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Recurrence Analysis as a Tool for Identifying Incipient Faults

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Abstract: In this work, a recurrence-based measure is introduced to identify incipient faults. The approach is demonstrated using a mass-spring-damper system, where the damper's degradation is simulated by reducing its value relative to the nominal condition. The proposed method offers two key advantages: it eliminates the need for an explicit model of the system being monitored, and it exhibits high sensitivity to minor faults. One disadvantage of the proposed method is its dependence on data. The effectiveness of recurrence measures can be heavily influenced by the quality and quantity of the data.

Keywords: Incipient faults, Recurrence analysis, Shannon entropy, Vibration analysis, Dynamical systems.

1. INTRODUCTION

Recurrence analysis is a nonlinear technique used to detect recurring or irregular patterns within a time series [Eckmann et al. (1987); Webber (2015)]. The goal of this method is to identify and quantify the repetition of patterns within a dynamical system. It is also valuable for comparing similarities and differences between two or more systems. The principal tool of this method is the recurrence plot, which visualizes the recurrence of patterns within a dynamical system [Eckmann et al. (1987)]. To quantify the structure of a recurrence plot, a set of measures based on the main diagonal is used. This set of measures is known as Recurrence Quantification Analysis (RQA) [Webber (2015)].

Recently, the recurrence plot and its quantification have been applied to study phenomena such as complexity, synchronization, bifurcation, and to distinguish between the behaviors of various dynamical systems [Marwan et al. (2007); Rysak et al. (2022); Braun et al. (2022); Hirata and Aihara (2011); Suresha et al. (2016)], as well as for identifying system failures [Syta et al. (2012); Kecik et al. (2022); Garcia et al. (2013)].

Among the different types of faults, the incipient fault is distinguished as the one that requires prognosis [Pilario et al. (2019); Samy and Gu (2011)]. Unlike abrupt faults, incipient faults begin with small magnitudes, but their severity gradually increases over time [Pilario et al. (2019); Samy and Gu (2011)]. If an incipient fault is not controlled, it can eventually lead to process degradation, which could result in complete system failure or an emergency situation [Pilario et al. (2019); Samy and Gu (2011)]. Therefore, monitoring incipient faults is crucial and must be managed using more advanced diagnostic and prognostic tools.

The present work analyzes a mass-spring-damper system, where small degradations in the damper are simulated to represent incipient faults. The system is first examined under normal conditions, and then compared with scenarios involving small changes in the damper. The case is studied through recurrence analysis, where changes in the measure of Shannon entropy are considered indicative of incipient faults. The objective of this work is to demonstrate that Shannon's entropy measure detects small changes in the mass-spring-damper system and can be a useful tool for existing methodologies in the detection of incipient faults.

2. MATERIALS AND METHODS

2.1 Recurrence plots and RQA

The Recurrence Plot (RP) method is a powerful technique for analyzing nonlinear and non-stationary signals. It transforms a one-dimensional time series into a two-dimensional graphical representation, revealing patterns and structures that may not be apparent through conventional analysis. The concept of recurrence can be mathematically expressed as the following equation [Marwan (2008); Marwan et al. (2007); Marwan and Webber Jr (2014)]:

$$\{x : \|x_i - x_j\| \leq \varepsilon\}, \tag{1}$$

where x_i and x_j represent the values of the time series at time points t_i and t_j, respectively, and ε is a predefined threshold that determines whether the points are considered sufficiently "close" to each other [Schinkel et al. (2008)]. The RP is constructed using an $N \times N$ matrix, where N is the total number of data samples. This matrix, denoted as R, contains binary values: ones (1) when $x_i = x_j$, and zeros (0) when $x_i \neq x_j$, as defined in equation

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(1). The RP features a prominent black diagonal, known as the *line of identity* (LOI) [Webber (2015)]. This diagonal is a key characteristic of all RPs, as it signifies where $R_{ij} = 1$, indicating that each point in the time series is identical to itself.

