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Arbitrary Change of the Relative Gain Array: Application to a Helicopter

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Abstract: In this work, the Relative Gain Array (RGA) is employed as a measure of input-output coupling in the operation of a two-degree-of-freedom (2DOF) helicopter platform. High levels of interaction between control channels typically affect the performance of multivariable control systems, especially when decentralized controllers are used. To address this, we propose a methodology that systematically modifies the RGA matrix to achieve a more favorable configuration, including decoupling, and improved closed-loop performance. The approach involves analyzing the system's steady-state gain matrix and applying a transformation that shifts the RGA values toward a desired target structure. Simulation results demonstrate that the proposed modification produces a significant change in system response, reducing or increasing coupling effects as required. These findings suggest a promising direction for the design of RGA-based controllers in practical multivariable systems, such as aerial platforms.

Keywords: Relative Gain Array, Multivariable control, Coupling, input-output interaction.

1. INTRODUCTION

Multivariable control systems often present significant challenges due to inherent input-output interactions, which can degrade the performance of decentralized control strategies. A widely used tool to evaluate such interactions is the Relative Gain Array (RGA), originally introduced by Bristol (1965), which provides insight into the steady-state coupling between manipulated and controlled variables. The RGA has become a standard method for variable pairing and assessing the feasibility of decoupling techniques.

In the context of aerial platforms, such as two-degree-of-freedom (2DOF) helicopters, the strong coupling between pitch and yaw dynamics presents challenges for controller design. Traditional control strategies may not adequately address these interactions, leading to suboptimal performance. Recent research has explored advanced control methodologies, including adaptive control Rodriguez et al.(2022), fault-tolerant control Zuñiga et al (2021), and model predictive control Zheng et al. (2024), to enhance the robustness and efficiency of such systems. For instance, studies have demonstrated the effectiveness of passive fault-tolerant control in 2DOF helicopters, as well as the application of adaptive neural control to manage input saturation and time-varying output constraints Wu, B. et al. (2022).

Moreover, the integration of RGA with modern control techniques has shown promise in improving multivariable system performance. For example, the use of RGA in conjunction with autoencoder-based machine learning has been proposed to enhance process control applications Martello, R.H. et al. (2024). Additionally, the development of multivariable PID control using iterative linear programming and decoupling strategies has been explored to address the complexities of multivariable systems Garrido, J. et al. (2024).

Despite these advancements, the RGA is purely used as a diagnostic measure. This paper explores a novel approach in which the RGA is intentionally modified to achieve a more favorable configurations for control purposes. Such approach could facilitate decoupling and improve closed-loop performance. This methodology actively modifies the RGA structure of a system, aiming to reduce coupling effects and enhance dynamic behavior. The proposed method involves analyzing the system's steady-state gain matrix and applying a compensator to steer the RGA towards a desired target structure. The proposed methodology was applied to Quanser's mathematical model of a two-degree-of-freedom helicopter, demonstrating the effectiveness of this approach in modifying system response.

The remainder of the paper is organized as follows: Section 2 presents the Bristol–Shinskey procedure step by step, from the state–space model to the steady–state transfer matrix, and explicitly applies the mathematical definitions of the RGA elements. The 2-DOF helicopter model is also presented. Section 3 describes the proposed RGA modification methodology. These calculations are then directly connected to the helicopter case study and validated through simulation, ensuring that the explanation is both consistent and sufficiently detailed to support the proposed methodology. Section 4 presents the simulation results and performance analysis of the 2-DOF helicopter validating the coupling modification. Finally, Section 5 concludes the paper and outlines future work.

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2. BRISTOL-SHINSKEY METHOD APPLIED TO THE PROPOSED SYSTEM

The objective of this section is to demonstrate the Bristol-Shinskey method for obtaining the degree of coupling of a two-degree-of-freedom Quanser helicopter model controlling pitch and yaw dynamics. The strong coupling in this system represents a challenge for controller design.

