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Convergence Time Decreasing of Extremum Seeking Control in a Distillation Column

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Abstract: In this work, the convergence time of the Extremum Seeking Control applied to ensure the optimal performance of a distillation column is studied and diminished in such a way that it is comparable to the one of a linear PI controller. The distillation column is one of the trays in continuous operation that separates an ideal binary mixture. Firstly, a sensitive-type analysis of the convergence time with respect to the ESC tuning parameters is carried out, locating the values for which the convergence time is minimum; next, a variation of the ESC is applied, which consists of the addition of a decreasing function to initially speed up the ESC performance, and a saturation block is also added to constrain likely large changes in the control inputs. The control problem is one of regulation, for which, for validation purposes, the optimal conditions for the testing cases are determined by a conventional sensitivity analysis of the output with respect to the inputs. The testing runs show not only the effectiveness of the ESC but also that the modified ESC has a reduced convergence time compared to the typical ESC, and it is even comparable to that of a linear PI controller.

Keywords: Extremum Seeking Control, Black Box Model, Regulatory Control, Distillation Column, Aspen Dynamics, Optimizing parameters, PI Controller.

1. INTRODUCTION

In the process industry, product purification is one of the most important stages in the production chain of a given product, as it decisively impacts the company's economics.

To select a suitable separation method, several aspects must be taken into account, including the key differences between the physical properties of the compounds that will be involved in the separation process and the purity requirements, while also doing so at the lowest possible economic cost. Among all the existing separation processes implemented to purify a compound of interest, one stands out: distillation carried out in a tray disillatuon column (Mustapa, et. al., 2007). This process is mainly based on the principle of the difference in boiling points between the components that make up the mixture to be separated.

Distillation columns are widely used in industry because they have several advantages that make them extremely attractive for the economics of a production plant. One advantage is that distillation columns can operate continuously, ensuring high production rates of the desired component. On the other hand, they can also guarantee extremely high purity of the component of interest, compared to some other separation processes. Not only that, but they are also capable of purifying multiple components simultaneously. However, not everything is so wonderful when it comes to the application of distillation columns, as their implementation presents a challenge.

A distillation column in its simplest form is shown in Figure 1; it has an inlet stream that can be fed into a specific column tray and two outlet streams: one at the dome, called the distillate stream, or simply the distillate, and one at the bottom, called the bottoms. Furthermore, a column must have a specific number of trays or stages, in which the purification process occurs gradually, stage by stage, until it reaches the dome; at this point, the most volatile component exits in higher concentrations while the least volatile component goes through the bottom.

Initially, work must be performed on designing the distillation columns, calculating the minimum plates using a method such as the one called Mcabe-Thiele or Ponchon-Savarit (Henley and Seader, 2011), and then determining the operating conditions that guarantee the required purity for the components of interest.

In this work, a distillation column whose structure has already been designed is considered; so, operation conditions must be determined, which typically consist of two inputs to reach the purity requirements in the output streams: (1) the reboiler duty and (2) the top reflux flow rate.

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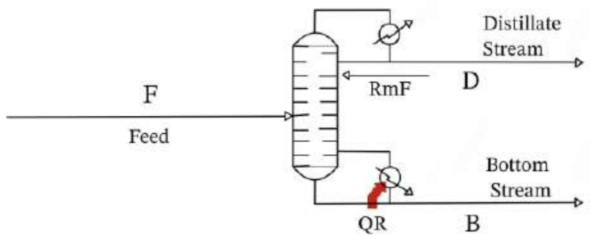

Fig. 1. Distillation column in continuous operation to separate an ideal binary mixture

It is important to note that a distillation column requires a significant amount of energy for its operation, sometimes exceeding half of the plant's operating cost; therefore, the reboiler duty, which is linked to the reflux flow rate, is crucial. In turn, the purity requirement is typically coupled to the minimum energy consumption, for which several techniques can be followed.

One of the most widely used techniques is response surface construction (e.g., BOX, G.E.P, et. al., 1951), which involves creating a mesh of the input variables (reboiler duty and reflux flow rate) and generating a surface based on an output variable, which in these cases could be, for example, the concentration of the most volatile component in the distillate stream. This enables finding the combination of input variables that guarantees the desired composition without going too far. This technique is effective but extremely laborious, as it requires a large number of runs to obtain a satisfactory mesh on the surface and a refined result. It also requires a mathematical model of the column to simulate each of the mesh points.

