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A Steady-State Methodology For Leak Diagnosis in Multi-branched Pipelines

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Abstract: This work proposes a general methodology to deal with the problem of leak diagnosis in a multi-branched pipeline. To achieve this, the flow rate and pressure measurements at the ends of the pipeline are assumed to be known, as well as the flow rate leaving each pipeline branch with a known position. The key of the proposed methodology consists of estimating the known position of each branch of the pipeline and generating a residual from the discrepancy between the estimated and the real position, due to the presence of a leak. Due to the sensitivity of the residuals to variations of flow and pressure, the sign of this set of residuals indicates the section in which the leak is located, and, knowing this, it is possible to localize it precisely through a simple mathematical expression using only known values. In particular, we present the equations for the cases of pipelines with two and three branches.

Keywords: Fault diagnosis, Residual, Monitoring, diagnostic model, Leak detection and isolation, branched pipeline.

1. INTRODUCTION

A branched pipeline system involves a main pipeline (or trunk) that splits into two or more branches to distribute fluid from a central source to multiple reservoirs or equipment at different locations. This configuration is common in water supply networks, irrigation systems, and industrial fluid distribution. As with all distribution systems, branched pipelines are susceptible to leaks due to various factors, including aging infrastructure, pressure variations, joint and connection failures, ground movement, manufacturing defects, and external damage. Therefore, diagnosing leaks in branched pipelines is essential for ensuring their integrity and safety. To address this challenge, several methodologies have been developed, utilizing approaches such as model-based approaches, transient-based methods, and data-based methods.

A recently presented model-based approach in the literature addresses the leak diagnosis problem in three stages to simplify the localization of a single leak in a pipeline with multiple branches: (1) leak detection, (2) leaky section search, and (3) leak localization. This method utilizes head loss and flow rate balances to effectively detect and locate the leak (Torres et al., 2021). Another approach for leak detection based on a physical model was presented by Anfinsen and Aamo (2022). This method estimates parametric uncertainties caused by leaks using adaptive observer techniques. It is formulated by mapping the pipe system model into a system of coupled, linear hyperbolic partial differential equations (PDEs) . A recent model-based methodology proposes a straightforward mass balance approach for leak detection, which subsequently activates an isolation process. This method introduces a loss function that incorporates both measured and estimated values to identify the candidate branch where the leak is occurring. Finally, the exact leak location is estimated as the distance from the upstream point of the identified candidate branch to the leak point (Navarro-Díaz et al., 2024).

Leak detection in branched pipelines can also be achieved using transient-based methods (TBMs) that analyze pressure and flow measurements at the inlet and outlet boundaries of the pipe system. Pan et al. (2022) presented an efficient transient-based method that has been developed for polymeric pipe parameter identification and leak detection in single and branched pipelines. This approach leverages transient wave analysis for viscoelastic parameter identification and leak detection in water-filled polymeric pipes. Ko et al. (2024) presented a detection approach using a high-order polynomial-based transient model to handle nonlinear valve maneuvers. This model significantly enhances transient simulations compared to traditional orifice-based approximations.

The organization of this article is as follows: Section 2 details the mathematical modeling for a pipeline with a branch and briefly reviews a steady-state method for detecting and localizing leaks in such pipelines. Section

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3 extends the modeling to pipelines with two and three branches and provides the equations used for leak localization. Section 4 presents numerical results for the case of a leak in a two-branch pipeline. Finally, Section 5 offers conclusions and suggestions for future work.

2. MATHEMATICAL MODELING OF THE FLUID IN A PIPELINE

Consider the following one-dimensional nonlinear model of distributed parameters for a generic fluid in a pipeline section (Chaudry (1979)):

$$\frac{\partial Q(z,t)}{\partial t} + gA\frac{\partial H(z,t)}{\partial z} + \mu Q(z,t)|Q(z,t)| = 0 \qquad (1)$$

$$\frac{\partial H(z,t)}{\partial t} + \frac{b^2}{gA}\frac{\partial Q(z,t)}{\partial z} = 0 \qquad (2)$$

which can be obtained by assuming a slightly compressible fluid and negligible convective changes in velocity.