The Quanser 2-DOF Helicopter experiment (Fig. 1), consists of a helicopter model mounted on a fixed base with two propellers that are driven by DC motors. The front propeller controls the elevation of the helicopter nose about the pitch axis and the back propeller controls the side to side motions of the helicopter about the yaw axis. The pitch and yaw angles are measured using high-resolution encoders.

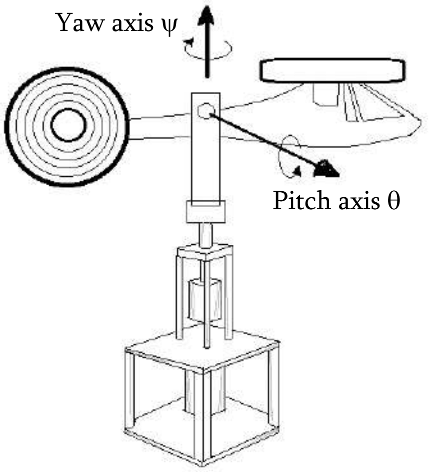

Figure 1. Quanser 2-DOF Helicopter.

Based on Quanser, Q. (2011), the state-space linear dynamics describing the position-tension-joint-angle of the system is:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \tag{1}$$

the state vector is considered as:

$$\dot{x}(t) = [\theta(t) \quad \psi(t) \quad \dot{\theta}(t) \quad \dot{\psi}(t)]^T \tag{2}$$

and the output equation is

$$y(t) = [\theta(t) \quad \psi(t)]^T \tag{3}$$

where $\theta(t)$ and $\psi(t)$ are the pitch and yaw angles, respectively, and the corresponding helicopter input vector $u(t) = [u_1 \quad u_2]^T$are control signals applied to pitch and yaw motors, respectively, and

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{-B_p}{J_{T_p}} & 0 \\ 0 & 0 & 0 & \frac{-B_y}{J_{T_y}} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{K_{pp}}{J_{T_p}} & \frac{K_{py}}{J_{T_p}} \\ \frac{K_{yp}}{J_{T_y}} & \frac{K_{yy}}{J_{T_y}} \end{bmatrix} \tag{4}$$

$$J_{T_p} = J_{eq\_p} + m_{heli} l_{cm}^2$$

$$J_{T_y} = J_{eq\_y} + m_{heli} l_{cm}^2$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The model parameters used in (4) are defined in the Table 1.

Table 1. Parameters of 2DOF Helicopter

K_{pp}	Pitch torque	0.204 N.m/V
K_{yy}	Yaw torque	0.072 N.m/V
K_{py}	Yaw on pitch torque	0.0068 N.m/V
K_{yp}	Pitch on yaw torque	0.0219 N.m/V
J_{eq_p}	Total pitch moment of inertia	0.0384 Kg.m^2
J_{eq_y}	Total yaw moment of inertia	0.0432 Kg.m^2
B_p	Pitch viscous damping	0.800 N
B_y	Yaw viscous damping	0.318 N
m_{heli}	Total moving mass	1.2872 Kg
l_{cm}	Center of mass length along helicopter body from pitch axis	0.186 m

This state space system is unstable but is controllable and observable. In order to analyze the coupling in a stable system it is used an arbitrary state feedback u(t)=-Kx(t) such that, assign the eigenvalues in $[-1.5 \quad -1.5 \quad -1 \quad -1]$ with $K = \begin{bmatrix} 0.6417 & -0.0640 & -2.8922 & 0.0421 \\ -0.1952 & 1.9193 & 0.8797 & -1.2631 \end{bmatrix}$.
The matrix transfer function $T(s) = C(sI - A + BK)^{-1}B$ is defined as

$$T(s) := \begin{matrix} y_1 \\ y_2 \\ \ \end{matrix} \begin{bmatrix} \frac{\theta(t)}{u_p} & \frac{\theta(t)}{u_y} \\ \frac{\psi(t)}{u_p} & \frac{\psi(t)}{u_y} \end{bmatrix}_{\substack{u_1 \qquad u_2}} \tag{10}$$