Other widely used and more sophisticated techniques are the ones called stochastic (e.g., Sun, J., et. al., 2023), which have been proven as very effective in optimal point searching; however, they have the disadvantage that there is no way to guarantee that the found optimal point corresponds to the actual optimum. Translated to the case of distillation columns, this means that satisfactory operating conditions that meet purity requirements can be found, but there is no way to guarantee that these conditions are the best existing throughout the system domain. Similar to the response surface method, it necessitates the use of a mathematical model.

This work follows a previous work (e.g., Hernández-Escoto, H., et. al., 2022) on the application of the technique Extremum Seeking Control to optimize the performance of a distillation column without requiring the mathematical model of the process. This technique is attractive since it can be used as a controller and an optimizer at the same time; however, its disadvantage lies in the long time of convergence and oscillating trajectories that tackle its on-line implementation in a real system. In the case the mathematical model is available, it can be used as an off-line optimization technique. It is worthy to recall that this technique has already been implemented to find the optimal operating conditions for bioreactores and chemical reactors (e.g., Ixbalank Torres-Zúñiga, et. al., 2021). In this work, an optimal study and performance of the ESC will be presented in two different cases, the first one optimizing the degrees of freedom of the ESC system and the second one adding a new variation of the ESC with an specific function to speed up the performance of the control system.

Regarding control for distillation columns, one of the most widely followed approaches is the implementation of conventional PI (proportional-integral) controllers, which is favored for the simplicity of their mathematical structure and ease of tuning, enabling rapid convergence times.

It should be noted that there are much more complex controllers that can achieve faster convergence times than a PI type controller, such as the SMC (e. g. Wang, H., et. al., 2020) and the MNPC (e. g. Jazaery-Rad H., 2004), but in this article our objective is not to demonstrate whether the ESC is capable of beating these controllers in convergence times, but rather to find the shortest convergence time that can be achieved using the ESC technique, by performing a sensitivity analysis on its tuning parameters, and comparing this minimum convergence time with that of the PI type controller, which would be the controller implemented in a column as simple as the one addressed in this work, which consists of a distillation column that separates a binary and ideal mixture.

The MNPC has been applied in severas cases to control a distillation column (e. g. Qian X., et. al., 2023) but all the cases reported have shown the need of the knowledge of the mathematical model.

So, as mentioned above, in this work a sensitivity analysis will be performed on various parameters that affect the operation of the Extremum Seeking Control control technique, to search for the parameters that minimize the convergence time of the system and compare this with the convergence times that would be obtained with the implementation of a conventional proportional-integral type controller.

However, this type of technique has the peculiarity that it depends on having full knowledge of the mathematical model and most of them have not been applied to distillation columns, because the mathematical model of a column is very extensive and has a large number of differential equations, most of which are non-linear in nature.

In the following section 2 the distillation column is described, as well as its construction in the commercial simulator Aspen Plus® and Aspen Dynamics® in section 3 the control problem is described and you will see the construction of the ESC system in Simulink for all cases and its link with Aspen Dynamics in Section 4. Section 5 presents the response surfaces of the process for the cases we will address, to later in section 6 show the results obtained. On section 7 the conclusions will be presented.

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2. DISTILLATION COLUMN IN ASPEN

Aspen Plus® is compelling software specialized in the chemical engineering field; in this software, the simulation of several chemical processes can be carried out.

The distillation column works getting focused on the difference between the boiling point of the components of the mixture. If it exists a considerable difference between the boiling points of the mixture components, then the application of this technique can be implemented or considered to purify the mixture.

The first step in Aspen Plus is to define the thermodynamic model to simulate the nature of the mixture, this is a very important part of the simulation, there are some mixtures that they present some extra molecular interactions and a thermodynamic model to consider these cases must be chosen.

For the mixture treated in this paper, an ideal and binary mixture of benzene and toluene, a regular thermodynamic model can be selected, for example the Peng – Robinson model.

A conventional distillation column has two degrees of freedom, which are generally the thermal load of the reboiler and the reflux flow of the distillate. By modifying them, we can obtain the desired purities in each of the column components.

It is considered a trays distillation column in continuous operation to separate an ideal binary mixture. It is recalled a configuration used as a benchmark in distillation column studies (Figure 1), which consist of a system of 30 trays, and a reboiler and a total condenser, where the feeding stream is in the stage 15 (the stages are numbered from top down to bottom). The feeding stream is saturated liquid of the equimolar mixture of benzene-toluene, to separate. It is assumed that the output streams are also saturated liquid, rich in their corresponding key components: benzene in the top and toluene in the bottom.

The mathematical model of a distillation column is too large and non-linear, that why we use the Aspen Plus software to fix the distillation column. To design, the application of the RadFrac module must be carried out and the specifications of the column must be stablished.