The following variables are used in this document: $Q(z,t)$ represents the volumetric flow (m^3/s), and $P(z)$ denotes the pressure head (Pa). The term $h(z)$ indicates the sea level, while $H(z,t)$ is the piezometric head, defined as $P(z)/\rho g + h(z)$, measured in meters (m). The variable ρ stands for density ($\mathrm{kg/m}^3$), and $z_i \in (0, L_i)$ is the spatial coordinate of a section i, measured in meters (m). The time coordinate is denoted by $t \in (0, \infty)$ (s). The cross-sectional area of a section is represented by A (m^2), and Q_{bi} is the flow of the lateral branch i (m^3/s). The position of the branch junction i is given by z_{bi} (m), while D refers to the diameter of a pipeline section (m). Gravity acceleration is denoted by g ($\mathrm{m/s}^2$), $\mu = f/2DA$ and f is the friction coefficient, which is dimensionless. The wave speed in the fluid is represented by b (m/s). The length of section i is given by L_i (m), and the complete pipeline length is $L = \sum_{i=1}^{n+1} L^i$ (m). The number of pipe sections is denoted by n, and r_i represents the residual for leak detection (m).

A pipeline of length L, with n branches, is composed of $n+1$ sections of length L_i, each one represented by (1-2), according to the profile and pipeline specifications. Hence, the dynamics of the complete pipeline would be represented by the $n+1$ couples of equations (1-2). This fact will be quite used in this document.

2.1 Steady-state model for a pipeline

According to González et al. (2017), under the assumption that the fluid is in a steady state, from (2) one gets

$$\frac{\partial Q(z,t)}{\partial z} = 0 \quad \Rightarrow Q \text{ is constant.} \qquad (3)$$

By combining (1) and (2) under the above condition, one obtains the ordinary differential equation

$$\frac{dH(z)}{dz} + \mu Q|Q| = 0 \qquad (4)$$

The analytical solution of (4) is then given by

$$H(z) = -\mu(Q)z + H(0) \quad \text{for} \quad 0 \le z \le L \qquad (5)$$

with $\mu(Q) = \mu Q|Q|$, and $H(0)$ being the piezometric head at the beginning of the section. Defining the boundary conditions of the pipeline in terms of the pressures (piezometric head) at the extremes, it follows:

$$H(z=0) := H_{in} \qquad H(z=L) := H_{out}, \qquad (6)$$

and, by substituting these variables in (5), one gets

$$H_{in} - H_{out} = \mu(Q)L. \qquad (7)$$

In the case of a pipeline with $n-1$ branch junctions (n sections) with the same parameters; since the boundary conditions between sections are related by

$$H_{(i,out)} = H_{(i+1,in)}, \qquad (8)$$

and using the expression (7), one gets the model

$$H_{(1,in)} - H_{(n,out)} = \sum_{i=1}^{n} L_i \mu_i(Q_i) \qquad (9)$$

in terms of the piezometric variable along a general pipeline of n sections. This pressure profile will be used in the next sections for leak diagnosis purposes.

2.2 Leak localization in a pipeline section

Consider the stage of a leak in a pipeline section, as shown in Fig. 1.

In several research works, it has been reported that

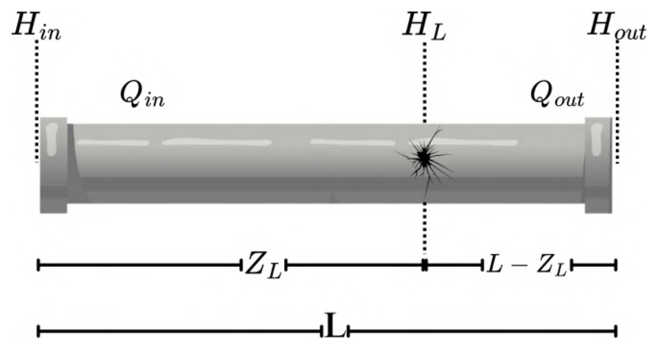

Fig. 1. A leaking pipeline.

the leak position, z_L, can be estimated through the expression:

$$z_L = \frac{\mu(Q_{out})L + H_{out} - H_{in}}{\mu(Q_{out}) - \mu(Q_{in})}. \qquad (10)$$

As it was reported in González et al. (2017) and Torres et al. (2020), the estimated value of z_L can be used as a *leak indicator signal* for a branched pipeline by the simple idea that, in the absence of leakage, when applying Eq. (10) will result in the known position of the branch junction. This fact is explained in the next section.