where

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$$\begin{aligned}\frac{\theta(t)}{u_p} &= \frac{2.361s^2 + 5.903s + 3.542}{s^4 + 5s^3 + 9.25s^2 + 7.5s + 2.25}\\ \frac{\psi(t)}{u_p} &= \frac{0.2402s^2 + 0.6004s + 0.3602}{s^4 + 5s^3 + 9.25s^2 + 7.5s + 2.25}\\ \frac{\theta(t)}{u_y} &= \frac{0.07871s^2 + 0.1968s + 0.1181}{s^4 + 5s^3 + 9.25s^2 + 7.5s + 2.25}\\ \frac{\psi(t)}{u_y} &= \frac{0.7895s^2 + 1.974s + 1.184}{s^4 + 5s^3 + 9.25s^2 + 7.5s + 2.25}\end{aligned} \tag{11}$$

To study the behavior of the system in steady state at step inputs, the final value theorem is applied to (10), where the behavior of the system is analyzed when s=0; therefore we have (12)

$$T(0) = \begin{bmatrix}1.5742 & 0.05251\\ 0.1601 & 0.5264\end{bmatrix}. \tag{12}$$

Rewriting (12) we obtain the following:

$$\begin{aligned}y_1 &= 1.5742u_1 + 0.05251u_2\\ y_2 &= 0.1601u_1 + 0.5264u_2\end{aligned} \tag{13}$$

The Bristol-Shinskey method is applied from (13) and will lead us to obtain a relative matrix array and with this matrix we can determine how each input affects the outputs Shinskey, F.G. (1996).

The first element of the relative gain matrix is λ_{11}, which gives us the coupling ratio of input one to output one and is obtained as in (14). For this relative gain, it is assumed that u_2 remains constant to a change in the variable u_1 of magnitude Δ_{y1} (which is the partial derivative of y_1), with this we obtain the numerator of the gain λ_{11}, now we keep the output y_2 constant and the rate of change of y_1 with respect to u_1 must be obtained, and this value will give us the denominator of equation (14)

$$\lambda_{11} = \frac{\left.\frac{\partial y_1}{\partial u_1}\right|_{u_2}}{\left.\frac{\partial y_1}{\partial u_1}\right|_{y_2}} \tag{14}$$

λ_{11} is a dimensionless number called the relative gain of the output y_1 to the input u_1. The remaining elements of the RGA are defined as Chen, H. L M. (1983):

$$\lambda_{12} = \frac{\left.\frac{\partial y_1}{\partial u_2}\right|_{u1}}{\left.\frac{\partial y_1}{\partial u_2}\right|_{y_2}} \quad \lambda_{21} = \frac{\left.\frac{\partial y_2}{\partial u_1}\right|_{u2}}{\left.\frac{\partial y_2}{\partial u_1}\right|_{y_1}} \quad \lambda_{22} = \frac{\left.\frac{\partial y_2}{\partial u_2}\right|_{u1}}{\left.\frac{\partial y_2}{\partial u_2}\right|_{y_1}} \tag{15}$$

The subscripts indicate which input and output are considered for the analysis. The relative gain matrix (RGA), considering (14),(15) is shown in (16). An alternative method to obtain RGA based on T(0) is $\Delta = T(0) \times (T(0)^T)^{-1}$ where $\times$ is evaluated element by element Albertos, P. y Antonio, S. (2006)

$$\Delta = \begin{bmatrix}\lambda_{11} & \lambda_{12}\\ \lambda_{21} & \lambda_{22}\end{bmatrix} \tag{16}$$

Applying the partial derivatives (14) and (15) to the 2-DOF Helicopter system, it is obtained the following relative gain array

$$\Delta = \begin{bmatrix}1.0102 & -0.0102\\ -0.0102 & 1.0102\end{bmatrix} \tag{17}$$

