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2D Adaptive Visual Servoing Problem: Relaxing the Excitation Requirements

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Abstract: Visual servoing incorporates visual information, from an external camera, in feedback control loops for position and motion control of autonomous robot manipulators. In practical implementations of these controllers, the intrinsic and extrinsic parameters of the camera have to be *a priori* calibrated. In this work we deal with the *uncalibrated* visual servoing problem. In particular, we report two new adaptive controllers that ensure that the tracking error globally asymptotically converges to zero. One controller is the certainty equivalent version of the known parameter controller, that requires some excitation conditions, and on the second scheme we relax such excitation conditions but it, possibly, needs to inject some high gain. The performance of the proposed controllers is illustrated with numerical simulations.

Keywords: Visual servoing, adaptive control.

1. INTRODUCTION

Visual servoing incorporates visual information, from an external camera, in feedback control loops for position and motion control of autonomous robot manipulators that perform tasks in unstructured environments (Hutchinson et al., 1996; Lizarralde et al., 2013; Wang et al., 2010; Piepmeier et al., 2004; Wang, 2015). An important, but tedious, issue in the practical applications of visual servoing is the calibration of the intrinsic and extrinsic parameters of the camera (Parra-Vega and Fierro-Rojas, 2003; Wang, 2015). In this paper we consider the problem of adaptive visual servoing of planar robot manipulators under a fixed-camera configuration, where the *camera orientation and the image scale factor are unknown*. The control goals are to place the robot end-effector in some desired constant position—or to make it track a trajectory—by using a vision system equipped with a fixed camera that is perpendicular to the plane where the robot evolves.

In (Kelly, 1996) it was shown that a (fixed parameter) PD-like controller ensures asymptotic *set-point regulation* of the full robot dynamics in spite of the uncertainty on the orientation parameter, which should however not be greater than $\frac{\pi}{2}$. It is well-known that the transient performance of PD-like schemes can be improved, particularly in tracking applications, adding an adaptation feature that has been, in particular, explored for camera calibration. The design of these adaptive controllers is unfortunately complicated because the unknown parameters enter nonlinearly into the system dynamics. One way to bypass the nonlinearity obstacle is to overparameterize the system, which has the undesirable consequence of increasing the number of parameters to be identified and hampering the possibility of parameter convergence (Sastry and Bodson, 1989). Several alternatives to avoid overparameterization have been reported in the literature but, to the best of the authors' knowledge, the design of a globally convergent adaptive controller remains an open problem—the reader is referred to (Astolfi et al., 2008; Zachi et al., 2006) for a review of these results.

In this paper we present two new, non-overparametrized, adaptive controllers that ensure that the tracking error globally asymptotically converges to zero. These controllers rely on the use of the same parameter estimator algorithm, which is derived introducing a new reparameterization of the systems mathematical model, and exploiting some structural properties of it, but they use this estimates in a different way. The first controller is the certainty equivalent version of the known parameter controller, hence has a very simple structure. However, to ensure global convergence, it requires some excitation conditions that impose some constraints on the reference trajectory. To relax the excitation assumption, a second version of the controller—which is now slightly more complicated and, possibly, needs to inject some high gain—is given. The performance of the proposed controllers is illustrated with numerical simulations.

2. PROBLEM FORMULATION

We consider an n-degrees of freedom (dof) robot manipulator that evolves in a plane, with $n \geq 2$. The vision system consists of a TV camera of CDD type that is fixed perpendicular to the plane where the robot evolves providing an image of the whole robot workspace, what includes the robot end-effector and the target—see Fig. 1. The image acquired by the camera supplies a two-dimensional array of brightness values from a three-dimensional scene.

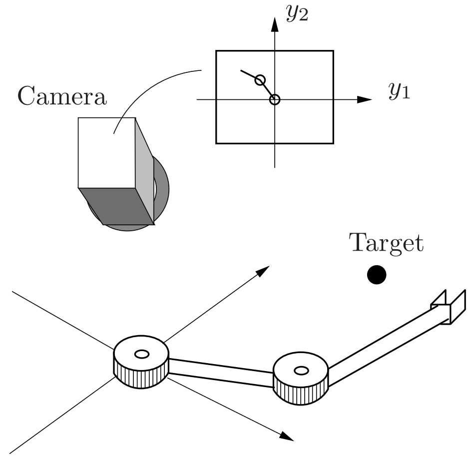

Fig. 1. Diagram of the visual servoing problem

Following the standard procedure Kelly (1996), we assume that the image features are the projection into the 2D image plane of 3D points in the scene space, hence we model the *action of the camera* as a static mapping from the joint robot positions $\mathbf{q} \in \mathbb{R}^n$ to the position (in pixels) of the robot tip in the image output, denoted $\mathbf{y} \in \mathbb{R}^2$. Such a mapping is described by

