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Scalable Design of a Distributed Proportional-Retarded Protocol for a Class of Multiagent Systems

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Abstract: In this paper, we present analytical tuning formulas for the parameters of a distributed Proportional-Retarded protocol operating in a class of double-integrator multiagent system. For undirected graphs, we use the theory of convex directions for quasipolynomials to show that the developed tuning technique remains valid independently of the number of agents in the network. An array of numerical examples are provided to demonstrate the relevance of the approach.

Keywords: Multiagent systems; Time-delay systems; Convex directions; Delay-based control.

1. INTRODUCTION

Distributed control in multiagent systems (MAS) has attracted significant attention in recent years across a wide range of applications (Lynch, 1996; Qin et al., 2016). However, extending these control strategies to large-scale MAS remains challenging. Although the conventional modal decomposition technique can reduce the control design task to the analysis of a finite set of subsystems (Ren and Cao, 2010), its validity breaks down at the collective level when the number of agents increases (Zhang, 2011).

It is well known that control tasks in MAS (such as formation, consensus, and synchronization) can be accelerated by incorporating derivative terms into the feedback law. However, while derivative action improves transient performance, it also amplifies measurement noise (Åström and Hägglund, 2001), which limits its practicality in real-world applications. This has motivated the development of derivative-free approaches (Suh and Bien, 1979; Berghuis and Nijmeijer, 1993), including delay-based controllers that introduce intentional delays to accelerate the system response. Despite their potential, such controllers pose significant challenges due to the infinite-dimensional nature of the resulting system dynamics, a difficulty that is exacerbated in MAS. Recent work has addressed these issues using convex optimization to ensure stability and robustness (Fridman and Shaikhet, 2016), while analytical methods have focuses mainly on single-input single-output systems with a single delay Ramírez et al. (2015), leaving MAS with multiple, but uniform, delays as an open problem that this work seeks to address.

Previous studies have explored the use of intentional delays in distributed control protocols to regulate agent behavior in noise-free dynamic networks (Cao and Ren, 2010; Huang et al., 2016; Yu et al., 2011; Meng et al., 2010; Yu et al., 2013; Meng et al., 2013; Song et al., 2015). In these approaches, delayed position measurements effectively act as derivative-like feedback, allowing the network to achieve consensus while avoiding direct velocity measurements. These studies demonstrate that delayed terms can accelerate convergence, attenuate noise, and improve performance while reducing control effort, see (Cao et al., 2008; Cao and Ren, 2010; Huang et al., 2016; Yu et al., 2011; Meng et al., 2010; Li et al., 2010).

Despite these promising results, a persistent challenge lies in deriving systematic tuning rules for delay-based distributed controllers, especially in large-scale MAS. Recent attempts (Ramírez et al., 2019; Ramírez and Sipahi, 2018b,a) have introduced schemes involving multiple non-commensurate delays and heterogeneous gains, but their complexity limits practical implementation. In contrast, in this paper, we develop a scalable design framework for a class of MAS, enabling both synthesis and deployment of a delay-based distributed control protocol.

2. PROBLEM STATEMENT

We consider a network of N vehicles moving in the (x, y) plane (Deshpande et al., 2013). Each vehicle is represented by two double-integrators of the form

$$\begin{aligned} \dot{x}_i(t) &= Ax_i(t) + Bu_i(t), \quad i = 1, \ldots, N \\ r_i(t) &= Cx_i(t), \end{aligned} \tag{1}$$

where $x_i^\top = (x_{i1}, x_{i2}, y_{i1}, y_{i2})$ is the state vector, with $r_i \coloneqq (x_{i1}, y_{i1})^\top$ the position vector and $v_i \coloneqq (x_{i2}, y_{i2})^\top$ the velocity vector of agent i. The control signal is $u_i(t)$ and the matrix coefficients are defined as

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$$A=\begin{pmatrix}0&1&0&0\\0&0&0&0\\0&0&0&1\\0&0&0&0\end{pmatrix},\ B=\begin{pmatrix}0&0\\1&0\\0&0\\0&1\end{pmatrix},\ C=\begin{pmatrix}1&0\\0&0\\0&1\\0&0\end{pmatrix}^{\top}. \quad (2)$$

According to C, in (2), the output is $r_i = (x_{i1},\, y_{i1})^{\top}$. The system (1) is subject to the Proportional-Retarded (PR) protocol:

$$u_i(t) = -k_1 r_i(t) + k_2(r_i(t-h) + \sigma_i(t-h) - d_i),$$
$$\sigma_i(t) = \Sigma_{j=1}^{N} a_{ij}(r_i(t) - r_j(t)), \quad (3)$$

where $d_i \in \mathbb{R}^2$ represents the relative displacement information between agents, delay $h > 0$ is intentionally introduced as an additional control parameter, and $k_1 \neq 0$ and $k_2 \neq 0$ are the controller gains. The scalars a_{ij} represent the elements of the adjacency matrix cassociated with the graph $\mathcal{G}$ that describes the network topology associated with (1). Notice that this approach eliminates the need for velocity measurements or estimates via observers.

