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Observer design for quasi-LPV systems with unmeasurable scheduling functions using the norm-$\mathcal{L}_2$ approach.

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Abstract: This paper presents a design of a generalized dynamic observer (GDO) for polytopic quasi-LPV (Linear Parameter varying) system where the parameters depend on unmeasured state variables. It generalizes the existing results on the proportional observers and proportional integral observers. Conditions for the existence and stability of the obtained observer are given through Lyapunov approach and $\mathcal{L}_2$ gain and its design is obtained in terms of a set of linear matrix inequalities (LMI). The performance and effectiveness of the proposed observer structure are illustrated by a numerical example.

Keywords: Generalized dynamic observer, quasi-LPV systems, unmeasurable scheduling functions

1. INTRODUCTION

The study of LPV (Linear Parameter varying) systems has recently received more attention because they can represent efficiently the dynamic behavior of some nonlinear systems (Shamma and Cloutier (1993); Shamma and Athans (1991)). With this characteristic, they can cope some nonlinear problems in different research areas such as controller design (López-Estrada et al. (2016); Balas (2002); Bianchi et al. (2005)), fault tolerant control (Rodrigues et al. (2007)), fault diagnosis (Chadli et al. (2017); Ortiz-Torres et al. (2016); Theilliol and Aberkane (2011)), modeling (De Caigny et al. (2011)) among others. In the LPV theory, the varying parameters depend on exogenous signals which are commonly measured and available. When the bounds of exogenous parameters are known, the LPV system can be reformulated in a convex linear combination of linear time invariant (LTI) systems. Therefore, LTI sytem theory can be used in LPV systems. On the other hand, if the nonlinearities are determined with endogenous signals, the system is referred as quasi-LPV and represents a large class of nonlinear systems.
Nevertheless, in some practical situations, the parameters are not accessible and could be estimated as presented in López-Estrada et al. (2015). There exist only few works considering quasi-LPV systems where the parameters depend on unmeasured or unknown states of the system. This problem has been addressed through the Input-to- Stability concept as in Millerioux et al. (2004), which is based on polyquadratic Lyapunov function with a bounded error estimation assumption. Likewise, in Bergsten et al. (2001); Ichalal et al. (2010); Lendek et al. (2009) the perturbation term is considered as Lipschitz to mitigate the influence of unmeasured parameters on the estimation error. Another approach to guarantee the estimation by using the $\mathcal{L}_2$ gain which minimize the effect of the unknown parameters on the estimation error (Ichalal et al. (2016); Nagy-Kiss et al. (2015); Bezzaoucha et al. (2013)). From the previous works, we can note that the study of quasi-LPV observer with unmeasured parameters is addressed in proportional observer (PO) (Ichalal et al. (2016)), proportional integral observers (PIO) (Ichalal et al. (2009)) and adaptive observers (AO) (Nagy-Kiss et al. (2015); Nagy-Kiss and Schutz (2013); Bezzaoucha et al. (2013)) structures.
More recently, the study of new observer structure, called generalized dynamic observer (GDO) has been introduced for descriptor systems (Osorio-Gordillo et al. (2016, 2015)), linear time invariant systems (Gao et al. (2016)) and LPV systems (Pérez-Estrada et al. (2018)). These observer structures are based on (Park et al. (2002); Marquez (2003)), where the principal idea is the additional dynamics in the observer and the degrees of freedom added to the structure, with the purpose of achieving steady state accuracy and improving robustness in estimation error against disturbances and parametric uncertainties. The obtained results from (Osorio-Gordillo et al. (2016); Gao et al. (2016)) generalize the existing results on the PO, the PIO and the dynamic observer (DO) and present their performances. From these results, it ts shown that

the DO has a better performances than the PIO and PO in the presence of uncertainties in the system.
The main contribution of this paper is the GDO design for quasi-LPV systems with unmeasured scheduling functions. The scheduling functions depend on unknown states and therefore should be estimated. The $\mathcal{L}_2$ norm is introduced to minimize the unmeasured scheduling functions effect on the state estimation. The GDO parameters are used to achieve steady state accuracy and improve robustness in estimation error against disturbances and parametric uncertainties. Also the GDO structure permits to unify the design of reduced and full order observer. At the end, the efficiency of this observer is shown through a numerical example.