$$\mathbf{y} = ae^{\mathbf{J}\theta}(\mathbf{f}(\mathbf{q}) - \boldsymbol{\vartheta}_1) + \boldsymbol{\vartheta}_2, \tag{1}$$

where $\theta \in \mathbb{D}$ is is the orientation of the camera with respect to the robot frame, $\boldsymbol{\vartheta}_1, \boldsymbol{\vartheta}_2 \in \mathbb{R}^2$ and $a > 0$ are the intrinsic camera parameters (scale factors, focal length and centre offset, respectively). The function $\mathbf{f} : \mathbb{R}^n \mapsto \mathbb{R}^2$ is the robot direct kinematics and

$$\mathbf{J} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \; e^{\mathbf{J}\theta} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}.$$

From the direct kinematics we have that

$$\dot{\mathbf{f}} = \left[\frac{\partial \mathbf{f}(\mathbf{q})}{\partial \mathbf{q}}\right]^\top \dot{\mathbf{q}} =: \mathcal{J}(\mathbf{q})\dot{\mathbf{q}},$$

where $\mathcal{J} : \mathbb{R}^n \mapsto \mathbb{R}^{2\times n}$ is the Jacobian matrix, which we assume is full rank. Differentiating (1) and replacing the identity above we get $\dot{\mathbf{y}} = ae^{\mathbf{J}\theta}\mathcal{J}(\mathbf{q})\dot{\mathbf{q}}$.

Invoking standard time-scale separation arguments and assuming an inner fast loop for the robot velocity control, we concentrate on the kinematic problem to generate the *references* for the robot velocities. The robot dynamics are then described by a simple integrator $\dot{\mathbf{q}} = \boldsymbol{\tau}$, where $\boldsymbol{\tau} \in \mathbb{R}^n$ represents the joint velocities references. Setting

$$\boldsymbol{\tau} = \mathcal{J}^\top(\mathbf{q}) \left[\mathcal{J}(\mathbf{q})\mathcal{J}^\top(\mathbf{q})\right]^{-1} \mathbf{u}$$

where $\mathbf{u} \in \mathbb{R}^2$ is a new input to be designed yields

$$\dot{\mathbf{y}} = ae^{\mathbf{J}\theta}\mathbf{u}. \tag{2}$$

Adaptive Calibration Problem. Given the vision system (2), with measurable $\mathbf{y}$, and a bounded target trajectory $\mathbf{y}_\star(t)$ with known bounded derivative $\dot{\mathbf{y}}_\star(t)$. Find a control signal $\mathbf{u}$ such that, in spite of the lack of knowledge of a and θ, all signals remain bounded and

$$\lim_{t\to\infty} |\tilde{\mathbf{y}}(t)| = 0 \tag{3}$$

where $\tilde{\mathbf{y}} := \mathbf{y} - \mathbf{y}_\star$ is the tracking error and $|\cdot|$ is the Euclidean norm.

Moreover, we are also interested in ensuring that the convergence of the tracking error happens in finite time. That is, ensure the existence of $T > 0$ such that $\tilde{\mathbf{y}}(t) = \mathbf{0}$ for all $t \geq T$.

3. CONTROLLER PARAMETERIZATION AND PARAMETER ESTIMATOR

In spite of the simplicity of the system dynamics (2), the task is complicated by its highly nonlinear dependence on the unknown parameters. Note that if a and θ were known, the ideal controller is given by

$$\mathbf{u}_\star = \frac{1}{a}e^{-\mathbf{J}\theta}\left(\dot{\mathbf{y}}_\star - \lambda\tilde{\mathbf{y}}\right), \tag{4}$$

with $\lambda > 0$. Resulting in the closed-loop system $\dot{\tilde{\mathbf{y}}} = -\lambda\tilde{\mathbf{y}}$. We make now the observation that the ideal controller (4) can be written in the form

$$\mathbf{u}_\star = \begin{bmatrix} \boldsymbol{\eta}^\top \\ -\boldsymbol{\eta}^\top\mathbf{J} \end{bmatrix}\left(\dot{\mathbf{y}}_\star - \lambda\tilde{\mathbf{y}}\right), \tag{5}$$

where we defined the vector

$$\boldsymbol{\eta} := \frac{1}{a}\begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix}. \tag{6}$$

and used the fact that

$$\frac{1}{a}e^{-\mathbf{J}\theta} = \begin{bmatrix} \boldsymbol{\eta}^\top \\ -\boldsymbol{\eta}^\top\mathbf{J} \end{bmatrix}. \tag{7}$$

The key feature of the ideal controller parameterization (5) is that it is *linear* in the unknown parameters $\boldsymbol{\eta}$.