2.3 Leak detection and location in a branched pipeline

In this section, we use the mentioned fact that in a branched pipeline, without leaks, if we apply Eq. (10), the known position of the branch will be obtained. To remark on this idea, consider the stage of a branch junction in a pipeline, as it is shown in Fig. 2. According to Wylie and Streeter (1978), any component connected to a pipeline -a valve, a pump, a branch, or a leak, for example- breaks the continuity of the variables $Q(z,t)$ and $H(z,t)$, the condition (2) is not met and new boundary conditions generated by the junction have to be satisfied (Mahgerefteh et al. (2006)).

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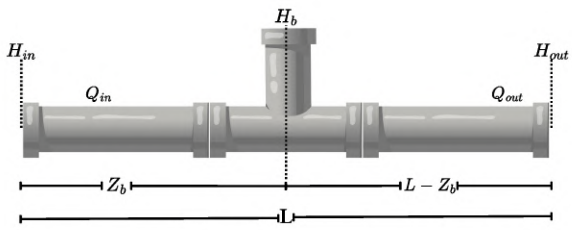

Fig. 2. A pipeline with a branch junction.

As González et al. (2017) mentioned, the separation losses terms due to friction and flow direction across a junction can be neglected. Moreover, the assumption that a junction without fluid storing the continuity of the three lines is ensured if

$$Q_{in} - Q_b - Q_{out} = 0, \tag{11}$$

where Q_b is the flow of the branch junction. Then, for a pipeline with a branch junction, the differential equation (4) becomes

$$\begin{aligned} \frac{dH(z)}{dz} - \mu(Q_{in}) \quad &= 0; \quad \text{for} \quad 0 \leq z \leq z_b \\ \frac{dH(z)}{dz} - \mu(Q_{out}) &= 0; \quad \text{for} \quad z_b < z \leq L, \end{aligned} \tag{12}$$

which describes the pressure head at any point z with a branch junction at z_b.

Since the set (12) has the same form as (4), their solutions have the form of (5). Therefore, considering the boundary conditions

BC1: $H(z = 0) = H_{in}$,

BC2: $H(z = L) = H_{out}$,

BC3: $Q_{in} = Q_{out} + Q_b$,

the solutions of the set (12) evaluated at the extremes of the section reduce to

$$\begin{aligned} \frac{H_{in} - H_b}{z_b} - \mu(Q_{in}) = 0 \\ \frac{H_b - H_{out}}{L - z_b} - \mu(Q_{out}) = 0. \end{aligned} \tag{13}$$

The variable z_b associated with the position of the branch junction can then be determined from (13) by algebraic operations, obtaining

$$z_b = \frac{\mu(Q_{out})L + H_{out} - H_{in}}{\mu(Q_{out}) - \mu(Q_{in})}, \tag{14}$$

which is clearly identical to Eq. (10).
Because z_b is associated with a fixed junction of the pipeline, any deviation of its estimation is a residual (Isermann (2006)) which indicates a discrepancy in the normal behavior of the fluid. Then, we can detect abnormal scenarios by using the residual

$$r(t) = z_b\big|_0 - \hat{z}_b(t), \tag{15}$$

where $\hat{z}_b(t)$ is estimated through (14).
As was remarked in González et al. (2017) after a sensitivity analysis, the signal of (15) is not arbitrary but indicates the direction (upstream or downstream) of the leakage section of the duct. Then, the use of (14) for leaks' detection is a key fact that will be exploited for multi-branch cases in the following sections.

Once the existence of a leak is known, it is necessary to locate it. In this sense, as we can see in Figs. 3-4, the corresponding leak could be located upstream or downstream of the branch. In both cases, the pipeline can be modeled as a three-section duct with new boundary conditions in between. For the upstream case, as in (13), in steady state

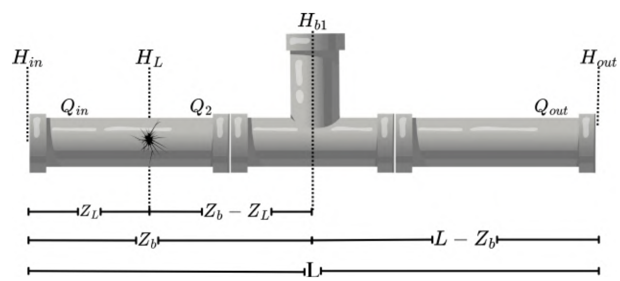

Fig. 3. Upstream leak in a pipeline with a branch junction.