Different types of Recurrence Plots (RPs) can be distinguished based on specific characteristics of the underlying process. A few common types are outlined below [Webber (2015)]:

- *Homogeneity*: The process is stationary, exhibiting consistent behavior over time.
- *White Banding Disruptions*: The process is nonstationary, showing sudden, irregular changes.
- *Periodic Patterns*: The process displays characteristic cyclic behavior, with the distance between repeating patterns corresponding to the period.
- *Single Isolated Points*: The states of the process are rare, transient, or exhibit large fluctuations that do not persist.
- *Diagonal Lines Parallel to the LOI*: The process is deterministic in a periodic sense (long diagonals) or chaotic in a short-term sense (short diagonals).

These typologies offer valuable insights into the dynamics of the system. However, they also present a potential drawback, as their interpretation relies heavily on the user's expertise and understanding of the patterns and structures within the recurrence plots. To mitigate the risk of misinterpretation, Zbilut and Webber introduced a method for quantifying the structure of the recurrence plot . They developed a set of measures based on the main diagonal, collectively known as RQA [Webber (2015)]. One of these measures is the *Shannon entropy*, which is defined by equation (2) [Webber (2015)].

$$ENTR = -\sum_{\ell=\ell_{min}}^{N} P(\ell) ln P(\ell), \tag{2}$$

where N represents the number of data points in the time series, and $P(l)$ refers to the histogram of the diagonal line lengths. Entropy quantifies the divergence of the orbits in a dynamical system as time progresses, making it a useful indicator of the system's complexity Webber (2015). Shannon entropy, in particular, is employed to measure the system's complexity, relying solely on the distances between points in the state space. One of its key advantages is that it can be computed efficiently. However, a potential limitation is its dependence on the threshold ε, which must be carefully chosen to ensure an accurate estimate.

2.2 Fault detection and isolation

Fault detection and isolation (FDI) is a critical process in dynamic and engineering systems, focused on accurately identifying and locating faults in equipment or processes before they lead to significant damage [Garcia and Frank (1999); Ding (2021); Samy and Gu (2011)]. By employing advanced monitoring and analysis techniques—such as sensors, mathematical algorithms, and predictive models—anomalies that indicate potential faults can be detected early [Ding (2021); Samy and Gu (2011)]. Once a fault is identified, fault isolation plays a vital role by determining its exact location, allowing for faster and more effective interventions. This approach not only optimizes operational efficiency but also reduces costs associated with repairs and unplanned downtime, thus enhancing the system's overall performance.

A crucial strategy that supports fault detection and isolation is information redundancy [Garcia and Frank (1999); Ding (2021); Samy and Gu (2011)]. This involves using additional elements, such as sensors, actuators, or even fully duplicated or triplicated components, to ensure that the system remains functional even if one of its parts fails. Moreover, analytical redundancy, which relies on mathematical models to provide extra insights into the measured signals, allows for detecting inconsistencies or faults without needing to duplicate physical components. Together, both physical and analytical redundancy significantly improve the system's reliability and diagnostic capabilities, offering multiple sources of information to verify and confirm the system's status. This layered approach further strengthens the process of fault detection and isolation, ensuring more accurate diagnoses and timely responses.

Generally, most FDI methods can be categorized into two main groups [Samy and Gu (2011)]:

(1) *Model-based methods*: These approaches rely on a plant model and use analytical redundancy techniques, such as:
   - Parity space
   - Observers
   - Parameter estimation
   - And others

(2) *Model-free methods*: These approaches do not depend on a plant model and use physical redundancy or other signal analysis techniques, including:
   - Physical redundancy
   - Limit value checking
   - Frequency analysis of measured signals
   - And others

A plant can typically be divided into three main subsystems: actuators, the process (i.e., components), and sensors. Consequently, faults in a plant can generally be classified into three categories [Samy and Gu (2011)]:

(1) *Actuator faults* (additive)
(2) *Process faults* (additive or multiplicative)
(3) *Sensor faults* (additive)

Beyond these fault categories, faults can be classified based on their nature [Samy and Gu (2011)]: they may either be *abrupt* (rapidly occurring) or *incipient* (gradually developing). Abrupt faults cause immediate and noticeable deviations from the system's normal, fault-free behavior. On the other hand, incipient faults evolve more slowly and tend to manifest as a gradual drift in performance.