To estimate the parameters $\boldsymbol{\eta}$ we follow the *input-error* formulation of adaptive control (Sastry and Bodson, 1989)—as opposed to the more classical output-error one. Towards this end, we first rewrite (2) in the equivalent form

$$\mathbf{u} = \begin{bmatrix} \boldsymbol{\eta}^\top \\ -\boldsymbol{\eta}^\top\mathbf{J} \end{bmatrix}\dot{\mathbf{y}}, \tag{8}$$

where we have used the identity (7). Now, define the following filtered signals

$$\begin{aligned} \dot{\mathbf{x}} &= -\mathbf{x} + \mathbf{y}, \quad \mathbf{x}(0) = \mathbf{y}(0) \\ \dot{\mathbf{v}} &= -\mathbf{v} + \mathbf{u}, \quad \mathbf{v}(0) = \mathbf{0} \\ \mathbf{z} &= -\mathbf{x} + \mathbf{y}, \end{aligned} \tag{9}$$

which are the state realizations of the linear time-invariant (LTI) filters [1]

$$\mathbf{z} = \frac{s}{s+1}\mathbf{y}, \quad \mathbf{v} = \frac{1}{s+1}\mathbf{u},$$

with some particular initial conditions, whose choice is explained below.

Applying the filter $\frac{1}{s+1}$ to (8) and using the definitions above, we get the key input-error parameterization

$$\mathbf{v} = \begin{bmatrix} \boldsymbol{\eta}^\top \\ -\boldsymbol{\eta}^\top\mathbf{J} \end{bmatrix}\mathbf{z}. \tag{10}$$

We underscore the fact that, because of the choice of initial conditions in (9), the identity (10) holds true for all $t \geq 0$. [2]

From (10), noting that

$$\frac{1}{a}e^{-\mathbf{J}\theta} = \frac{1}{a}\left[\cos(\theta)\mathbf{I} - \sin(\theta)\mathbf{J}\right] = \begin{bmatrix} \boldsymbol{\eta}^\top \\ -\boldsymbol{\eta}^\top\mathbf{J} \end{bmatrix}$$

[1] With the standard abuse of notation, the Laplace transform symbol s is used also to denote the derivative operator.

[2] Otherwise, the identity holds true up to an additive, exponentially decaying term—see Remark 1.

and after some simple calculations, we see that

$$\begin{bmatrix} \mathbf{z}^\top \mathbf{v} \\ \mathbf{z}^\top \mathbf{J}\mathbf{v} \end{bmatrix} = |\mathbf{z}|^2 \boldsymbol{\eta}, \tag{11}$$

where $\boldsymbol{\eta}$ is defined in (6). Since $\mathbf{v}$ and $\mathbf{z}$ are measurable signals, (11) defines a *linear regression* for the unknown parameters $\boldsymbol{\eta}$, to which standard gradient-descent or least-squares algorithm can be applied. For simplicity, in the paper we consider the former estimator, namely

$$\dot{\hat{\boldsymbol{\eta}}} = \gamma \left(\begin{bmatrix} \mathbf{z}^\top \mathbf{v} \\ \mathbf{z}^\top \mathbf{J}\mathbf{v} \end{bmatrix} - |\mathbf{z}|^2 \hat{\boldsymbol{\eta}} \right), \tag{12}$$

where $\gamma > 0$ is an adaptation gain.

The estimator (12) verifies the following property.

Proposition 1. Consider the system (2) and the estimator (12) with $\mathbf{v}, \mathbf{z}$ generated via (9). Define the parameter error vector $\tilde{\boldsymbol{\eta}} := \hat{\boldsymbol{\eta}} - \boldsymbol{\eta}$, where $\boldsymbol{\eta}$ is given in (6). For any $\mathbf{y}(0) \in \mathbb{R}^2$ and any $\hat{\boldsymbol{\eta}}(0) \in \mathbb{R}^2$ the following equivalence is true:

$$\mathbf{z}(t) \notin \mathcal{L}_2 \iff \lim_{t\to\infty} |\tilde{\boldsymbol{\eta}}(t)| = 0, \tag{13}$$

where $\mathcal{L}_2$ is the space of square-integrable functions.

