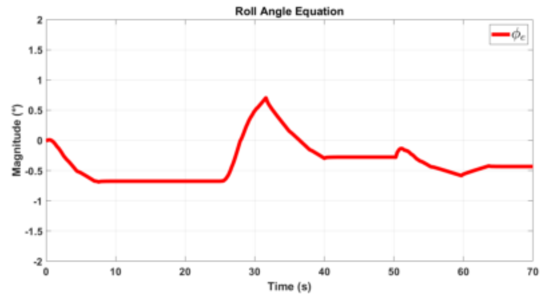

Fig. 9. Roll angle equation ϕ_e.

Additionally, Fig. 9 shows the variation of the angular position of the roll ϕ_e is presented as a result of the programmed position changes in the angle of the handlebar δ. The results show small variations around its operating point, concluding that the vehicle is stable.

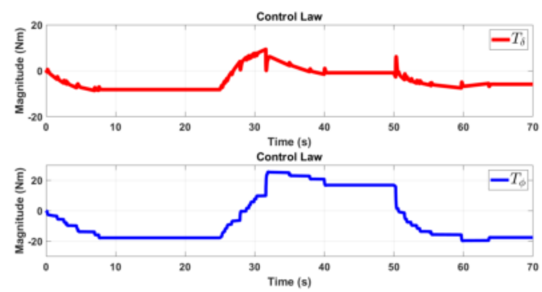

Fig. 10. Control laws T_ϕ y T_δ.

Fig. 10, on the other hand, presents the control laws applied to the observer and the ϕ differential equation, while only T_δ (generalized input force associated with the crank) is applied to the real system. Consequently, from the angular positions programmed to the handlebar angle, the effect of the control law to reach that reference is observed.

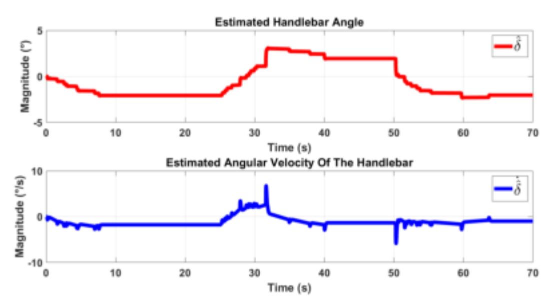

Fig. 11. Results of δ generated by the observer.

Fig. 11 shows the results of δ generated by the observer. In this case, $\hat{\delta}$ takes equal values according to the reference changes, thus validating the observer's design.

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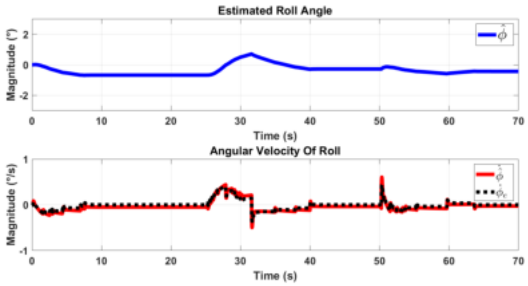

Fig. 12. Results of ϕ generated by the observer.

Finally, Fig. 12 presents a comparison of the observer's results and the ϕ differential equation. Note that, the roll angle product of the observer takes a behavior equal to that generated by the differential equation of ϕ.

7. CONCLUSION

This work addressed the design of a control system with the objective of stabilizing a real two-wheeled vehicle. Its development involves the approximation of the particular mathematical model of the selected two-wheeled vehicle, the design of a state observer from the information obtained from the physical prototype and the consideration of hardware in the loop for the overall controller design.
From the observed results, it can be assumed that the vehicle is stable, given that the values remain approximately close to the reference and to its operating point (0°). With all of the above, the control system could consider new features in order to obtain new results.
Finally, the important contributions addressed in this work are focused on the design of the control laws because previous researches do not consider the complete mathematical model, i.e., they only stabilize the vehicle with the generalized input force associated to the handlebars T_δ. In the same way, the implementation of hardware in the loop through the solution of the differential equation of T_ϕ turns out to be a new solution methodology for the design of control systems.

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Appendix A. PARAMETERS OF THE MODEL

Table A.1. Parameters.

Parameter	Symbol	Value
Wheelbase	w	0.706 m
Trail	c	0.025 m
Front frame angle	α	70°
Steer axis tilt	λ	$\pi/2 - \alpha$
Gravity	g	9.81 m/s^2
Rear wheel ***R***		
Radius	r_R	0.16 m
Mass	m_R	2.32 Kg
Mass moments of inertia	(I_{Rxx}, I_{Ryy})	(0.03, 0.07) $Kg\ m^2$
Rear body and frame assembly ***B***		
Center of mass position	(x_B, z_B)	(0.06, 0.07) m
Mass	m_B	35 Kg
Mass moments of inertia	$\begin{bmatrix} I_{Bxx} & 0 & I_{Bxz} \\ 0 & I_{Byy} & 0 \\ I_{Bxz} & 0 & I_{Bzz} \end{bmatrix}$	$\begin{bmatrix} 0.41 & 0 & 0.45 \\ 0 & 1.35 & 0 \\ 0.45 & 0 & 0.98 \end{bmatrix} Kg\ m^2$
Front handlebar and fork assembly ***H***		
Center of mass position	(x_H, z_H)	(0.15, -0.27) m
Mass	m_H	2.0001 Kg
Mass moments of inertia	$\begin{bmatrix} I_{Hxx} & 0 & I_{Hxz} \\ 0 & I_{Hyy} & 0 \\ I_{Hxz} & 0 & I_{Hzz} \end{bmatrix}$	$\begin{bmatrix} 0.13 & 0 & 0.03 \\ 0 & 0.10 & 0 \\ 0.03 & 0 & 0.04 \end{bmatrix} Kg\ m^2$
Front wheel ***F***		
Radius	r_F	0.16 m
Mass	m_F	1.87 Kg
Mass moments of inertia	(I_{Fxx}, I_{Fyy})	(0.03, 0.05) $Kg\ m^2$