The distillation column works to separate an ideal, equimolar and binary mixture or benzene and toluene, that why the Peng-Robinson thermodynamical model must be chosen.

The column has 30 stages, where the feed stream gets in the fifteenth stage, the operation pressure of the column is 1 atm and the pressure drop in the column is 0.7 atm. This column works with a total condenser and a kettle reboiler. The initial operation conditions are a reflux ratio of 1 and a reboiler duty of $4.8\,\frac{GJ}{h}$. The feed stream has a molar flow of $100\,\frac{kmol}{h}$ and gets in the column at $T = 25\,°C$ and $P = 2\,atm$.

The dynamic model must be activated, and the next specification are required to finish with the specification of the distillation column on Aspen Plus. We need to indicate the stages where the dynamic model will be applied. Theses stages are from stage 2 to stage 29. The distillate drum dimensions are required to run the dynamic simulation, the drum height is 3 feet, and the drum diameter is 2 feet.

With this specification the simulation is run and the program in Aspen Dynamics is generated and ready to link with Simulink.

3.THE CONTROL PROBLEM.

The objective is to identify the operational conditions of the reflux flow rate that will optimize purity and recovery for both components. The reboiler duty of the distillation column in this case is $4.8\,\frac{GJ}{h}$.

The control problem lies in the fact that the conventional Extremum Seeking Control technique is much slower than a conventional PI controller, even though its operating conditions have been highly refined to ensure minimum convergence times. In this case, a modification to the conventional Extremum Seeking Control technique is proposed, which considerably accelerates the technique's performance, bringing it close to matching that of a conventional PI controller.

4. EXTREMUM SEEKING CONTROL IN SIMULINK

The Extremum Seeking Control is built on the program MATLAB® in the section called Simulink, where some modules are needed to set the control diagram. Three different cases are shown: the first one is the conventional ESC diagram, the second one is a new variation of the conventional ESC, and the third one is the PI controller system. These modules are presented in Table 1, and the control diagrams built on Simulink are presented in Figures 2, 3, and 4.

Table 1. Simulink Modules used to fix the control diagram.

Module: AM Simulink	This module is the link between the Aspen Dynamics file and MATLAB. The control inputs and outputs are assigned.
Module: Function	This module is used to define an objective function to guide the Extremum seeking control technique to the desired set point of the system.
Module: Transfer Function	This module is used to build a high pass filter to separate the oscillations of the signal that comes from the Function Module. As general rule, the frequency of the filter must be at least 10 times lower than the frequency w of the sine wave.
Module: Product	This module is used to multiply the signals.

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Module: Integrator	This module is used to calculate the area under the curve of the signal and transform it into a number.
Module: Gain	This module is a signal amplifier. The signal is amplified k times.
Module: Sine Wave	This module is used to create the oscillations on the system, the amplitude A and the frequency w must be specified.
Module: Constant	This module is used to set the initial values of the control inputs.
Module: Mux	This module is used to define the module function for n variables.
Module: Sum	This module is used to add or subtract signals.
Module: Abs	This module is used to get the absolute value for a signal.
Module: Saturation	This module is used to keep the signal within a specific range of values.

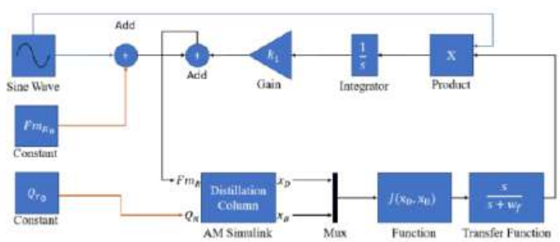

Fig 2. Conventional ESC system fixed in Simulink.

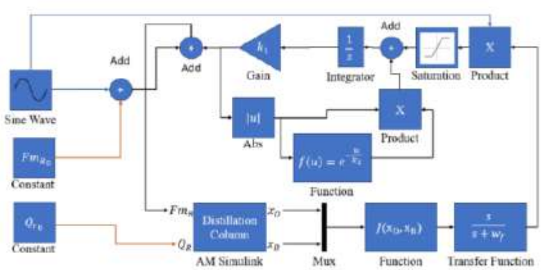

Fig 3. Variation of ESC system fixed in Simulink.

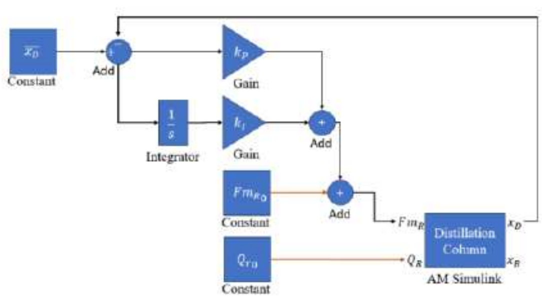

Fig 4. PI Controller fixed on Simulink.