This work specifically focuses on *incipient faults*, which are often linked to factors such as temperature variations, calibration errors, or equipment wear. In the short term, these faults might not significantly degrade system performance, meaning there is often less urgency for their immediate detection. However, if left unaddressed for an

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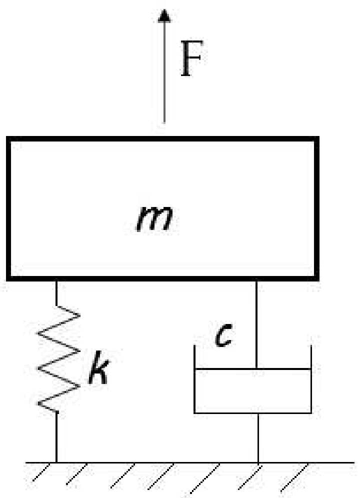

Fig. 1. Basic schematic representation of a mass-spring-damper system

extended period, incipient faults can lead to catastrophic failures.

2.3 mass-spring-damper system

When a vibrating system is modeled using a combination of discrete masses m_i, dampers c_i, and springs k_i, it is referred to as a *lumped parameter model* or *discrete model* of the system [Jazar (2013); Szymiec (2016)]. In this type of model, the physical properties such as mass, stiffness, and damping are assumed to be concentrated at specific points rather than distributed continuously throughout the structure, simplifying the analysis of dynamic behavior [Jazar (2013); Szymiec (2016)].

In the field of engineering, the mass-spring-damper system represents a fundamental model for the analysis of dynamic systems. This system ideally consists of three elements: a mass that can move freely in one dimension, a spring that provides a restoring force proportional to the displacement, and a damper that introduces a dissipative force proportional to the velocity. Together, these components allow for the simulation of the oscillatory behavior of a wide variety of mechanical, electrical, and structural systems [Jazar (2013); Szymiec (2016)].

From an engineering perspective, this system is crucial for understanding and designing structures and mechanisms subjected to vibrations or dynamic forces. Its main characteristics include the ability to represent both free and forced behavior, the influence of damping on energy dissipation, and the system's response to different initial conditions or external excitations [Jazar (2013); Szymiec (2016)]. Fig. 1 illustrates a basic schematic representation of a mass-spring-damper system.

Of the three components that make up the mass-spring-damper system, the *damper* is generally the most prone to damage or failure over time [Wang et al. (2021); Gao et al. (2024)]. This is because its operation relies on complex internal mechanisms and the use of fluids (such as hydraulic oil or gas) to dissipate energy [Wang et al. (2021); Gao et al. (2024)].

One of the main factors contributing to damper wear is *prolonged use*, as its internal components—such as seals and valves—can deteriorate over time. Additionally, *fluid leaks* are common, which significantly reduce the damper's ability to absorb vibrations [Wang et al. (2021); Gao et al. (2024)].

For these reasons, the damper requires *more frequent maintenance* and generally has a shorter service life compared to the spring or the mass. Its proper functioning is essential to ensure the stability and control of the system under vibrations or undesired movements.

3. RESULTS

From the Fig. 1, the mass is subjected to a harmonic excitation of the form $Z(t) = F_0 \sin(\Omega t)$. The numerical solution of the mass-spring-damper system from the Fig. 1 is represented as a system of two first-order differential equations, as follows [Majewski (2017)]:

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= \frac{F_0(k\sin(\Omega t) + c\Omega\cos(\Omega t)) - cx_2 + kx_1}{m}. \end{aligned} \tag{3}$$

where x_1 represents the displacement of the mass and x_2 the velocity of the mass. The nominal values of the parameters are: $m = 250\,\text{kg}$; $c = 700\,\text{kg/s}$; $k = 1000\,\text{N/m}$; $F = 10\,\text{N}$; $\Omega = 30\,\text{rad/s}$.