In RGA, we observe couplings where each element has a meaning. The different values that an RGA can take are listed below.

a) If we find a value with $\lambda_{ij} = 0$, there is no relationship between the manipulated variable j and the controlled variable i.

b) If we find a value with $\lambda_{ij} = 1$, there is no interaction with other links.

c) If we find a value with $0 < \lambda_{ij} < 1$, it means there is interaction; that is, a change in one manipulated variable will alter the other controlled variables.

d) If we find a value with $\lambda_{ij} < 0$, it indicates that there will be slow and poor dynamic responses, and this interaction forces the controlled variable to respond in the opposite direction of the direct response. As a result, the controlled variable moves in one direction and then, to a greater extent, in the opposite direction.

e) If we find a value with $\lambda_{ij} = \infty$, it indicates that both variables cannot be controlled simultaneously.

In our work, the stabilizing feedback was intentionally introduced only as a preliminary step to enable steady–state coupling analysis. However, as shown by the RGA results, the strong input–output interactions remain after stabilization, which is precisely the motivation for our proposal to modify the RGA and improve the conditions for subsequent controller design.

3. PROPOSED RGA MODIFICATION METHODOLOGY.

In this section, the proposed method that performs the arbitrary change of the coupling degree is shown, applying it to the 2-DOF helicopter system. First, the steady-state analysis of a system is computed by applying the final value theorem when s=0, therefore we will have a system described in a general way by (18)

$$\begin{aligned}y_1 &= au_1 + bu_2\\ y_2 &= cu_1 + du_2\end{aligned} \tag{18}$$

Considering (18), the relative gains (14) and (15) are the following:

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$$\lambda_{11} = \frac{a}{\left(a - \frac{bc}{d}\right)} = \frac{\left.\frac{\partial y_1}{\partial u_1}\right|_{u_2}}{\left.\frac{\partial y_1}{\partial u_1}\right|_{y_2}} \quad \lambda_{12} = \frac{b}{\left(b - \frac{ad}{c}\right)} = \frac{\left.\frac{\partial y_1}{\partial u_2}\right|_{u1}}{\left.\frac{\partial y_1}{\partial u_2}\right|_{y_2}}$$
$$\lambda_{21} = \frac{c}{\left(c - \frac{da}{b}\right)} = \frac{\left.\frac{\partial y_2}{\partial u_1}\right|_{u2}}{\left.\frac{\partial y_2}{\partial u_1}\right|_{y_1}} \quad \lambda_{22} = \frac{d}{\left(d - \frac{cb}{a}\right)} = \frac{\left.\frac{\partial y_2}{\partial u_2}\right|_{u1}}{\left.\frac{\partial y_2}{\partial u_2}\right|_{y_1}} \tag{19}$$

Rewriting (19), where the variables x, y, w, and z are defined to indicate the values of the degree of coupling that are desired. The proposed relative gain array is

$$\Delta_p = \begin{bmatrix} \frac{ad}{ad - bc} = x & \frac{bc}{bc - ad} = y \\ \frac{bc}{bc - ad} = w & \frac{ad}{ad - bc} = z \end{bmatrix} \tag{20}$$

From (20) the elements of this matrix can be rewritten as

$$a = \frac{-xbc}{(d - xd)}, \quad b = \frac{-yad}{(c - yc)}$$
$$c = \frac{-wad}{(b - wb)}, \quad d = \frac{-zbc}{(a - za)} \tag{21}$$

where, it is observed that the elements of (21) are a function of the proposed relative gains.