5. RESPONSE SURFACES

The response surface technique is used to determine the best conditions of the control outputs with respect to the control inputs, so the figure of the column described in section 2 was obtained in the simulator Aspen Plus®. In Section 3, the figures illustrate how the distillate and bottom compositions of the light component vary with changes in the mass reflux flow and reboiler duty. In this work we use the response surface technique to find the optimal conditions maintaining the reboiler duty of the column working at a reboiler duty of 4.8 $\frac{GJ}{h}$.

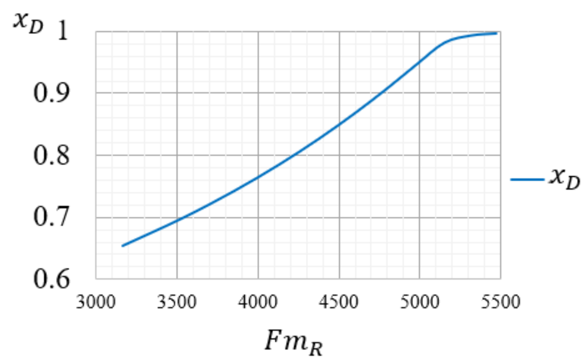

Fig 5. Distillate composition for the light component.

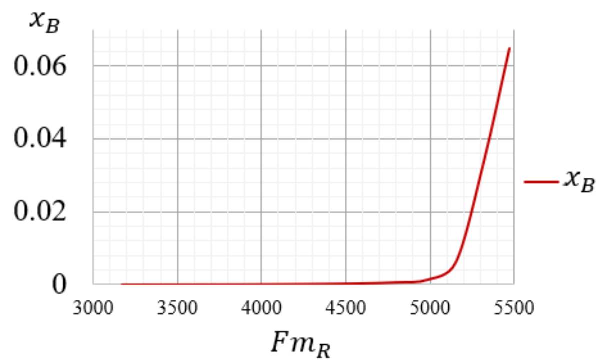

Fig 6. Bottom composition for the light component.

As it can be observed, the trajectories for the light component in distillate and bottom streams have opposite behaviors; therefore, an objective function with a critical value on the desired conditions must be established. The objective function defined is:

$$J = (abs(1 - x_D - x_B))(-10) \quad (1)$$

Trajectory of the objective function is shown in Figure 7.

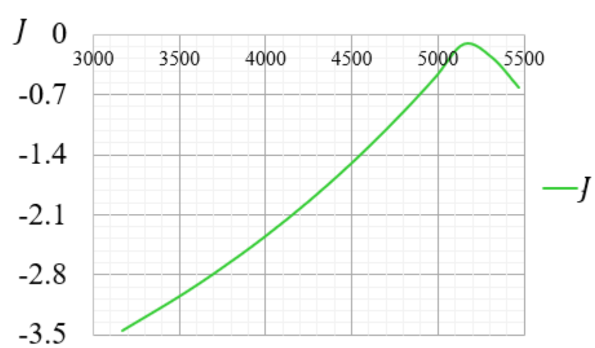

Fig 7. Objective function.

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The optimal conditions found are presented on Table 3.

Table 3. Optimal conditions found on the response surface.

Fm_R	$5200\,\frac{kg}{h}$
Q_R	$4.8\,\frac{GJ}{h}$
x_D	0.988
x_B	0.012

6. RESULTS

In the first simulation, a conventional ESC system was built to achieve optimal conditions at a specific time; however, in several cases, the ESC parameters were modified to minimize convergence time. In the second simulation a modified ESC system was built, and the optimal conditions were obtained substantially faster than in the conventional case. Finally, the third case, where a PI controller guides the system to its optimal conditions, is simulated.

The results of these cases are presented in Figures 8 to 11.

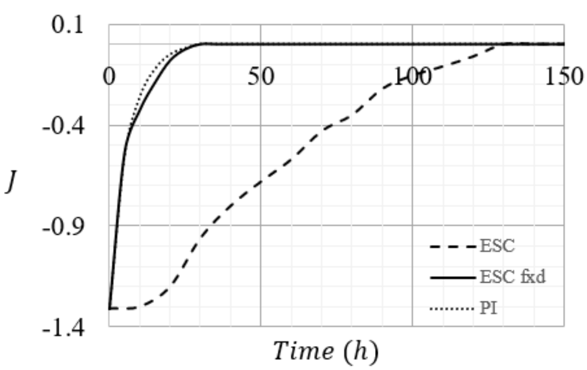

Fig 8. Objective function vs time.