2. PRELIMINARIES

The following lemma will be used in the sequel of this paper, its proof is presented in Skelton et al. (1997).

Lemma 1: Let matrices $\mathcal{B}$, $\mathcal{C}$, $D = \mathcal{D}^T$ be given, then the following statements are equivalent:

(S1) There exists a matrix $\mathcal{X}$ satisfying

$$\mathcal{B}\mathcal{X}\mathcal{C} + (\mathcal{B}\mathcal{X}\mathcal{C})^T + \mathcal{D} < 0$$

(S2) The following two conditions hold

$$\mathcal{B}^{\perp}\mathcal{D}\mathcal{B}^{\perp T} < 0 \quad \text{or} \quad \mathcal{B}\mathcal{B}^T > 0$$
$$\mathcal{C}^{T\perp}\mathcal{D}\mathcal{C}^{T\perp T} < 0 \quad \text{or} \quad \mathcal{C}^T C > 0$$

Suppose that the statement (S1) holds. Let r_b and r_c be the ranks of $\mathcal{B}$ and $\mathcal{C}$, respectively, and ($\mathcal{B}_l$, $\mathcal{B}_r$) and ($\mathcal{C}_l$, $\mathcal{C}_r$) be any full rank factors of $\mathcal{B}$ and $\mathcal{C}$, i.e. $\mathcal{B} = \mathcal{B}_l\mathcal{B}_r$ and $\mathcal{C} = \mathcal{C}_l\mathcal{C}_r$.
Then the matrix $\mathcal{X}$ in statement (S1) is given by

$$\mathcal{X} = \mathcal{B}_r^+\mathcal{K}\mathcal{C}_l^+ + \mathcal{Z} - \mathcal{B}_r^+\mathcal{B}_r\mathcal{Z}\mathcal{C}_l\mathcal{C}_l^+$$

where $\mathcal{Z}$ is an arbitrary matrix and

$$\mathcal{K} = \mathcal{R}^{-1}\mathcal{B}_l^T\vartheta\mathcal{C}_r^T(\mathcal{C}_r\vartheta\mathcal{C}_r^T)^{-1} + \mathcal{S}^{1/2}\phi(\mathcal{C}_r\vartheta\mathcal{C}_r^T)^{-1/2}$$
$$\mathcal{S} = \mathcal{R}^{-1} - \mathcal{R}^{-1}\mathcal{B}_l^T[\vartheta - \vartheta\mathcal{C}_r^T(\mathcal{C}_r^T\vartheta\mathcal{C}_r^T)^{-1}\mathcal{C}_r\vartheta]\mathcal{B}_l\mathcal{R}^{-1}$$

where ϕ is an arbitrary matrix such that $||\phi|| < 1$ and $\mathcal{R}$ is an arbitrary positive definite matrix such that

$$\vartheta = (\mathcal{B}_l\mathcal{R}^{-1}\mathcal{B}_l^T - D)^{-1} > 0$$

3. PROBLEM FORMULATION

Let us consider the following quasi-LPV system

$$\dot{x}(t) = \sum_{i=1}^{r}\mu_i(x(t))(A_i x(t) + B_i u(t)) \tag{2a}$$
$$y(t) = Cx(t) \tag{2b}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ the input vector and $y(t) \in \mathbb{R}^p$ is the output vector. Matrices $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$ are constant matrices. r is the number of local models. The scheduling functions $\mu_i(x(t))$ depend on the unmeasured state $x(t)$, and they have the following convex properties:

$$\sum_{i=1}^{r}\mu_i(x(t)) = 1 \quad \text{and} \quad \mu_i(x(t)) \geq 0. \tag{3}$$

Now, let us consider the following GDO for system (2).