A simulated degradation of the damper was performed by reducing its value relative to the nominal condition. Table 1 shows the variation in the Shannon entropy throughout the service life of the damper. The results obtained are based on the underlying principles of the algorithm described in the article [Baghdadi et al. (2021)].

Table 1. Shannon entropy values at different stages of damper life.

Damper Life (%)	Shannon Entropy
100	1.7280 (Nominal)
98	1.7286
95	1.7452
90	1.7342
80	1.7653
70	1.7598
50	1.7041

Next, the results are analyzed and interpreted.

3.1 Interpretation results

100%–95% Life: Slight Increase in Entropy

- Entropy increases slightly from 1.7280 to 1.7452.
- An early rise in entropy corresponds to normal operation.

Fig. 2 presents the velocity of the mass along with its recurrence plot under nominal conditions.

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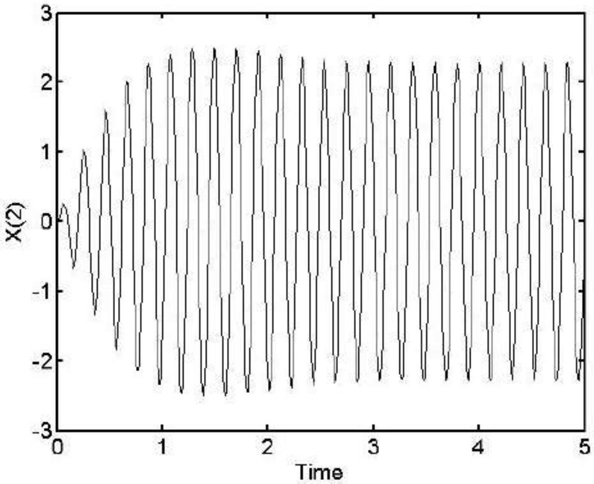
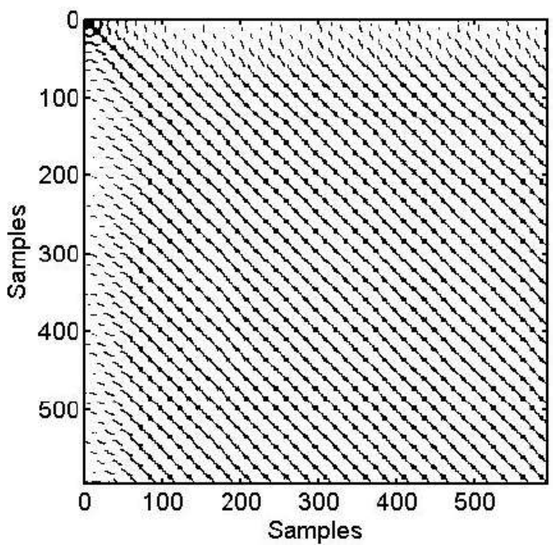
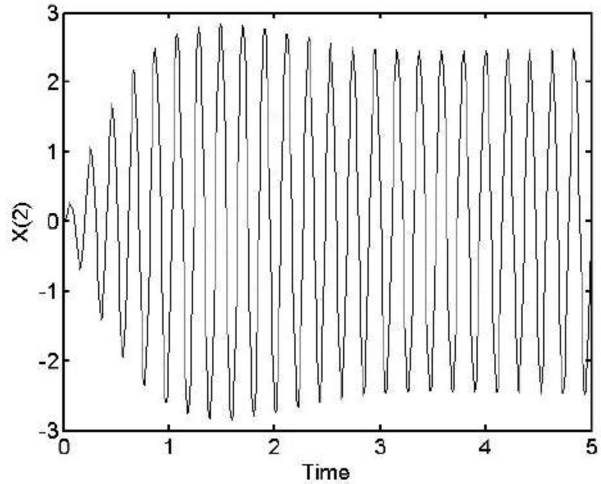


(a) Velocity of the mass at its nominal value.