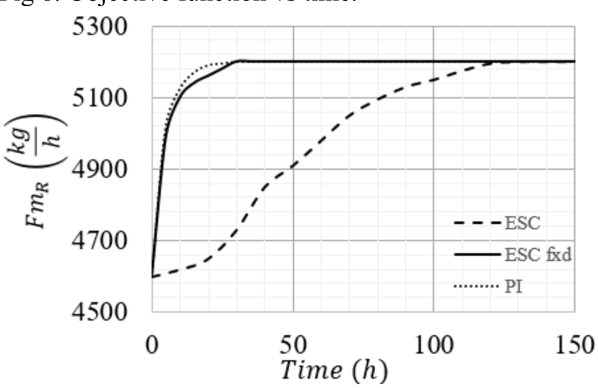

Fig 9. Reflux mass flow vs time.

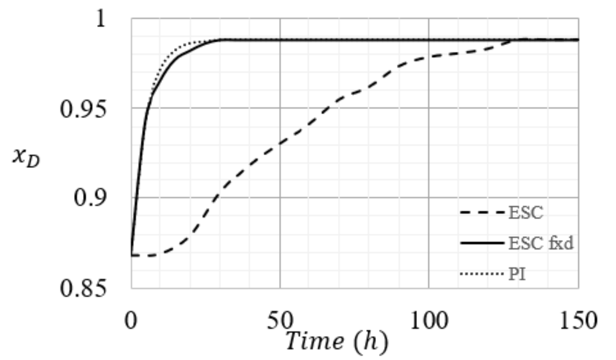

Fig 10. Distillate composition vs time.

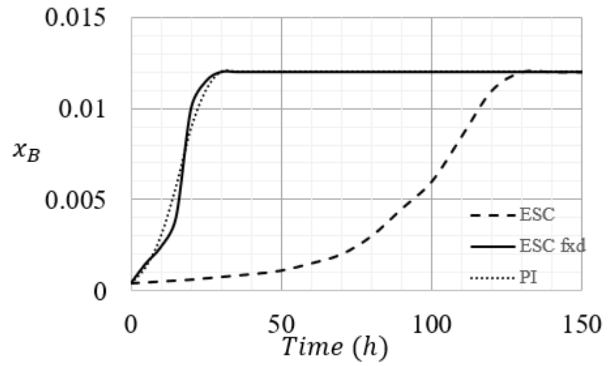

Fig 11. Bottoms composition vs time.

All the cases developed are reported in Table 4 and Table 5.

Table 4. Convergence time for several cases modifying the frequency ω for an integral gain of $k_1 = 10$.

ω	$Time\ (h)$
$\frac{\pi}{3}$	No convergent
$\frac{\pi}{2}$	No convergent
π	130
$\frac{4\pi}{3}$	150
$\frac{3\pi}{2}$	170
$\frac{5\pi}{3}$	> 200
2π	> 200

Table 5. Convergence time for several cases modifying the integral gain for a frequency of $\omega = \pi$.

k_1	$Time\ (h)$
1	> 200
3	> 200
5	185
8	135
10	130

The conventional ESC has the disadvantage that it is too slow to be able to compete against a PI controller. In cases one and three, the last sentence was confirmed; the

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conventional ESC exhibited a settling time nearly four times longer than that of the PI controller.

In case two, a new variation of the conventional ESC performed as it was expected, and a nice performance is exhibited in Figures 8 to 11. The settling time reached in cases two and three is the same; this means that the variant of the ESC proposed can compete in the settling time, and it is ready to be tested in future situations for more complex cases.

Additionally, studying the ESC parameters is important for maximizing a typical ESC; this step is essential for tuning the variant of the ESC before proceeding to implement the new version. It is recommended to use a frequency system measured in hours, as the system has a normal deviation; tuning to a low frequency may result in a non-converging system. The integral gain in the ESC is also important; this procedure is the easy way to accelerate the process and to reduce settling as much as possible.

7. CONCLUSIONS.

This study shows that a mathematical free-model control technique can be used to determine optimal operating conditions for a distillation column in real-time, achieving convergence speeds like those of a PI controller.

Three different cases were developed comparing the settling time of a conventional ESC, a new modification proposed for the ESC system, and a PI controller system, where effective results were obtained. In the three cases, the optimal point was reached and corroborated with the one obtained from the response surface technique.

The implementation of the modified Extremum Seeking Control should be viewed as a significant improvement in control and optimization of distillation columns, as it can consistently determine conditions that maximize a user-defined objective function, regardless of the process' circumstances.

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