$$\dot{\zeta}(t) = \sum_{i=1}^{r}\mu_i(\hat{x}(t))(N_i\zeta(t) + H_i v(t) + F_i y(t) + J_i u(t)) \tag{4a}$$
$$\dot{v}(t) = \sum_{i=1}^{r}\mu_i(\hat{x}(t))(S_i\zeta(t) + L_i v(t) + M_i y(t)) \tag{4b}$$
$$\hat{x}(t) = P\zeta(t) + Qy(t) \tag{4c}$$

where $\zeta(t) \in \mathbb{R}^{q_0}$ represents the state vector of the observer, $v(t) \in \mathbb{R}^{q_1}$, is an auxiliary vector and $\hat{x}(t)$ is the estimate of $x(t)$. Matrices N_i, F_i, J_i, H_i, L_i, M_i, P and Q are unknown matrices of appropriate dimensions which must be determined such that $\hat{x}(t)$ asymptotically converges to $x(t)$. In order to facilitate the comparison between the system (2) and the GDO (4), the system can be written with scheduling functions depending on the unmeasured state vector as:

$$\dot{x}(t) = \sum_{i=1}^{r}\mu_i(\hat{x}(t))(A_i x(t) + B_i u(t)) + \omega(t) \tag{5a}$$
$$y(t) = Cx(t) \tag{5b}$$

where

$$\omega(t) = \sum_{i=1}^{r}(\mu_i(x(t)) - \mu_i(\hat{x}(t)))(A_i x(t) + B_i u(t)) \tag{6}$$

The term $\omega(t)$ can be considered as a perturbation of finite energy and its effect on the estimation error must be minimized.

***Remark 1*:**

i) The GDO (4) is in general form and generalizes the existing observers. In fact:
   - If $H_i = 0$, $S_i = 0$, $M_i = 0$ and $L_i = 0$ then the observer reduces to the PO for LPV systems.
   $$\dot{\zeta}(t) = \sum_{i=1}^{r}\mu_i(\hat{x}(t))(N_i\zeta(t) + F_i y(t) + J_i u(t))$$
   $$\hat{x}(t) = P\zeta(t) + Qy(t)$$
   - For $L_i = 0$, $S_i = -CP$ and $M_i = -CQ + I$ then the following PIO for LPV systems is obtained
   $$\dot{\zeta}(t) = \sum_{i=1}^{r}\mu_i(\hat{x}(t))(N_i\zeta(t) + H_i v(t) + F_i y(t) + J_i u(t))$$
   $$\dot{v}(t) = y(t) - C\hat{x}(t)$$
   $$\hat{x}(t) = P\zeta(t) + Qy(t)$$

ii) The GDO design unifies the reduced-order observer if $q_o = n - p$ and the full-order one if $q_o = n$.

The problem of the observer design is to guarantee the convergence of the estate estimation error toward zero when $\omega(t)$ is null and minimizing the $\mathcal{L}_2$ gain, for $\omega(t) \neq 0$, on the estimation error ($e(t) = \hat{x}(t) - x(t)$) formulated as $\|e(t)\|_2 < \gamma\|\omega(t)\|_2$. The following lemma gives existence conditions of the observer (4) under the assumption $\omega(t) = 0$.
Before of the observer design, we can give the following lemma.
Lemma 2: For $\omega(t) = 0$, there exists an observer of the form (4) for the system (5) if the following two statements hold.

1. There exists a matrix T of appropriate dimension such that the following conditions are satisfied
   (a) $N_iT + F_iC - TA_i = 0$
   (b) $J_i = TB_i$
   (c) $S_iT + M_iC = 0$
   (d) $PT + QC = I_n$
2. The system $\dot{\varphi}(t) = \sum_{i=0}^{r} \mu_i(\hat{x}(t)) \begin{bmatrix} N_i & H_i \\ S_i & L_i \end{bmatrix} \varphi(t)$ is asymptotically stable.