Subsequently, the degree of coupling that is desired to have in the system is proposed according to some desired performance of interaction between inputs and outputs. In the helicopter example, there was a relative gain matrix as in (17), in which input 1 dominates over output 1 and input 2 over output 2. Just to show the methodology without pursuing a specific control objective, it is arbitrarily proposed that this interaction changes as shown in (22)

$$\Delta_p = \begin{bmatrix} -0.0002 & 1.0002 \\ 1.0002 & -0.0002 \end{bmatrix}. \tag{22}$$

That is, input 2 dominates more over output 1 and input 1 over output 2. In reality, with this methodology, any positive, negative, large or small interaction can be assigned, modifying the original interaction of the system. This is important because, in process control, the use of RGA is a good way to analyze and propose input-output pairings. However, if the system exhibits weak or inadequate coupling, a controller design will have to deal with poor coupling.

Comparing (22) with (20), and $\boldsymbol{x}$ and b=c=d=1 then

$$a = \frac{-xbc}{(d - xd)} = 0.0002 \tag{23}$$

Based on (18) and (23)

$$y_1 = 0.0002u_1 + u_2$$
$$y_2 = u_1 + u_2 \tag{24}$$

Representing (24) in a matrix way, we have the matrix of (25), which we will call $M_{proposal}$

$$M_{proposal} = \begin{bmatrix} 0.0002 & 1 \\ 1 & 1 \end{bmatrix} \tag{25}$$

Since the helicopter's steady-state response is (12) it will now be defined as $M_{original}$

$$M_{original} \coloneqq T(0) = \begin{bmatrix} 1.5742 & 0.05251 \\ 0.1601 & 0.5264 \end{bmatrix} \tag{26}$$

It is proposed that there exists a compensator C such that relates (25) and (26) as

$$C = \left[M_{original}\right]^{-1} * \left[M_{proposal}\right] \tag{27}$$

Therefore,

$$C = \begin{bmatrix} 1.5742 & 0.05251 \\ 0.1601 & 0.5264 \end{bmatrix}^{-1} \begin{bmatrix} 0.0002 & 1 \\ 1 & 1 \end{bmatrix}$$
$$C = \begin{bmatrix} -0.0638 & 0.5778 \\ 1.9192 & 1.7241 \end{bmatrix} \tag{28}$$

It is proposed that this compensator can be applied to the original system of (10),

$$T_{\text{proposal}}(s) = [T(s)] * [C] \tag{29}$$

$$\mathrm{T_{modified}}(\mathrm{s}) = \begin{bmatrix} \frac{\mathrm{u}_1}{\mathrm{y}_1} & \frac{\mathrm{u}_2}{\mathrm{y}_1} \\ \frac{\mathrm{u}_1}{\mathrm{y}_2} & \frac{\mathrm{u}_2}{\mathrm{y}_2} \end{bmatrix} \tag{30}$$

where

$$\frac{u_1}{y_1} = \frac{0.0002999\,s^6 + 0.00225\,s^5 + 0.006974\,s^4}{s^8 + 10\,s^7 + 43.5\,s^6 + 107.5\,s^5 + 165.1\,s^4} \cdots$$
$$\cdots \frac{+ 0.01144\,s^3 + 0.01046\,s^2 + 0.005061\,s + 0.001012}{+ 161.2\,s^3 + 97.87\,s^2 + 33.75\,s + 5.062}$$
$$\frac{u_1}{y_2} = \frac{1.5\,s^6 + 11.25\,s^5 + 34.87\,s^4}{s^8 + 10\,s^7 + 43.5\,s^6 + 107.5\,s^5 + 165.1\,s^4} \cdots$$
$$\cdots \frac{+ 57.19\,s^3 + 52.31\,s^2 + 25.31\,s + 5.062}{+ 161.2\,s^3 + 97.87\,s^2 + 33.75\,s + 5.062}$$
$$\frac{u_2}{y_1} = \frac{1.5\,s^6 + 11.25\,s^5 + 34.87\,s^4}{s^8 + 10\,s^7 + 43.5\,s^6 + 107.5\,s^5 + 165.1\,s^4} \cdots \tag{31}$$
$$\cdots \frac{+ 57.19\,s^3 + 52.31\,s^2 + 25.31\,s + 5.062}{+ 161.2\,s^3 + 97.87\,s^2 + 33.75\,s + 5.062}$$
$$\frac{u_2}{y_2} = \frac{1.5\,s^6 + 11.25\,s^5 + 34.87\,s^4}{s^8 + 10\,s^7 + 43.5\,s^6 + 107.5\,s^5 + 165.1\,s^4} \cdots$$
$$\cdots \frac{+ 57.19\,s^3 + 52.31\,s^2 + 25.31\,s + 5.062}{+ 161.2\,s^3 + 97.87\,s^2 + 33.75\,s + 5.062}$$