Proof. Replacing (11) in (12) yields

$$\dot{\tilde{\boldsymbol{\eta}}} = -\gamma |\mathbf{z}|^2 \tilde{\boldsymbol{\eta}}. \tag{14}$$

Thus

$$\tilde{\boldsymbol{\eta}}(t) = e^{-\gamma \int_0^t |\mathbf{z}(\tau)|^2 d\tau} \tilde{\boldsymbol{\eta}}(0), \tag{15}$$

from which the proof is completed. □

Remark 1. One sense of the implication of Proposition 1 holds true for arbitrary initial condition on the filters (9). Indeed, in this case, equation (10) becomes

$$\mathbf{v} = \begin{bmatrix} \boldsymbol{\eta}^\top \\ -\boldsymbol{\eta}^\top \mathbf{J} \end{bmatrix} \mathbf{z} + \boldsymbol{\varepsilon}_t,$$

where $\boldsymbol{\varepsilon}_t$ is an exponentially decaying term depending on the system and filter initial conditions. The parameter error equation (14), in its turn, becomes

$$\dot{\tilde{\boldsymbol{\eta}}} = -\gamma |\mathbf{z}|^2 \tilde{\boldsymbol{\eta}} + \begin{bmatrix} \mathbf{z}^\top \\ \mathbf{z}^\top \mathbf{J} \end{bmatrix} \boldsymbol{\varepsilon}_t.$$

Invoking Lemma 1 of the Appendix, whose proof is given in (Aranovskiy et al., 2015), we can show that

$$\mathbf{z}(t) \notin \mathcal{L}_2 \implies \lim_{t\to\infty} |\tilde{\boldsymbol{\eta}}(t)| = 0.$$

Notice that, in contrast with (13), the later is not an equivalence anymore.

Remark 2. Imposing on $\mathbf{z}$ the condition of persistence of excitation (PE), that is, existence of $T > 0$ and $\delta > 0$ such that

$$\int_t^{t+T} |\mathbf{z}(\tau)|^2 d\tau > \delta, \ \forall t \geq 0,$$

the convergence of the parameter errors is *exponential* (Sastry and Bodson, 1989). This rather strict condition is, however, not required for our analysis. See (Aranovskiy et al., 2017) for a discussion on the relation between non square-integrability and PE of regressor signals.

4. MAIN RESULTS

This section presents our novel adaptive controllers that solve the adaptive calibration problem. The first one is the certainty equivalent version of (5), but relies on parameter convergence hence, in view of (13), imposes some excitation requirements on the closed-loop signals. The second one, overcomes the latter assumption at the prize of requiring a more complicated controller and, possibly, needing to inject some high gain.

4.1 Controller relying on excitation

Proposition 2. Consider the system (2) in closed-loop with the adaptive controller

$$\mathbf{u} = \begin{bmatrix} \hat{\boldsymbol{\eta}}^\top \\ -\hat{\boldsymbol{\eta}}^\top \mathbf{J} \end{bmatrix} (\dot{\mathbf{y}}_\star - \lambda \tilde{\mathbf{y}}), \tag{16}$$

with (9) and (12). For any $\mathbf{y}(0) \in \mathbb{R}^2$ and any $\hat{\boldsymbol{\eta}}(0) \neq \mathbf{0}$ we have that (3) holds with all signals bounded provided that $\mathbf{z}(t) \notin \mathcal{L}_2$.

Proof. Using (5) and (16) yields the closed-loop system

$$\begin{aligned} \dot{\tilde{\mathbf{y}}} &= -\lambda \tilde{\mathbf{y}} + a e^{\mathbf{J}\theta} (\mathbf{u} - \mathbf{u}_\star) \\ &= -\lambda \tilde{\mathbf{y}} + a e^{\mathbf{J}\theta} \begin{bmatrix} (\hat{\boldsymbol{\eta}} - \boldsymbol{\eta})^\top \\ (\boldsymbol{\eta} - \hat{\boldsymbol{\eta}})^\top \mathbf{J} \end{bmatrix} (\dot{\mathbf{y}}_\star - \lambda \tilde{\mathbf{y}}) \\ &= -\lambda \tilde{\mathbf{y}} + a e^{\mathbf{J}\theta} \begin{bmatrix} \tilde{\boldsymbol{\eta}}^\top \\ -\tilde{\boldsymbol{\eta}}^\top \mathbf{J} \end{bmatrix} (\dot{\mathbf{y}}_\star - \lambda \tilde{\mathbf{y}}). \end{aligned} \tag{17}$$

Notice that $\tilde{\boldsymbol{\eta}}$ is defined by (15) hence it is non-increasing, bounded and, under the standing assumption on $\mathbf{z}$, converges to zero.