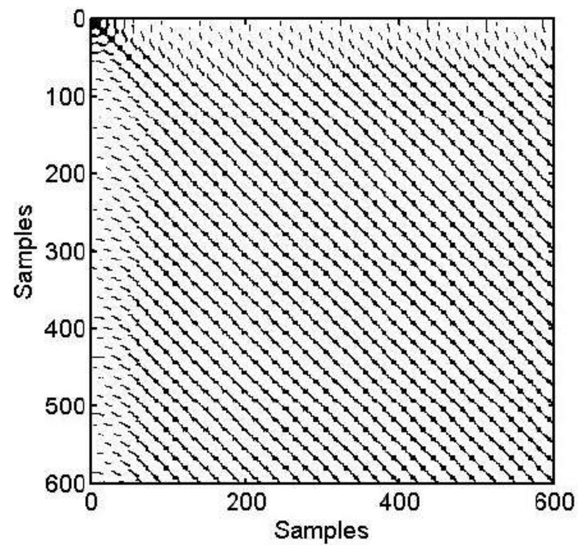


(b) Recurrence plot at its nominal value.

Fig. 2. Damper life at 100% with Shannon entropy of 1.7280.



(a) Velocity of the mass with 80% of life.



(b) Recurrence plot with 80% of life.

Fig. 3. Damper life at 80% with Shannon entropy of 1.7653.

90%–70% Life: Entropy Fluctuates at a Higher Level

- Entropy remains elevated between 1.73 and 1.76.
- Suggests nonlinear degradation with high dynamic variability.
- Entropy peak near 80% life indicates the highest level of system complexity.
- Damper may be experiencing unstable or chaotic internal dynamics.
- High dynamic variability suggests normal aging.
- Likely reflects early-stage wear or internal micro-dynamic changes.
- Peak and drop behaviors may inform predictive maintenance thresholds.

Fig. 3 presents the velocity of the mass along with its recurrence plot at 80 % of its service life. Between 100 % and 70 % of the service life of the damper, the recurrence plots exhibit similar patterns.

50% Life: Drop in Entropy

- Entropy decreases to 1.7041.
- Indicates a possible behavioral transition:
    - · A more rigid or degraded dynamic state.
    - · Structural or functional changes (e.g., fluid deterioration or component wear).
    - · Entropy drop at 50% life may represent a transition toward failure or a more stable (but degraded) state.
- Sudden changes in entropy could signal the need for inspection or replacement.

Fig. 4 presents the velocity of the mass along with its recurrence plot at 50 % of its service life. Fig. 4 shows a disruption in the pattern compared to the previous plots (Fig. 2 and 3). This disruption appears within the recurrence plot and may indicate a transition toward failure. Fig. 5 represents a magnified view of Figure 4, allowing for a clearer visualization of the pattern disruption.

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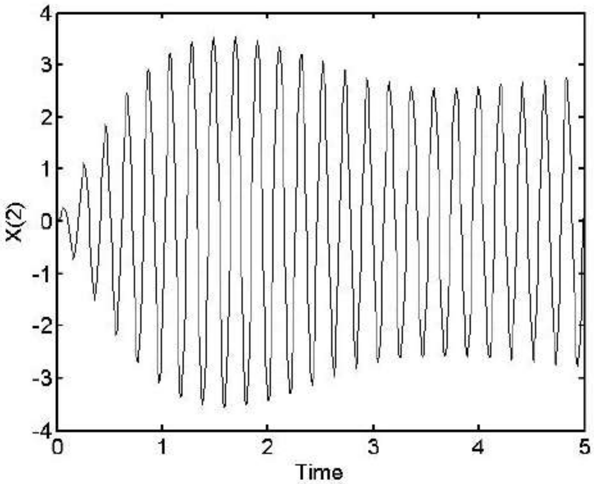
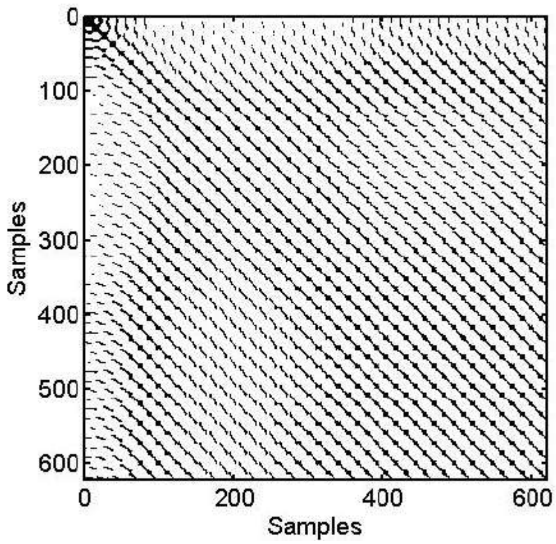