Assumption 1. The graph $\mathcal{G}$ associated with the system (1) is undirected and connected.

The problem that we aim to solve is as follows.

Problem 1. Find the parameters k_1, k_2 and h that guarantee the fastest speed of response for the closed-loop system consisting of the MAS (1) and the PR protocol (3).

3. ANALYSIS OF THE PR PROTOCOL

3.1 Modal decomposition of the system

Following (Deshpande et al., 2013), let us write (3) in compact form as:

$$U(t) = -(I_N \otimes K_1C)X(t) + (I_N \otimes K_2)D + ((I_N + \mathcal{L}) \otimes K_2C)X(t-h), \quad (4)$$

where $\mathcal{L}$ is the graph Laplacian, $K_1 = k_1 I_2$, $K_2 = k_2 I_2$, $D = \frac{1}{k_2}((I \otimes K_1C) - (I_N + \mathcal{L}) \otimes K_2C)X_f$, and

$$X_f = \begin{bmatrix} x_1^f & 0 & y_1^f & 0 & \cdots & x_N^f & 0 & y_N^f & 0 \end{bmatrix}^{\top}.$$

Here, (x_i^f, y_i^f) is the steady-state position of the i-th agent and $X(t)$ is formed by concatenating the states of the agents. Similarly, $U(t)$ is formed by concatenating the control inputs. With the above, (1) with (3) is

$$\dot{X}(t) = \overline{A}_0 X(t) + \overline{A}_1 X(t-h) + (I_N \otimes BK_2)D, \quad (5)$$

where $\overline{A}_0 = I_N \otimes (A - BK_1C)$, and $\overline{A}_1 = (I_N + \mathcal{L}) \otimes BK_2C$. Introducing next the transformation $Z(t) = X(t) - X_f$ gives

$$\dot{Z}(t) = \overline{A}_0 Z(t) + \overline{A}_1 Z(t-h) + (\overline{A}_0 + \overline{A}_1)X_f + (I_N \otimes BK_2)D. \quad (6)$$

The goal now is to ensure that $(\overline{A}_0 + \overline{A}_1)X_f + (I_N \otimes BK_2)D = 0$ by choosing D. Suppose that K_1 and K_2 are given, and the compensation vector D is chosen to satisfy

$$(I_N \otimes K_2)D = ((I_N \otimes K_1C) - ((I_N + \mathcal{L}) \otimes K_2C))X_f. \quad (7)$$

Since $K_2 = k_2 I_2$, $(I_N \otimes K_2) = k_2 I_{2N}$ and $k_2 \neq 0$, then (7) has a unique solution

$$D = {k_2}^{-1}((I_N \otimes K_1C) - ((I_N + \mathcal{L}) \otimes K_2C))X_f. \quad (8)$$

It is clear that for the matrix A in (1) and any X_f, we have $(I_N \otimes A)X_f = 0$. Premultiplying the term $(I_N \otimes B)$ on both sides of (7) gives $(I_N \otimes B)(I_N \otimes K_2)D = (I_N \otimes B)((I_N \otimes K_1C) - ((I_N + \mathcal{L}) \otimes K_2C))X_f$ which, by properties of the Kronecker product, is equivalent to $(I_N \otimes BK_2)D = (I_N \otimes BK_1C)X_f - (I_N \otimes B)((I_N + \mathcal{L}) \otimes K_2C)X_f$.