The closed-loop system (17) is a perturbed, linear time-varying (LTV) system of the form

$$\dot{\tilde{\mathbf{y}}} = \mathbf{A}(t) \tilde{\mathbf{y}} + \mathbf{b}(t)$$

where

$$\mathbf{A}(t) := \lambda \mathbf{I} - \lambda a e^{\mathbf{J}\theta} \begin{bmatrix} \tilde{\boldsymbol{\eta}}^\top \\ -\tilde{\boldsymbol{\eta}}^\top \mathbf{J} \end{bmatrix}$$

$$\mathbf{b}(t) := a e^{\mathbf{J}\theta} \begin{bmatrix} \tilde{\boldsymbol{\eta}}^\top \\ -\tilde{\boldsymbol{\eta}}^\top \mathbf{J} \end{bmatrix} \dot{\mathbf{y}}_\star.$$

We recall now that an LTV system $\dot{\mathbf{x}} = \mathbf{A}(t)\mathbf{x}$, with $\mathbf{A}(t)$ continuous, is globally asymptotically stable if the eigenvalues of $\mathbf{A}(\infty)$ exists and they have negative real parts, Lemma 3.8 of (Tomas-Rodriguez and Banks, 2010). Moreover, this property is preserved in the presence of an asymptotically decaying additive perturbation. Applying this lemma to the system (17) completes the proof. □

4.2 Controller with asymptotic convergence

To avoid the need of imposing an excitation assumption on the signals of the system, which is difficult to verify *a priori*, we propose in this subsection a new adaptive controller. To streamline the presentation of the main result we introduce the following assumption, which essentially requires that $\mathbf{y}(t)$ is not a constant function—a reasonable assumption in all practical scenarios.

Assumption 1. Fix a small constant $\mu \in (0, 1)$. There exists a time $t_c > 0$ such that

$$\int_0^{t_c} |\mathbf{z}(\tau)|^2 d\tau \geq -\frac{1}{\gamma} \ln(1 - \mu). \tag{18}$$

Proposition 3. Consider the system (2) in closed-loop with the adaptive controller

$$\mathbf{u} = \begin{bmatrix} \boldsymbol{\ell}^\top \\ -\boldsymbol{\ell}^\top \mathbf{J} \end{bmatrix} (\dot{\mathbf{y}}_\star - \lambda \tilde{\mathbf{y}}), \tag{19}$$

where

$$\boldsymbol{\ell} = \frac{1}{1-w_c} (\hat{\boldsymbol{\eta}} - w_c \hat{\boldsymbol{\eta}}(0)), \tag{20}$$

with $\hat{\boldsymbol{\eta}}$ generated via (12), w is the solution of

$$\dot{w} = -\gamma |\mathbf{z}|^2 w, \ w(0) = 1, \tag{21}$$

and the clipped function w_c is defined as

$$w_c = \begin{cases} w & \text{if } \ w < 1-\mu \\ 1-\mu & \text{if } \ w \geq 1-\mu, \end{cases} \tag{22}$$

with $\mu \in (0,1)$ a small constant. If $\mathbf{z}(t)$ verifies Assumption 1, then for all $\mathbf{y}(0) \in \mathbb{R}^2$ and all $\hat{\boldsymbol{\eta}}(0) \neq \mathbf{0}$, we have that (3) holds with all signals bounded.

Proof. First, notice that the definition of w_c in (22) ensures the control law (19), (20) is well-defined. This, together with the boundedness of $\hat{\boldsymbol{\eta}}$, ensure that trajectories cannot escape in finite time. Now, from (15) and (21) we have that

$$\tilde{\boldsymbol{\eta}} = w \tilde{\boldsymbol{\eta}}(0).$$

Clearly, this is equivalent to

$$(1-w)\boldsymbol{\eta} = \hat{\boldsymbol{\eta}} - w\hat{\boldsymbol{\eta}}(0). \tag{23}$$

On the other hand, under Assumption 1, we have that $w_c(t) = w(t), \ \forall t \geq t_c$.

Consequently, from (20) and (23) we conclude that $\boldsymbol{\ell}(t) = \boldsymbol{\eta}, \ \forall t \geq t_c$, and from (5) and (19) we get that

$$\mathbf{u}(t) = \mathbf{u}_\star(t), \ \forall t \geq t_c,$$

with $\mathbf{u}_\star$ given in (5). Finally, since trajectories cannot escape in finite time, this completes the proof. □

5. SIMULATIONS

The proposed adaptive schemes have been simulated for a robot manipulator with 2-dof, whose kinematics is given by

$$\mathbf{f}(\mathbf{q}) = \begin{bmatrix} L_1 \cos(q_1) + L_2 \cos(q_1+q_2) + O_1 \\ L_1 \sin(q_1) + L_2 \sin(q_1+q_2) + O_2 \end{bmatrix},$$

where L_1 and L_2 are the link lengths and O_1, O_2 are the robot base coordinates. The Jacobian matrix is

$$\mathcal{J}(\mathbf{q}) = \begin{bmatrix} -L_1 \sin(q_1) - L_2 \sin(q_1+q_2) & -L_2 \sin(q_1+q_2) \\ L_1 \cos(q_1) + L_2 \cos(q_1+q_2) & L_2 \cos(q_1+q_2) \end{bmatrix}$$