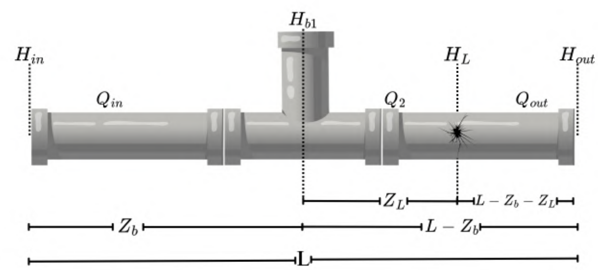

Fig. 4. Downstream leak in a pipeline with a branch junction.

conditions, the solution of the mathematical description of the pipeline becomes

$$\begin{aligned} \frac{H_{in} - H_L}{z_L} - \mu(Q_{in}) = 0 \\ \frac{H_L - H_b}{z_b - z_{L_u}} - \mu(Q_2) = 0 \\ \frac{H_b - H_{out}}{L - z_b} - \mu(Q_{out}) = 0. \end{aligned} \tag{16}$$

with $Q_2 = Q_{out} + Q_b$, which can be solved for z_{L_u} as

$$z_{L_u} = \frac{H_{out} - H_{in} + \mu(Q_{out})(L - z_b) + \mu(Q_2)z_b}{\mu(Q_2) - \mu(Q_{in})}. \tag{17}$$

Equivalently, for the downstream case, we have

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$$\frac{H_{in} - H_b}{z_b} - \mu(Q_{in}) = 0$$
$$\frac{H_b - H_L}{z_L} - \mu(Q_2) = 0 \qquad (18)$$
$$\frac{H_L - H_{out}}{L - z_b - z_L} - \mu(Q_{out}) = 0.$$

with $Q_2 = Q_{in} - Q_b$, which can be solved for z_L as

$$z_{L_d} = \frac{H_{out} - H_{in} + \mu(Q_{in})(z_b) + \mu(Q_{out})(L - z_b)}{\mu(Q_{out}) - \mu(Q_2)}. \qquad (19)$$

Then, Eqs. (17)-(19) represent the upstream and downstream leak position respectively. In a complementary way with Eq. (14), we use this expression for leak detection and localization in the event of one leak in a branched pipeline, keeping in mind the idea of applying the method in a multi-branched pipeline, which will be discussed in next sections.

3. MULTI BRANCHED PIPELINE MODELS FOR LEAK DIAGNOSIS PURPOSES

In the last section, as was reported in González et al. (2017), the problem of leak diagnosis in a pipeline with one branch junction was solved. This was achieved by using flow and pressure measurements of the ends and the knowledge of the position and flow in the branch. In that case, the leak can be located upstream or downstream of the branch, but; What happens if we have two or more branches? Will it be possible to systematize a methodology to detect and locate a leak regardless of the number of branches? This section seeks to answer this question. Let's start by analyzing the models of the fluid in the pipeline for the cases of two and three branches, obtaining the leak indicator signals (residues) and their position in each case.

3.1 Mathematical models for two branches in a pipeline

Consider initially the stage of a pipeline with two branches, which in case of a leak, can be located in three possible sections, as shown in Fig. 5. As we did before, to obtain a fluid model of this branched pipeline, boundary conditions at the intersection point are required Chaudry (1979).

First, we are interested in residual generation for this new scenario. It can be seen that the case of two branches with no leaks is equivalent to having a three-section pipeline. Then, from (1)-(2), in steady state conditions, this stage can be modeled as

$$\frac{dH_i(z_i)}{dz_i} - \mu(Q_{in}) = 0; \quad \text{for} \quad 0 \le z_i \le z_{b_1}$$
$$\frac{dH_i(z_i)}{dz_i} - \mu(Q_2) = 0; \quad \text{for} \quad z_{b_1} < z_i \le z_{b_2}, \qquad (20)$$
$$\frac{dH_i(z_i)}{dz_i} - \mu(Q_{out}) = 0; \quad \text{for} \quad z_{b_2} < z_i \le L,$$

where the corresponding solutions can be obtained, as in (13):