The transfer matrix (30) already has the new relative gain array. This is verified through the application of the Bristol-Shinskey method, which begins with the steady-state response s=0 of (30), that satisfies the following system of equations:

$$y_1 = 0.0002u_1 + u_2$$
$$y_2 = u_1 + u_2 \tag{32}$$

It is important to note that (32) is equal to (24). Now, calculating the relative degrees corresponding to (32):

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$$\lambda_{11}=\frac{\left.\frac{\partial y_1}{\partial u_1}\right|_{u_2}}{\left.\frac{\partial y_1}{\partial u_1}\right|_{y_2}}=-0.0002 \quad \lambda_{12}=\frac{\left.\frac{\partial y_1}{\partial u_2}\right|_{u1}}{\left.\frac{\partial y_1}{\partial u_2}\right|_{y_2}}=1.0002$$
$$\lambda_{21}=\frac{\left.\frac{dy_2}{du_1}\right|_{u2}}{\left.\frac{dy_2}{du_1}\right|_{y_1}}=1.0002 \quad \lambda_{22}=\frac{\left.\frac{\partial y_2}{\partial u_2}\right|_{u1}}{\left.\frac{\partial y_2}{\partial u_2}\right|_{y_1}}=-0.0002 \tag{33}$$

Representing these relative degrees in a matrix form, it is possible to see the relative gain matrix of the modified system as follows:

$$\Delta=\begin{bmatrix}-0.0002 & 1.0002\\ 1.0002 & -0.0002\end{bmatrix}=\Delta_p \tag{34}$$

Finally, it can be seen that the compensator (28) is capable of modifying the system (10) in the form of (30) affecting its interaction between inputs and outputs as it is required.

4. SIMULATION RESULTS AND PERFORMANCE ANALYSIS

This section shows the behavior of the 2-DOF helicopter stable with a RGA as in (17) and with a modified RGA as in (22). The block diagram in Fig. 2 was implemented in Matlab Simulink.

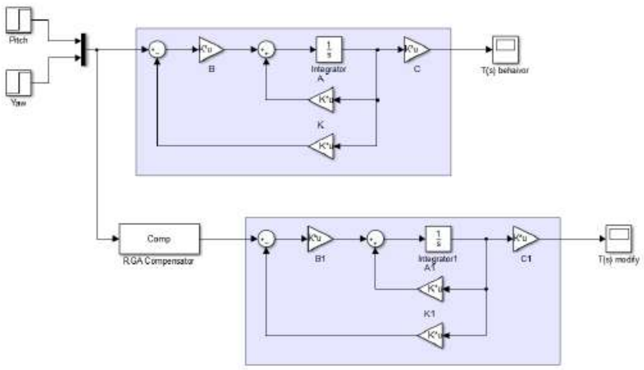

Figure 2. Simulink to evaluate RGA.

The simulations shown in Fig. 2 are indeed based on the linear state–space model presented in equation (4), stabilized with state feedback to enable steady–state analysis. The Simulink block diagram implements the corresponding state and output equations, allowing us to calculate and compare the original and modified RGA. Thus, the verification directly corresponds to the model described in the paper and supports the proposed methodology.