The simulations have been carried-out in the same conditions as in Astolfi et al. (2002). Hence, we set $L_1 = 0.8$m, $L_2 = 0.5$m $O_1 = -0.666$m and $O_2 = -0.333$m. Further, the orientation of the camera is set to $\theta = 1$rad and the scaling factor $a = 0.7$. This yields $\boldsymbol{\eta} = [0.7719, 1.2021]^\top$. The robot initial conditions are $\mathbf{q}(0) = 1.3[1, -1]^\top$rad.

The desired trajectory $\mathbf{y}_\star(t)$ is obtained from the first order system

$$\dot{\mathbf{y}}_\star = -\alpha_1(\mathbf{y}_\star + \mathbf{r}), \tag{24}$$

where $\mathbf{r}$ is the reference signal given by

$$\mathbf{r} = \begin{bmatrix} \alpha_2 \sin(w_r t) + \alpha_2 \sin(1.5 w_r t) + c \\ \alpha_2 \sin(w_r t + d) + \alpha_2 \sin(1.5 w_r t + d) + c \end{bmatrix},$$

with $\mathbf{y}_\star(0) = [-0.5, 0.5]^\top$, $c = 0.1$ and $d = 1$rad.

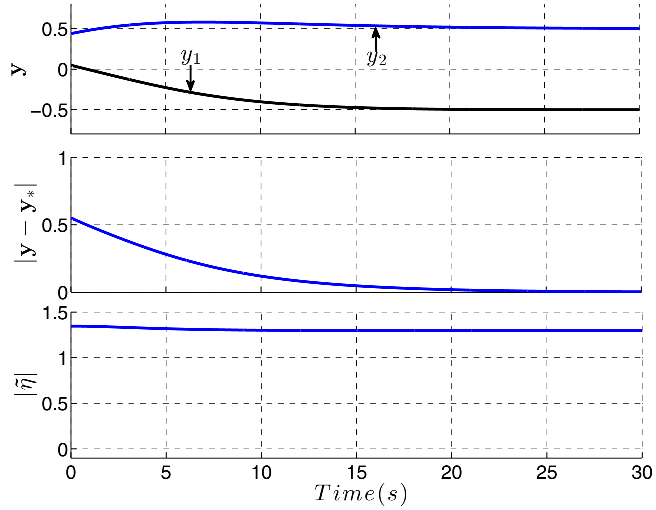

Fig. 2. Simulation results for the controller relying on excitation (16), with $\dot{\mathbf{y}}_\star = \mathbf{0}$.

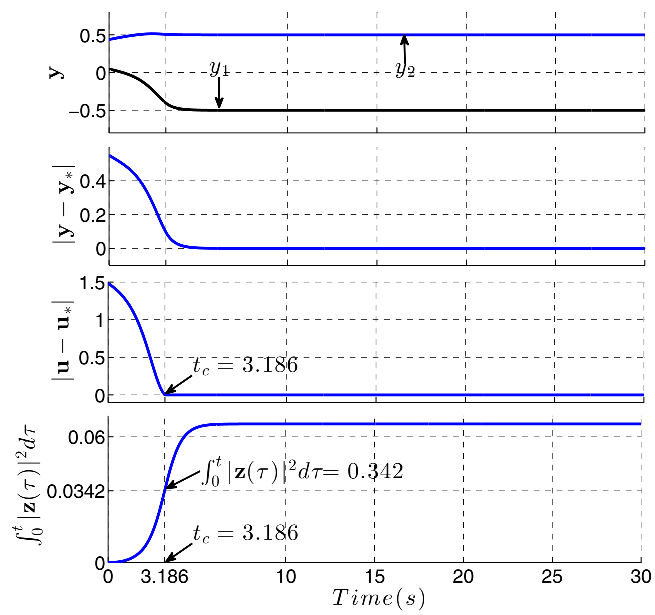

Fig. 3. Simulation results for the controller with asymptotic convergence (19) and with $\dot{\mathbf{y}}_\star = \mathbf{0}$.