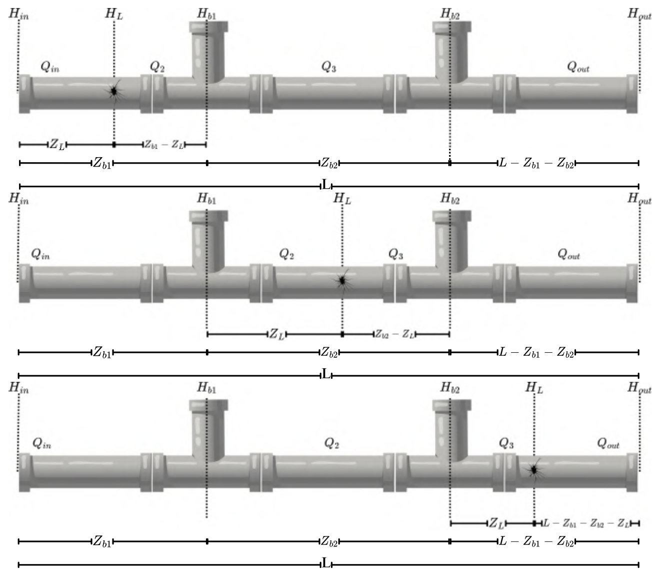

Fig. 5. Possible leak cases in a two-branched pipeline.

$$\frac{H_{in} - H_{b_1}}{z_{b_1}} - \mu(Q_{in}) = 0$$
$$\frac{H_{b_1} - H_{b_2}}{z_{b_2}} - \mu(Q_2) = 0 \qquad (21)$$
$$\frac{H_{b_2} - H_{out}}{L - z_{b_1} - z_{b_2}} - \mu(Q_{out}) = 0,$$

then, having in mind to use the estimation of the known positions of the branches as in (15), from (21) we obtain

$$z_{b_1} = \frac{H_{out} - H_{in} + \mu(Q_2)(z_{b_2}) + \mu(Q_{out})(L - z_{b_2})}{\mu(Q_{out}) - \mu(Q_{in})} \qquad (22)$$

and

$$z_{b_2} = \frac{H_{out} - H_{in} + \mu(Q_{in})z_{b_1} + \mu(Q_{out})(L - z_{b_1})}{\mu(Q_{out}) - \mu(Q_2)}, \qquad (23)$$

which represent the corresponding branch positions in the considered stage.

3.2 Mathematical models for three branches in a pipeline

Through an analogous procedure, the three branches scene -as shown in Fig. 6-9-, when the pipeline is free of leaks, the fluid in the pipeline can be modeled as in (20), where the corresponding solutions are represented by

$$\frac{H_{in} - H_{b1}}{z_{b_1}} - \mu(Q_{in}) = 0$$
$$\frac{H_{b_1} - H_{b_2}}{z_{b_2}} - \mu(Q_2) = 0$$
$$\frac{H_{b_2} - H_{b_3}}{z_{b_3}} - \mu(Q_3) = 0 \qquad (24)$$
$$\frac{H_{b_3} - H_{out}}{L - z_{b_1} - z_{b_2} - z_{b_3}} - \mu(Q_{out}) = 0,$$

with $Q_2 = Q_{in} - Q_{b_1}$ and $Q_3 = Q_{out} + Q_{b_3}$.

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From 24, assuming again that the system is in steady-state conditions, the positions of each branch can be easily obtained, resulting in:

$$z_{b_1} = \frac{\Delta H + \mu(Q_2)z_{b_2} + \mu(Q_3)z_{b_3}}{\mu(Q_{out}) - \mu(Q_{in})} + \frac{\mu(Q_{out})(L - z_{b_2} - z_{b_3})}{\mu(Q_{out}) - \mu(Q_{in})}, \quad (25)$$

$$z_{b_2} = \frac{\Delta H + \mu(Q_{in})z_{b_1} + \mu(Q_3)z_{b_3}}{\mu(Q_{out}) - \mu(Q_2)} + \frac{\mu(Q_{out})(L - z_{b_1} - z_{b_3})}{\mu(Q_{out}) - \mu(Q_2)}, \quad (26)$$