Subtracting the null term $(I_N \otimes A)X_f = 0$ results in

$$(I_N \otimes BK_2)D = -(\overline{A}_0 + \overline{A}_1)X_f. \quad (9)$$

Thus, using (9) into (6), the system (5) can be rewritten as

$$\dot{Z}(t) = \overline{A}_0 Z(t) + \overline{A}_1 Z(t-h). \quad (10)$$

Since the graph $\mathcal{G}$ is undirected, the Laplacian matrix $\mathcal{L}$ is symmetric and positive semidefinite, implying that $I_N + \mathcal{L}$ is symmetric and positive definite. By spectral decomposition, $I_N + \mathcal{L} = V\Lambda V^{\top}$, where V is a unitary matrix whose columns are the eigenvectors of $I_N + \mathcal{L}$, and $\Lambda = \mathrm{diag}(\mu_1, \ldots, \mu_N)$ is a diagonal matrix containing its eigenvalues. Given that $\mathcal{L}$ has a zero eigenvalue, it follows that $\mu_i \geq 1$ for $i = 1, \ldots, N$.

Assumption 2. The eigenvalue $\mu_1 = 1$ of $I_N + \mathcal{L}$ is simple with eigenvector $(1, \ldots, 1)^{\top} \in \mathbb{R}^N$.

Consider the orthogonal state transformation $Z \to (V^{\top} \otimes I_4)Z = \xi$. In the new coordinates, (10) becomes

$$\dot{\xi}(t) = (I_N \otimes (A - BK_1C))\xi(t) + (\Lambda \otimes BK_2C)\xi(t-h), \quad (11)$$

where the matrix coefficients are in block-diagonal form. Based on (11), the characteristic function of (1) with (3) can be expressed as the product of N factors:

$$\overline{P}(s) = \prod_{j=1}^{N} \overline{p}_j(s) = \prod_{j=1}^{N} (s^2 + k_1 - k_2\mu_j e^{-sh})^2 \quad (12)$$

where $s \in \mathbb{C}$ and μ_j denotes the eigenvalues of $I_N + \mathcal{L}$.

Based on Assumption 1, the factor $\overline{p}_j(s)$ with the smallest eigenvalue is related to the consensus dynamics, and the rest of the factors are related to the disagreement dynamics. As a convention, throughout the rest of this paper, we adopt the following ordering

$$1 = \mu_1 < \mu_2 \leq \cdots \leq \mu_N. \quad (13)$$

Here, we are interested in improving the system's dynamic response. To achieve this, we make the change of variable $s \to s - \gamma$, $\gamma > 0$, in $\overline{p}_j(s)$ and study the stability of the shifted factors

$$p_j(s) = (s^2 - 2s\gamma + \gamma^2 + k_1 - k_2 e^{\gamma h}\mu_j e^{-sh})^2. \quad (14)$$

Therefore, the quasi-polynomial used to study stability with respect to γ of (1) with (3) is

$$P(s) = \prod_{j=1}^{N} p_j(s) = 0. \quad (15)$$

We say that the MAS achieves consensus exponentially fast, with a convergence rate γ, if and only if all the roots of (15) have real parts less than or equal to $-\gamma$.

Note that analyzing the stability of the shifted factors $p_j(s)$ is equivalent to analyzing $\gamma-$stability of the MAS. A problem that arises is that the number of factors increases with the number of agents. We face two technical challenges. The first is that we have only three parameters, h, k_1, and k_2, to design a total of N factors. This restriction limits our design options. The second is that designing these parameters for some μ_j can place the dominant roots of factor p_j in the desired location. However, the same parameter values may not produce the desired dominant

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roots for the rest of the factors. Thus, the competition between the dominant roots of the different factors deserves particular attention.

3.2 Decomposition of the parameter space (h, k_2)

As a preliminary step to characterize the maximum value of γ, we decompose the parameter space

$$\mathcal{D} = \{(h, k_2) : h \in \mathbb{R}_{>0}, k_2 \in \mathbb{R}\}. \tag{16}$$

For this, we use the D-subdivision method (Neimark, 1949) to obtain a partition of $\mathcal{D}$, namely,

$$\mathcal{D} = \bigcup_{\upsilon \in \mathbb{N}} \mathcal{D}(\upsilon), \tag{17}$$

where υ is the number of unstable roots associated with each region $\mathcal{D}(\upsilon)$ in the partition. In particular, we are interested in the stable region denoted by $\mathcal{D}(0)$.

Definition 1. We say that a collection of points $(h, k_2) \in \mathcal{D}$ is a stability crossing boundary if for every point in this collection, (14) has at least one root on the imaginary axis. Each of these points is known as a crossing point.

Definition 2. The curve $\mathcal{K}$ is the boundary of the stability region, namely, $\mathcal{K} = \partial\mathcal{D}(0)$.