It should be underscored that the immersion and invariance adaptive scheme in (Astolfi et al., 2002) has a steady state error that increases when tracking high frequency signals. Although in (Astolfi et al., 2002) the simulations have been carried out with $w_r = 0.07$rad/s and $\alpha_2 = 0.04$, to show the performance improvement of our proposal, here we set $w_r = 1$rad/s and $\alpha_2 = 0.4$. Moreover, compared to the works in (Hsu et al., 2015; Yang et al., 2016), the schemes proposed here can deal with constant desired trajectories and we can achieve asymptotic convergence of the tracking error to zero.

For the controller (16) we selected $\gamma = 1.5$ and $\lambda = 2$, with the same gains, plus $\mu = 0.05$, for the controller (19). These choices yield $-\frac{1}{\gamma}\ln(1-\mu) = 0.0342$.

The controllers have been simulated under the same conditions and for two different scenarios, namely for a

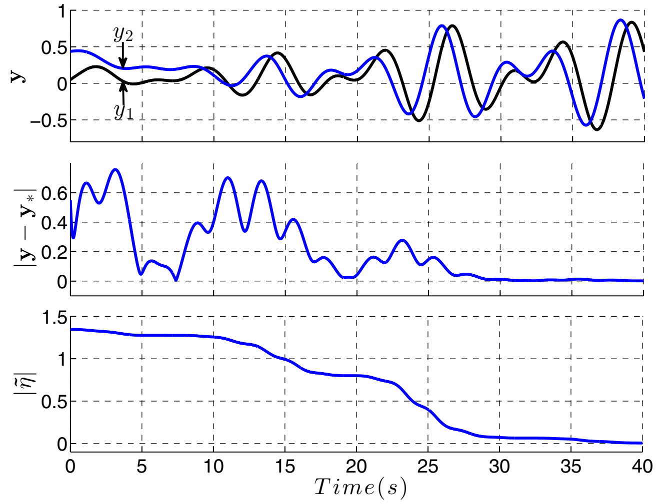

Fig. 4. Simulation results for the controller relying on excitation (16), with $\dot{\mathbf{y}}_\star \neq \mathbf{0}$ and with $w_r = 1$rad/s.

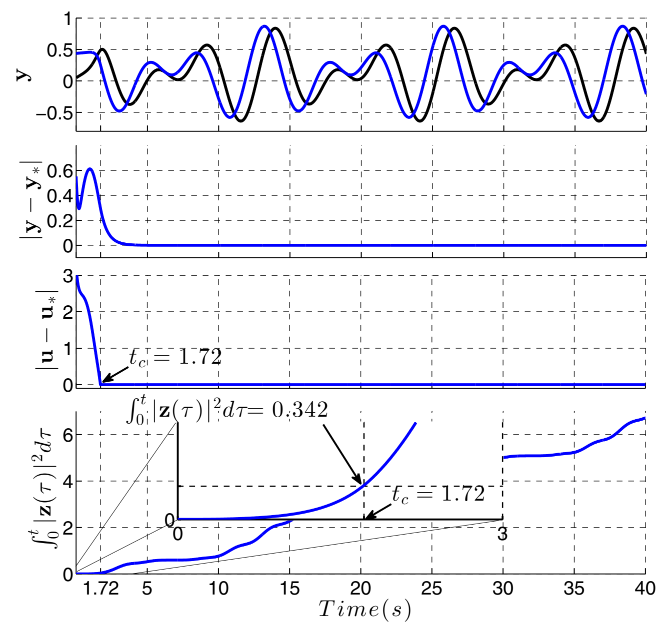

Fig. 5. Simulation results for the controller with asymptotic convergence (19), with $\dot{\mathbf{y}}_\star \neq \mathbf{0}$ and with $w_r = 1$rad/s.

constant desired trajectory, *i.e.*, $\dot{\mathbf{y}}_\star = \mathbf{0}$, and for a time-varying trajectory, *i.e.*, $\dot{\mathbf{y}}_\star$ as in (24), with $\alpha_1 > 0$.

The first set of simulations is for the constant desired trajectory case, hence $\alpha_1 = 0$ in (24). In Fig. 2 we show the behavior of the controller (16). Note that, as expected because the lack of excitation, the estimation error does not converge to zero. However, in spite of this fact, position tracking is ensured. The behavior of controller (19) is shown in Fig. 3. This scheme also ensures output regulation with a faster transient than (16). Clearly, signal $\mathbf{z}$ satisfies Assumption 1 with $t_c = 3.186$ seconds.