and

$$z_{b_3} = \frac{\Delta H + \mu(Q_{in})z_{b_1} + \mu(Q_2)z_{b_2}}{\mu(Q_{out}) - \mu(Q_3)} \frac{+\mu(Q_{out})(L - z_{b_1} - z_{b_2})}{\mu(Q_{out}) - \mu(Q_3)}, \quad (27)$$

with $\Delta H = H_{out} - H_{in}$. As we mentioned in past sections, a sensitivity analysis and simulation tests, show that the sign of the residuals generated by these expressions (22-23 and 25-27) indicates invariably the region (pipeline section) in which the abnormal condition or leak is located.

3.3 Leak localization

Once the section in which the leak is located is known, it is necessary to obtain a more precise location of it. This is done by adding a section of pipe to each of the previously analyzed models and following the same methodology, from which we obtain:

For the two branches stage:

$$z_{L_1} = \frac{\Delta H + \mu(Q_2)z_{b_1} + \mu(Q_3)z_{b_2} + \mu(Q_{out})z_3}{\mu(Q_2) - \mu(Q_{in})}, \quad (28)$$

$$z_{L_2} = \frac{\Delta H + \mu(Q_{in})z_{b_1} + \mu(Q_3)z_{b_2} + \mu(Q_{out})z_3}{\mu(Q_3) - \mu(Q_2)}, \quad (29)$$

and

$$z_{L_3} = \frac{\Delta H + \mu(Q_{in})z_{b_1} + \mu(Q_2)z_{b_2} + \mu(Q_{out})z_3}{\mu(Q_{out}) - \mu(Q_3)}, \quad (30)$$

for a leak located in the first, second, or third section, respectively, with $z_3 = L - z_{b_1} - z_{b_2}$.

For the three branches stage.

$$z_{L_1} = \frac{\Delta H + \mu(Q_2)z_{b_1} + \mu(Q_3)z_{b_2} + \mu(Q_4)z_{b_3}}{\mu(Q_2) - \mu(Q_{in})} \frac{+\mu(Q_{out})(L - z_{b_1} - z_{b_2} - z_{b_3})}{\mu(Q_2) - \mu(Q_{in})} \quad (31)$$

with $Q_2 = Q_{out} + Q_{b_1} + Q_{b_2} + Q_{b_3}$,

$$z_{L_2} = \frac{\Delta H + \mu(Q_{in})z_{b_1} + \mu(Q_3)z_{b_2} + \mu(Q_4)z_{b_3}}{\mu(Q_3) - \mu(Q_2)} \frac{+\mu(Q_{out})(L - z_{b_1} - z_{b_2} - z_{b_3})}{\mu(Q_3) - \mu(Q_2)}, \quad (32)$$

with $Q_2 = Q_{in} - Q_{b_1}$, $Q_3 = Q_{out} + Q_{b_2} + Q_{b_3}$, and $Q_4 = Q_{out} + Q_{b_3}$

$$z_{L_3} = \frac{\Delta H + \mu(Q_{in})z_{b_1} + \mu(Q_2)z_{b_2} + \mu(Q_4)z_{b_3}}{\mu(Q_4) - \mu(Q_3)} \frac{+\mu(Q_{out})(L - z_{b_1} - z_{b_2} - z_{b_3})}{\mu(Q_4) - \mu(Q_3)}, \quad (33)$$

with $Q_2 = Q_{in} - Q_{b_1}$, $Q_3 = Q_{in} + Q_{b_1} + Q_{b_2}$, and $Q_4 = Q_{out} + Q_{b_3}$

and

$$z_{L_4} = \frac{\Delta H + \mu(Q_{in})z_{b_1} + \mu(Q_2)z_{b_2} + \mu(Q_3)z_{b_3}}{\mu(Q_{out}) - \mu(Q_4)} \frac{+\mu(Q_{out})(L - z_{b_1} - z_{b_2} - z_{b_3})}{\mu(Q_{out}) - \mu(Q_4)}, \quad (34)$$

with $Q_2 = Q_{in} - Q_{b_1}$, $Q_3 = Q_{in} - Q_{b_1} - Q_{b_2}$, and $Q_4 = Q_{in} - Q_{b_1} - Q_{b_2} - Q_{b_3}$ for a leak located in the first, second, third or fourth section, respectively, with $z_3 = L - z_{b_1} - z_{b_2}$.