Proposition 3. Given $\mu_j \geq 1$, $\gamma > 0$, $\omega > 0$, and $k_1 \in \mathbb{R}$, the crossing points that generate the stability boundaries associated with the j-th factor $p_j(s)$ in (14) are given by

$$h(\omega) = \frac{1}{\omega}\tan^{-1}\left(\frac{2\omega\gamma}{-\omega^2 + \gamma^2 + k_1}\right) + \frac{n\pi}{\omega}, \tag{18}$$

$$k_2(\omega, \mu_j) = \frac{-\omega^2 + \gamma^2 + k_1}{e^{\gamma h}\mu_j \cos(\omega h)}. \tag{19}$$

For the case $\omega = 0$, the equation

$$k_2(0, \mu_j) = \frac{\gamma^2 + k_1}{e^{\gamma h}\mu_j}, \tag{20}$$

generates a crossing point for any $h > 0$.

Proof. According to the D-subdivision method, we set $s = 0$ in (14) to obtain $p_j(0) = \gamma^2 + k_1 - k_2 e^{\gamma h}\mu_j = 0$. Solving the above equation for k_2, we obtain (20). On the other hand, with $s = i\omega$, we have: $p_j(i\omega) = -\omega^2 - 2i\omega\gamma + \gamma^2 + k_1 - k_2 e^{\gamma h}\mu_j(\cos(\omega h) - i\sin(\omega h)) = 0$ The above equation is true if and only if $\Re\{p_j(i\omega)\} = \Im\{p_j(i\omega)\} = 0$, from which (19) and (18) follow. □

3.3 Smoothness of the stability crossing boundaries

Considering the factorization of the characteristic equation of the system, the stability region can be decomposed as

$$\mathcal{D}(0) = \bigcap_{j=1}^{N} \mathcal{D}(0, \mu_j), \tag{21}$$

where $\mathcal{D}(0, \mu_j)$ is the stability region associated with each of the factors. Moreover, for each $\mathcal{D}(0, \mu_j)$, we use $\mathcal{K}(\mu_j)$ to denote its boundary; that is, $\mathcal{K}(\mu_j) = \partial\mathcal{D}(0, \mu_j)$. Before proceeding, we make the following decomposition for the j-th factor in real and imaginary parts:

$$-\frac{\partial p_j}{\partial h} = R_1 + iI_1,\ -\frac{\partial p_j}{\partial k_2} = R_2 + iI_2,\ \frac{\partial p_j}{\partial \omega} = R_0 + iI_0. \tag{22}$$

Proposition 4. The tangent to $\mathcal{K}(\mu_j)$ associated with the factor p_j in (14) is given by:

$$\begin{pmatrix} \frac{\mathrm{d}h}{\mathrm{d}\omega} \\ \frac{\mathrm{d}k_2}{\mathrm{d}\omega} \end{pmatrix} = \frac{1}{R_1 I_2 - R_2 I_1}\begin{pmatrix} I_2 R_0 - I_0 R_2 \\ I_0 R_1 - I_1 R_0 \end{pmatrix}. \tag{23}$$

Proof. Let $\gamma = \gamma^*$ be fixed. On the shifted imaginary axis, p_j describes an analytic function of (ω, h, k_2); therefore:

$$i\frac{\partial p_j}{\partial \omega} + \frac{\partial p_j}{\partial h}\frac{\mathrm{d}h}{\mathrm{d}\omega} + \frac{\partial p_j}{\partial k_2}\frac{\mathrm{d}k_2}{\mathrm{d}\omega} = 0. \tag{24}$$

Substituting (22) into (24) yields

$$\begin{pmatrix} R_0 \\ I_0 \end{pmatrix} = \begin{pmatrix} R_1 & R_2 \\ I_1 & I_2 \end{pmatrix}\begin{pmatrix} \frac{\mathrm{d}h}{\mathrm{d}\omega} \\ \frac{\mathrm{d}k_2}{\mathrm{d}\omega} \end{pmatrix}.$$

From which (23) follows. □

4. DESIGN OF THE DISTRIBUTED PR PROTOCOL

4.1 Scalability over undirected graphs

Scalability in MAS is often defined as the ability to maintain stability despite perturbations as the number of agents increases (Xie et al., 2021; Besselink and Knorn, 2018). Here, we define scalability as the ability to preserve the PR protocol's tuning regardless of the number of agents. To illustrate the problem addressed in this paper, we include a simple illustrative example, not intended for general conclusions.