The second set of simulations deal with the time-varying trajectory, and hence we set $\alpha_1 = 5$ in (24). The results are illustrated in Figs. 4 and 5. In this case, the desired trajectory is rich enough to ensure $\mathbf{z}(t) \notin \mathcal{L}_2$ guaranteeing

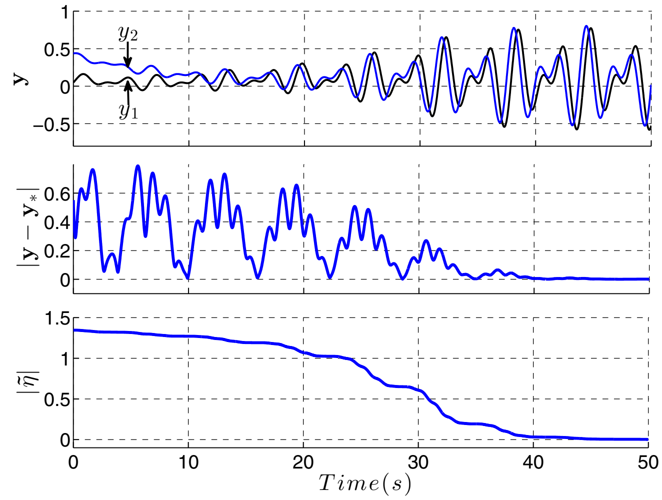

Fig. 6. Simulation results for the controller relying on excitation (16), with $\dot{\mathbf{y}}_\star \neq \mathbf{0}$ and with $w_r = 2$rad/s.

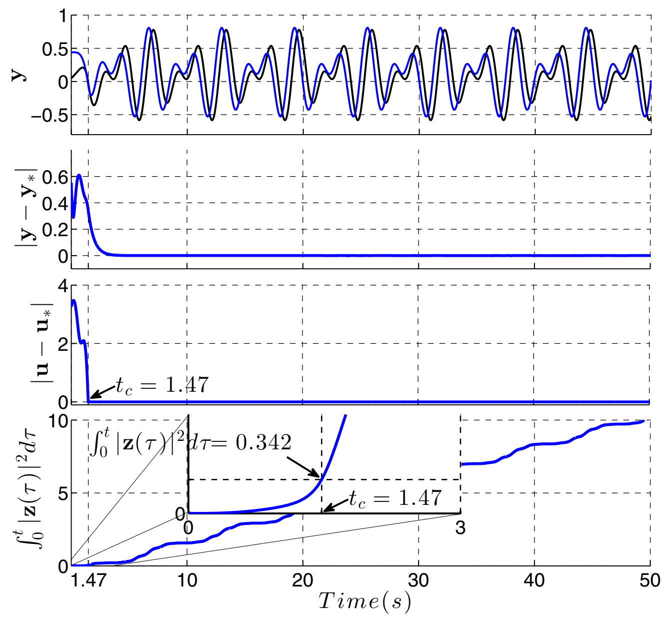

Fig. 7. Simulation results for the controller with asymptotic convergence (19), with $\dot{\mathbf{y}}_\star \neq \mathbf{0}$ and with $w_r = 2$rad/s.

that the estimation error converges to zero. Again, the performance of controller (19) is better than the performance of controller (16). Observe that the tracking error converges around 4 seconds for controller (19), while for controller (16) the convergence happens around 30 seconds. Furthermore, signal $\mathbf{z}$ satisfies Assumption 1 with $t_c = 1.72$ seconds.

The third set of simulations aims at showing that, with the same gain setting, when we increase the frequency of the desired trajectory, from $w_r = 1$rad/s to $w_r = 2$rad/s, both schemes ensure trajectory tracking but the performance of controller (16) is deteriorated. However, for controller (19), t_c decreases and it tracks the trajectory *almost* at the same time as with the previous case. These conclusions can be verified in Figs. 6 and 7.

6. CONCLUDING REMARKS AND FUTURE WORK

Using a novel parameterization of the camera action mapping, and exploiting some structural properties of it, we have developed new controllers that solve the visual servoing problem for an uncalibrated camera. The first controller is a standard certainty equivalence version of the known parameter controller, on the other hand, the second one uses in a novel way the estimates generated by the parameter estimator, which is the same for both controllers. Another difference between the controllers is that, while the first one relies on excitation to ensure its correct behavior, this requirement is obviated the second. It is shown through simulations that the second controller outperforms the other, both in regulation and tracking scenarios.

Future research involves the inclusion of the robot dynamics and the extension to 3D scenarios.

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APPENDIX

Lemma 1. (Aranovskiy et al., 2015) Consider the scalar, linear time-varying, system defined by $\dot{x} = -a^2(t)x + b(t)$, where $x \in \mathbb{R}$, $a(t)$ and $b(t)$ are piecewise continuous functions. If $a(t) \notin \mathcal{L}_2$ and $b(t) \in \mathcal{L}_1$ then $\lim_{t\to\infty} x(t) = 0$.