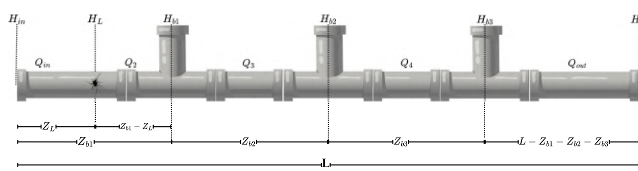

Fig. 6. Leak in the first section in a three-branched pipeline.

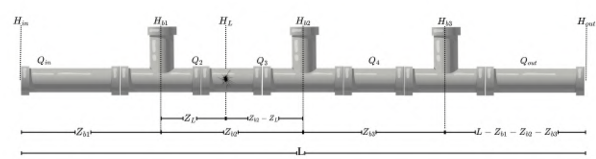

Fig. 7. Leak in the second section in a three-branched pipeline.

4. RESULTS

In this section, simulation test are presented to illustrate the efficiency of the leak detection methodology. To do this, a 200 m of length pipeline is considered, where the parameters of the simulated system are: $H_{in} = 20\ m$, $H_{out} = 5.7\ m$ $g = 9.81\ m/s^2$, $D = 0.1016\ m$, $b = 1284\ m/s$ and $f = 0.022$.

For the two branches stage, as in Eq. 15, it is possible to generate:

$$r_1(t) = z_{b_1} - \hat{z}_{b_1}(t), \quad (35)$$

and

$$r_2(t) = z_{b_2} - \hat{z}_{b_2}(t), \quad (36)$$

as leak indicator signals.

As you can see in Table 1, cases 1 and 2 correspond to a leak located downstream of both branches. So that, $r_1(t)$ and $r_2(t)$ are both negative. In tests 3 and 4, the leak is simulated between the branches and it can be seen that $r_1(t)$ is negative while $r_2(t)$ is positive. Finally, in tests 5 and 6 it has been simulated a leak upstream of both branches and, in consequence, $r_1(t)$ and $r_2(t)$ are both positive, indicating the leaking section.

Table 1. Residual generation for a two-branched pipeline

#	z_L	z_{b_1}	$\hat{z}_{b_1}$	r_1	z_{b_2}	$\hat{z}_{b_2}$	r_2
1	130	50	72.09	−22.09	100	112.18	−12.18
2	190	50	77.35	−27.35	100	121.39	−21.39
3	60	50	61.94	−11.94	150	112.18	37.82
4	140	50	85.68	−35.68	150	135.56	14.44
5	10	100	49.73	50.27	150	61.92	88.08
6	90	100	98.21	1.79	150	131.61	18.39

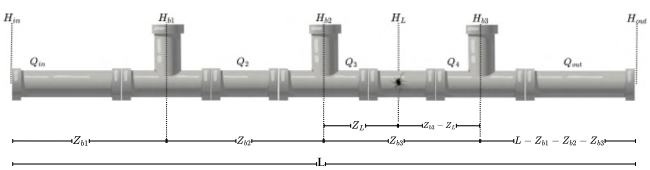

Fig. 8. Leak in the third section in a three-branched pipeline.

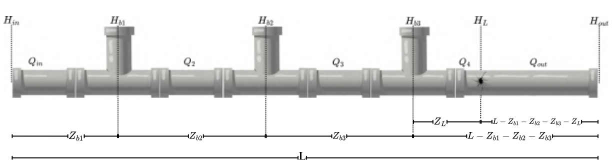

Fig. 9. Leak in the fourth section in a three-branched pipeline.

5. CONCLUSION

A steady-state approach to detect and locate leaks in a steady state in a branched pipeline has been proposed. The requirements to do that are the measurements of pressure and flow at the extremes of the pipeline and the flow through the known extractions (branches). The keys of the method are:

(1) The position error of the known branch junction if abnormal conditions occur is used as a residual to indicate an abnormal condition.
(2) The equations to calculate the leak position upstream or downstream of a specific branch junction.

Although it has not been completed in this work, it is expected to demonstrate that the sign of the residue generated through mass balance is invariant to the number of branches, which would result in a great industrial application due to the low cost and the simplicity of its implementation.

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