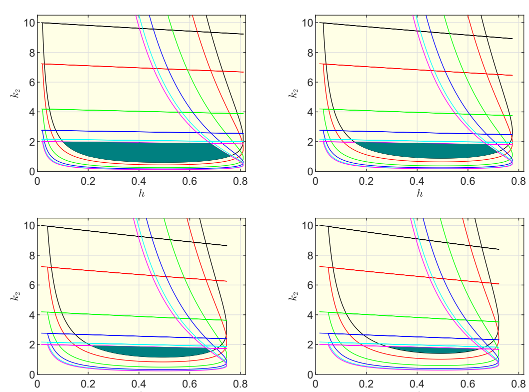

Fig. 1. Boundaries of γ-stability for the characteristic factors $p_j(s)$ in (14) for $\gamma \in \{0.10, 0.15, 0.20, 0.25\}$ (top-left to bottom-right). The green region denotes the overall stability region. Colored boundaries correspond to: – $j = \{1\}$, – $j = \{2, 3\}$, – $j = \{4, 5\}$, – $j = \{6, 7\}$, – $j = \{8, 9\}$, and – $j = \{10\}$.

Illustrative Example: Consider a circular graph with 10 agents. Using Proposition 3, Figure 1 shows the stability domain $\mathcal{D}(0)$ (in green) for various values of γ. As γ increases, this domain shrinks, reaching its maximum at the point where $\mathcal{D}(0, \mu_1)$ and $\mathcal{D}(0, \mu_N)$ collapse into a single point tangent to $\mathcal{K}(\mu_1)$ and $\mathcal{K}(\mu_N)$. At this point, the following conditions hold:

$$k_2(\omega, \mu_1) = k_2(0, \mu_N) \tag{25}$$

$$\frac{\mathrm{d}k_2}{\mathrm{d}h}(\omega, \mu_1) = \frac{\mathrm{d}k_2}{\mathrm{d}h}(0, \mu_N) \tag{26}$$

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Note that the factors in (12) have the form:

$$p_j(s) = q_0(s) + q_1(s)\mu_j e^{-sh}, \quad j = 1, \ldots, N \tag{27}$$

where $\deg(q_1(s)) \leq \deg(q_0(s))$, $q_0(s) = s^2 - 2s\gamma + \gamma^2 + k_1$ and $q_1(s) = -k_2 e^{\gamma h}$. Using the edge theorem and the concept of convex directions for quasipolynomials introduced in (Kharitonov and Zhabko, 1994), the following proposition from (Gomez and Ramírez, 2022) reduces the stability analysis of the full MAS to studying only two characteristic factors, rather than all N.

Proposition 5. Under Assumption 2, the MAS (1) with (3) is stable if and only if both $p_1(s)$ and $p_N(s)$ in (12) are Hurwitz stable.

Proof. The proof follows from the fact that $\deg q_1(s) = 0$ and Corollary 1 of (Gomez and Ramírez, 2022). □

According to Proposition 5, the stability region of the complete system is given by the intersection of the stability regions of the extreme polynomials p_1 and p_N.

4.2 Tuning

The following proposition provides a scalable design method for the PR parameters that ensures the fastest velocity of response for the MAS.

Proposition 6. Let a desired exponential decay $\gamma > 0$ be given. Then, the real part of the dominant roots of the MAS (1) with (3) is placed at $-\gamma$ with the following tuning of the PR protocol parameters:

$$h = \frac{1}{\omega} \sin^{-1}(\theta), \tag{28}$$

$$k_1 = \frac{2\omega\, \gamma\, \mu_N}{\mu_1 \sin(\omega\, h)} - \gamma^2, \tag{29}$$

$$k_2 = \frac{\gamma^2 + k_1}{e^{\gamma h} \mu_N}, \tag{30}$$

where

$$\omega = \frac{\gamma}{\mu_1}\sqrt{2}\sqrt{-\Delta_\mu + \sqrt{-\mu_N^2 \Delta_\mu}}, \tag{31}$$

$$\theta = 2\gamma \frac{\omega\mu_N + \sqrt{\omega^2\mu_1^2 + 4\gamma^2\Delta_\mu}}{\mu_1\left(4\gamma^2 + \omega^2\right)}, \tag{32}$$

with $\Delta_\mu = \mu_1^2 - \mu_N^2$, where μ_1 and μ_N denote, respectively, the smallest and largest eigenvalues of $I_N + \mathcal{L}$.

Proof. The proof follows from the derivative and gain conditions in (26) and (25), respectively.

Proposition 6 provides the conditions under which the stability domain collapses for a desired value of γ. In other words, it ensures that the exponential decay rate of the full system can be arbitrarily assigned by appropriately tuning the parameters of the PR distributed control protocol.