(a) Velocity of the mass with 50% of life.



(b) Recurrence plot with 50% of life.

Fig. 4. Damper life at 50% with Shannon entropy of 1.7041.

4. CONCLUSION

The evolution of entropy throughout the damper's service life reveals valuable insights into its dynamic behavior and degradation process. During the early stage (100%–95%), a slight increase in entropy reflects normal operation. From 90% to 70% of life, sustained elevated entropy levels and increased variability suggest nonlinear aging and heightened internal complexity—potentially linked to early wear or chaotic dynamics. Notably, the entropy peak near 80% may serve as an early indicator for predictive maintenance. At 50% life, a noticeable drop in entropy implies a transition to a more rigid or degraded state, possibly signaling structural or functional deterioration. This entropy decline could mark the onset of failure or a less adaptive, yet temporarily stable, condition. Overall, entropy trends provide a useful framework for diagnosing component health and identifying optimal intervention points.

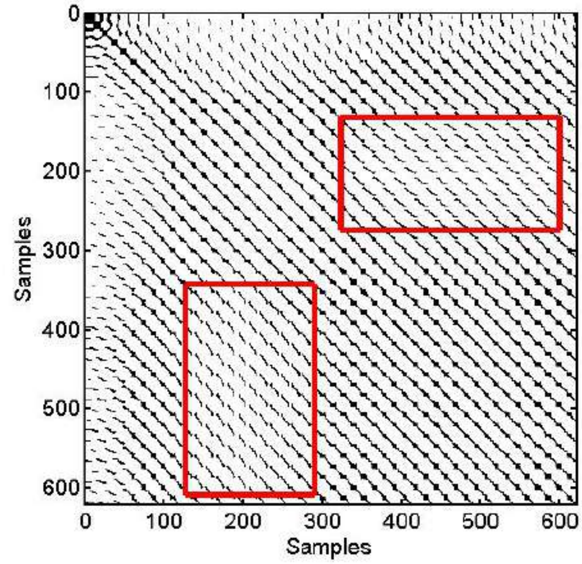


Fig. 5. Magnified View of Pattern Disruption (Fig. 4)

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REFERENCES

Baghdadi, G., Amiri, M., Falotico, E., and Laschi, C. (2021). Recurrence quantification analysis of eeg signals for tactile roughness discrimination. *International Journal of Machine Learning and Cybernetics*, 12, 1–22. doi:10.1007/s13042-020-01224-1.

Braun, T., Kraemer, K.H., and Marwan, N. (2022). Recurrence flow measure of nonlinear dependence. *The European Physical Journal Special Topics*, 232. doi: 10.1140/epjs/s11734-022-00687-3.

Ding, S.X. (2021). *Advanced methods for fault diagnosis and fault-tolerant control*, volume 184. Springer.

Eckmann, J.P., Kamphorst, S.O., and Ruelle, D. (1987). Recurrence plots of dynamical systems. *Europhysics Letters*, 4(9), 973. doi:10.1209/0295-5075/4/9/004. URL `https://dx.doi.org/10.1209/0295-5075/4/9/004`.

Gao, H., Sun, J., E, S., and Chi, M. (2024). Cavitation induced hydraulic yaw damper failure and its effect on railway vehicle dynamic stability. *Engineering Failure Analysis*, 161, 108318. doi: https://doi.org/10.1016/j.engfailanal.2024.108318.