In this simulation the system (4) is considered with K such that the stable feedback system (10) is obtained. Zero initial conditions are considered in the helicopter, and two step inputs are applied. The first input affects the pitch angle which at the beginning is zero and in 20 seconds changes to 0.3 rad. The second input is a control signal applied to Yaw motor in which a step input is applied where at the beginning it is zero and after 60 seconds it changes to 0.5 rad. The two feedback systems in Fig.2 are the same, considering that the upper system has the calculated coupling (17), while the lower system includes the compensator that modifies the coupling as in (22). In Fig.3, it is shown the response of the stable system that has the RGA (17) and in Fig. 4 the response of the system with the modified RGA (22).

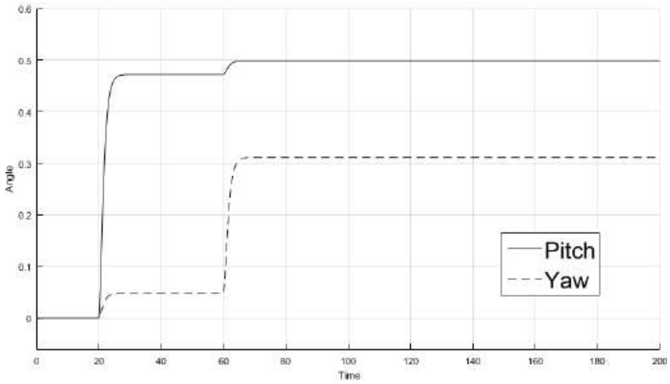

Figure 3. Helicopter response with RGA (17).

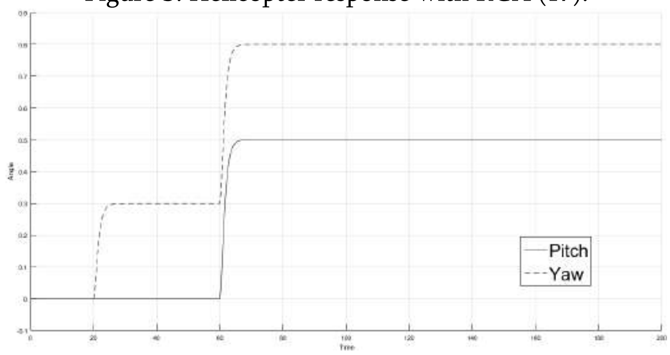

Figure 4. Helicopter response with RGA (22).

Based on the obtained responses, it can be observed that the interaction between inputs and outputs can be modified. This is evident in Fig. 3, where Input 1, applied at second 20, has a greater effect on the pitch output angle than on the yaw output angle, which is consistent with (17). Subsequently, when Input 2 is applied, the pitch output angle remains largely unaffected, whereas the yaw output angle exhibits a significant response, consistent with equation (17).

In the modified system shown in Fig 4, it can be seen that Input 1 barely affects the pitch output angle, which aligns with (22). Although, it slightly affects the yaw angle, it is still consistent with (22). Then, with Input 2, there is a noticeable effect on the pitch output angle, as expected in (22), and in this case, the yaw output angle is also affected, as shown in the coupling matrix.

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It is important to note that the coupling of the system is being modified, and its behavior analyzed, but no control strategy is being applied to meet specific control objectives such as reference tracking or others. It is interesting that this work allows for the assignment of a relative gain matrix (RGA) that can help achieving more appropriate input-output interaction response for controller design.

5. CONCLUSIONS

The proposed method provides a way to arbitrarily modify the degree of coupling in a system, allowing for the freedom to choose how inputs interact with outputs. The advantage of manipulating this coupling is that it enables the selection of input-output interactions in processes where coupling affects system performance, such as very large, negative, or very small couplings. These types of couplings are found in systems like refrigeration units, distillation columns, helicopters, aircraft, etc.

The ability to modify the coupling allows the engineer to start from a better-conditioned system in terms of input-output interaction, facilitating controller design. It is recommended to analyze different coupling configurations and their impact on controller performance. As future work, this coupling modification approach will be explored with different controllers.

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