Remark 7. The tuning in Proposition 6 is valid independently of the number of agents, requiring only knowledge of the eigenvalues μ_1 and μ_N, thus ensuring the scalability of the PR protocol. We note that choosing the value of γ must take into account the physical limits of the actuators.

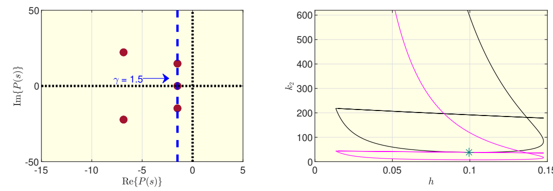

Fig. 2. Roots of $\overline{P}(s)$ in (12) and γ-stability boundaries associated with $p_j(s)$ in (14) for $\gamma = 1.5$ and $(h, k_1, k_2) = (0.0995, 220.4770, 38.3704)$.

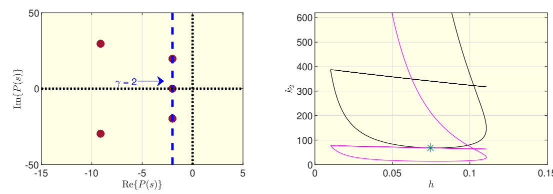

Fig. 3. Roots of $\overline{P}(s)$ in (12) and γ-stability boundaries associated with $p_j(s)$ in (14) for $\gamma = 2$ and $(h, k_1, k_2) = (0.0746, 391.9592, 68.2140)$.

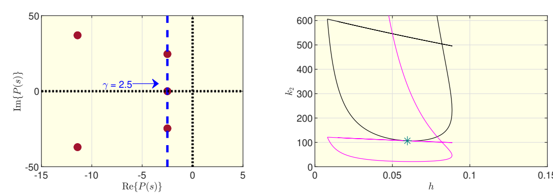

Fig. 4. Roots of $\overline{P}(s)$ in (12) and γ-stability boundaries associated with $p_j(s)$ in (14) for $\gamma = 2.5$ and $(h, k_1, k_2) = (0.0597, 612.4362, 106.5844)$.

5. CASE STUDY

We present an example with four agents and Laplacian:

$$\mathcal{L} = \begin{pmatrix} 2 & -1 & 0 & -1 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ -1 & 0 & -1 & 2 \end{pmatrix}.$$

To illustrate the tuning procedure from the previous subsection, we consider three values of $\gamma \in \{1.5, 2, 2.5\}$. For each case, the system roots and corresponding γ-stability boundaries are computed using the parameters (h, k_1, k_2) obtained from Proposition 6.

Figures 2–4 illustrate the results for each γ value. On the left, the roots of the quasipolynomial $\overline{P}(s)$ in (12) are shown, with the desired dominant roots at γ indicated by the dashed line – –. On the right, the γ-stability boundaries of the characteristic factors $p_j(s)$ in (14) are displayed. The point of tangency between $\mathcal{K}(\mu_1)$ and $\mathcal{K}(\mu_N)$ is marked with *. Boundary colors: — for $j = 1$ and — for $j = 4$. Is it worthy of mention that the same qualitative behavior is observed for $N = 1000$. The results are omitted here due to space restrictions.

It can be observed that with the tuning based on Proposition 6, the stability domain of the complete system $\mathcal{D}(0)$ collapses at the calculated point (h, k_1, k_2). Thus, the maximum exponential decay is guaranteed, which in turn will ensure that the speed of response of all agents is maximized.

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6. CONCLUDING REMARKS

The results demonstrate that the proposed tuning technique successfully enforces the collapse of the system's stability region at the computed point (h, k_1, k_2) for the desired γ. This collapse corresponds to the tangency of the curves $\mathcal{K}(\mu_1)$ and $\mathcal{K}(\mu_N)$ and marks the point at which the entire MAS attains its maximum allowable exponential decay rate. As a result, the network not only remains stable but also achieves the fastest possible transient response across all agents under the given protocol structure. This performance is obtained without derivative feedback or velocity measurements, relying instead on intentional delays to emulate derivative-like behavior.

The approach is scalable in that the same tuning remains valid regardless of the number of agents, provided the graph structure and eigenvalue bounds are preserved. This contrasts with previous methods, which require noise-free conditions or velocity measurements, or are restricted to small-scale networks or single-delay systems. Overall, the methodology offers a practical and analytically grounded framework for delay-based distributed control in large-scale MAS, with guaranteed stability and performance based on a minimal set of spectral properties.

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