Garcia, E.A. and Frank, P. (1999). A novel design of structured observer-based residuals for fdi. In *Proceedings of the 1999 American Control Conference (Cat. No. 99CH36251)*, volume 2, 1341–1345. IEEE.

Garcia, S., Miguel, R., and Figueroa-Nazuno, J. (2013). Characterization of ground motions using recurrence plots. *Geofísica internacional*, 52, 209–227. doi: 10.1016/S0016-7169(13)71473-9.

Hirata, Y. and Aihara, K. (2011). Erratum: Devaney's chaos on recurrence plots. *Physical Review E - PHYS REV E*, 83.

Jazar, R.N. (2013). *Advanced vibrations*. Springer.

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Kecik, K., Smagala, A., and Liubytska, K. (2022). Ball bearing fault diagnosis using recurrence analysis. *Materials*, 15, 5940. doi:10.3390/ma15175940.

Majewski, T. (2017). *VIBRACIONES EN SISTEMAS FÍSICOS*.

Marwan, N. (2008). A historical review of recurrence plots. *Eur. Phys. J. Special Topics*, 164, 3–12. doi:10.1140/epjst/e2008-00829-1. URL https://doi.org/10.1140/epjst/e2008-00829-1.

Marwan, N., Carmen Romano, M., Thiel, M., and Kurths, J. (2007). Recurrence plots for the analysis of complex systems. *Physics Reports*, 438(5), 237–329. doi: https://doi.org/10.1016/j.physrep.2006.11.001.

Marwan, N. and Webber Jr, C.L. (2014). Mathematical and computational foundations of recurrence quantifications. In *Recurrence quantification analysis: Theory and best practices*, 3–43. Springer.

Pilario, K.E.S., Cao, Y., and Shafiee, M. (2019). Incipient fault detection, diagnosis, and prognosis using canonical variate dissimilarity analysis. In A.A. Kiss, E. Zondervan, R. Lakerveld, and L. Özkan (eds.), *29th European Symposium on Computer Aided Process Engineering*, volume 46 of *Computer Aided Chemical Engineering*, 1195–1200. Elsevier. doi:https://doi.org/10.1016/B978-0-12-818634-3.50200-9.

Rysak, A., Sedlmayr, M., and Gregorczyk, M. (2022). Revealing fractionality in the rössler system by recurrence quantification analysis. *The European Physical Journal Special Topics*, 232, 1–16. doi:10.1140/epjs/s11734-022-00740-1.

Samy, I. and Gu, D.W. (2011). *Fault Detection and Flight Data Measurement*, volume 419. Springer Berlin, Heidelberg. doi:10.1007/978-3-642-24052-2.

Schinkel, S., Dimigen, O., and Marwan, N. (2008). Selection of recurrence threshold for signal detection. *The European Physical Journal Special Topics*, 164, 15–53. doi:10.1140/epjst/e2008-00833-5.

Suresha, S., Sujith, R., Emerson, B., and Lieuwen, T. (2016). Nonlinear dynamics and intermittency in a turbulent reacting wake with density ratio as bifurcation parameter. *Physical Review E*, 94. doi: 10.1103/PhysRevE.94.042206.

Syta, A., Jonak, J., Jedliński, , and Litak, G. (2012). Failure diagnosis of a gear box by recurrences. *Journal of Vibration and Acoustics*, 134. doi:10.1115/1.4005846.

Szymiec, T.M. (2016). *Vibraciones En Sistemas Físicos*. ALFAOMEGA, 1 edition.

Wang, W., Huang, B., Zhou, Z., and Du, J. (2021). Wear mechanism and failure analysis of a high-speed train hydraulic damper using cfd approach. *Journal of Physics: Conference Series*, 1877, 012006. doi:10.1088/1742-6596/1877/1/012006.

Webber, C. (2015).

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https://doi.org/10.58571/CNCA.AMCA.2025.009