Proof. Let $T \in \mathbb{R}^{q_o \times n}$ be a parameter matrix and consider the transformed error $\varepsilon(t) = \zeta(t) - Tx(t)$, then its derivative is given by:

$$\dot{\varepsilon}(t) = \sum_{i=1}^{r} \mu_i(\hat{x}(t))(N_i\varepsilon(t) + (N_iT + F_iC - TA_i)x(t) + H_iv(t) + (J_i - TB_i)u(t)) \tag{7}$$

by using the definition of $\varepsilon(t)$, equations (4b) and (4c) can be written as:

$$\dot{v}(t) = \sum_{i=1}^{r} \mu_i(\hat{x}(t))(S_i\varepsilon(t) + (S_iT + M_iC)x(t) + L_iv(t)) \tag{8}$$

$$\hat{x}(t) = P\varepsilon(t) + (PT + QC)x(t) \tag{9}$$

If conditions (a)-(d) of Lemma 2 are satisfied, the following observer error dynamics is obtained from (7) and (8)

$$\underbrace{\begin{bmatrix} \dot{\varepsilon}(t) \\ \dot{v}(t) \end{bmatrix}}_{\dot{\varphi}(t)} = \sum_{i=1}^{r} \mu_i(\hat{x}(t)) \underbrace{\begin{bmatrix} N_i & H_i \\ S_i & L_i \end{bmatrix}}_{\mathbb{A}_i} \underbrace{\begin{bmatrix} \varepsilon(t) \\ v(t) \end{bmatrix}}_{\varphi(t)} \tag{10}$$

From (9), we have

$$\hat{x}(t) - x(t) = e(t) = P\varepsilon(t) \tag{11}$$

in this case if system (10) is asymptotically stable then

$$\lim_{t \to \infty} e(t) = 0. \qquad \square$$

4. GDO DESIGN

4.1 Parameterization of the observer

In this section, we shall give the parameterization of the algebraic constraint equations (a)-(d) of Lemma 2. Let $E \in \mathbb{R}^{q_0 \times n}$ be any full row rank matrix such that the matrix $\Sigma = \begin{bmatrix} E \\ C \end{bmatrix}$ is of full column rank and let $\Omega = \begin{bmatrix} I_n \\ C \end{bmatrix}$. Conditions (c) and (d) can be written as:

$$\begin{bmatrix} S_i & M_i \\ P & Q \end{bmatrix} \begin{bmatrix} T \\ C \end{bmatrix} = \begin{bmatrix} 0 \\ I_n \end{bmatrix} \tag{12}$$

the necessary and sufficient condition for equation (12) to have a solution is:

$$\text{rank} \begin{bmatrix} T \\ C \end{bmatrix} = \text{rank} \begin{bmatrix} T \\ C \\ 0 \\ I_n \end{bmatrix} = n \tag{13}$$

Now, since rank $\begin{bmatrix} T \\ C \end{bmatrix} = n$, there always exist matrices $T \in \mathbb{R}^{q_0 \times n}$ and $K \in \mathbb{R}^{q_0 \times p}$ such that $T + KC = E$, which can be written as:

$$[\, T \;\; K \,]\, \Omega = E \tag{14}$$

and since rank(Ω)=rank$\begin{bmatrix} \Omega \\ E \end{bmatrix}$. The general solution of equation (14) is given by:

$$[\, T \;\; K \,] = E\Omega^+ - Y_1(I_{n+p} - \Omega\Omega^+) \tag{15}$$

Equation (15) is equivalent to:

$$T = T_1 - Y_1T_2 \tag{16}$$

$$K = K_1 - Y_1K_2 \tag{17}$$

where $T_1 = E\Omega^+ \begin{bmatrix} I_n \\ 0 \end{bmatrix}$, $T_2 = (I_{n+p} - \Omega\Omega^+) \begin{bmatrix} I_n \\ 0 \end{bmatrix}$, $K_1 = E\Omega^+ \begin{bmatrix} 0 \\ I_p \end{bmatrix}$ and $K_2 = (I_{n+p} - \Omega\Omega^+) \begin{bmatrix} 0 \\ I_p \end{bmatrix}$.
Now, inserting the equivalence T from (14) into condition (a) it leads to:

$$N_iE + \tilde{K}_iC = TA_i \tag{18}$$

where $\tilde{K}_i = F_i - N_iK$ and equation (18) can be written as:

$$[\, N_i \;\; \tilde{K}_i \,]\, \Sigma = TA_i \tag{19}$$

The general solution of (19) is given by:

$$[\, N_i \;\; \tilde{K}_i \,] = TA_i\Sigma^+ - Z_i(I_{n+p} - \Sigma\Sigma^+) \tag{20}$$

by replacing matrix T from equation (16) into equation (20) it gives:

$$N_i = N_{1,i} - Y_1N_{2,i} - Z_iN_3 \tag{21}$$

$$\tilde{K}_i = \tilde{K}_{1,i} - Y_1\tilde{K}_{2,i} - Z_i\tilde{K}_3 \tag{22}$$

where $N_{1,i} = T_1A_i\Sigma^+ \begin{bmatrix} I_{q_0} \\ 0 \end{bmatrix}$, $N_{2,i} = T_2A_i\Sigma^+ \begin{bmatrix} I_{q_0} \\ 0 \end{bmatrix}$, $N_3 = (I_{q_0+p} - \Sigma\Sigma^+) \begin{bmatrix} I_{qo} \\ 0 \end{bmatrix}$, $\tilde{K}_{1,i} = T_1A_i\Sigma^+ \begin{bmatrix} 0 \\ I_p \end{bmatrix}$, $\tilde{K}_{2,i} = T_2A_i\Sigma^+ \begin{bmatrix} 0 \\ I_p \end{bmatrix}$, $\tilde{K}_3 = (I_{qo+p} - \Sigma\Sigma^+) \begin{bmatrix} 0 \\ I_p \end{bmatrix}$ and Z_i are arbitrary matrices of appropriate dimension. As matrices N_i, T, K, $\tilde{K}_i$ are known, we can deduce the matrix F_i as:

$$F_i = F_{1,i} - Y_1F_{2,i} - Z_iF_3 \tag{23}$$

where $F_{1,i} = T_1A_i\Sigma^+ \begin{bmatrix} K \\ I_p \end{bmatrix}$, $F_{2,i} = T_2A_i\Sigma^+ \begin{bmatrix} K \\ I_p \end{bmatrix}$, $F_3 = (I_{n+p} - \Sigma\Sigma^+) \begin{bmatrix} K \\ I_p \end{bmatrix}$.
On the other hand from equation (14) we obtain:

$$\begin{bmatrix} T \\ C \end{bmatrix} = \begin{bmatrix} I_{qo} & -K \\ 0 & I_p \end{bmatrix} \Sigma \tag{24}$$

inserting equation (24) into the equation (12) we get:

$$\begin{bmatrix} S_i & M_i \\ P & Q \end{bmatrix} \begin{bmatrix} I_{qo} & -K \\ 0 & I_p \end{bmatrix} \Sigma = \begin{bmatrix} 0 \\ I_n \end{bmatrix} \tag{25}$$

Since matrix Σ is of full column rank and

$$\begin{bmatrix} I_n & -K \\ 0 & I_{n_y} \end{bmatrix}^{-1} = \begin{bmatrix} I_{qo} & K \\ 0 & I_p \end{bmatrix}$$

the general solution to equation (25) is given by:

$$\begin{bmatrix} S_i & M_i \\ P & Q \end{bmatrix} = \left(\begin{bmatrix} 0 \\ I_n \end{bmatrix} \Sigma^+ - \begin{bmatrix} U_{1,i} \\ U_2 \end{bmatrix} (I_{qo+p} - \Sigma\Sigma^+) \right) \times \begin{bmatrix} I_{qo} & K \\ 0 & I_p \end{bmatrix} \tag{26}$$

where $U_{1,i}$ and U_2 are arbitrary matrices of appropriate dimensions.
Then matrices S_i, M_i, P and Q can be determined as:

$$S_i = -U_{1,i}N_3 \tag{27}$$

$$M_i = -U_{1,i}F_3 \tag{28}$$

$$P = \Sigma^+ \begin{bmatrix} I_{qo} \\ 0 \end{bmatrix} \tag{29}$$

$$Q = \Sigma^+ \begin{bmatrix} K \\ I_p \end{bmatrix} \tag{30}$$

where for the sake of simplicity $U_2 = 0$. Now, by using (21) and (27) the observer error dynamics (10) can be rewritten as:

$$\dot{\varphi}(t) = \sum_{i=1}^{r} \mu_i(x(t))((\mathbb{A}_i - \mathbb{Y}_i\mathbb{A}_2)\varphi(t)) \tag{31a}$$

$$e(t) = \mathbb{P}\varphi(t) \tag{31b}$$

where $\mathbb{A}_i = \begin{bmatrix} N_{1,i} - Y_1N_{2,i} & 0 \\ 0 & 0 \end{bmatrix}$, $\mathbb{A}_2 = \begin{bmatrix} N_3 & 0 \\ 0 & -I_{qo} \end{bmatrix}$, $\mathbb{Y}_i = \begin{bmatrix} Z_i & H_i \\ U_{1,i} & L_i \end{bmatrix}$ and $\mathbb{P} = [\, P \;\; 0 \,]$.

4.2 GDO stability

This section shows the results to design the GDO observer considering the LPV system (5) with $\omega(t) \neq 0$. From the Lemma 2, the dynamic of error $\varepsilon(t)$ becomes

$$\dot{\varepsilon}(t) = \sum_{i=1}^{r} \mu_i(\hat{x}(t))(N_i\varepsilon(t) + H_iv(t) - T\omega(t)) \tag{32}$$

the dynamic of variable $v(t)$

$$\dot{v}(t) = \sum_{i=1}^{r} \mu_i(\hat{x}(t))(S_i\varepsilon(t) + L_iv(t)) \tag{33}$$

and the estimation error $e(t)$ equals to

$$e(t) = P\varepsilon(t). \tag{34}$$

With the previous equations, we obtain the following system:

$$\dot{\varphi}(t) = \sum_{i=1}^{r} \mu_i(\hat{x}(t))((\mathbb{A}_i - \mathbb{Y}_i\mathbb{A}_2)\varphi(t) + \Gamma\omega(t)) \tag{35a}$$

$$e(t) = \mathbb{P}\varphi(t) \tag{35b}$$

where $\Gamma = \begin{bmatrix} -T^T & 0 \end{bmatrix}^T$ and $\mathbb{P} = [\, P \;\; 0 \,]$. Matrices $\mathbb{A}_i$, $\mathbb{A}_2$ and $\mathbb{Y}_i$ are defined in (31a).
Consequently, the observer design is based to the analysis of the system (35) to determine the matrices $\mathbb{Y}_i$ through $\mathcal{L}_2$ gain condition. The following theorem gives the existence conditions of the GDO observer using lemma 1.

Theorem 1: System (35) is asymptotically stable and $\|e(t)\|_2 < \gamma\|\omega(t)\|_2$ if there exists the parameter matrices $\mathbb{Y}_i$, an attenuation level $\gamma > 0$ and symmetric positive definite matrices X_1 and X_2 such that the following LMIs are satisfied.

$$X_2 > X_1 \tag{36}$$

$$\begin{bmatrix} \Pi_i & -N_3^{T\perp}X_1T_1 + N_3^{T\perp}WT_2 \\ (*) & -\gamma^2 I_n \end{bmatrix} < 0 \tag{37}$$

where

$$\Pi_i = N_3^{T\perp}(N_{1,i}^TX_1 - N_{2,i}^TW^T + X_1N_{1,i} - WN_{2,1} + P^TP)N_3^{T\perp T} \tag{38}$$

In this case, matrix $Y_1 = X_1^{-1}W$ and the matrices $\mathbb{Y}_i$ are parameterized as

$$\mathbb{Y}_i = X^{-1}(\mathcal{B}_r^+\mathcal{K}_i\mathcal{C}_l^+ + \mathcal{Z} - \mathcal{B}_r^+\mathcal{B}_r\mathcal{Z}\mathcal{C}_l\mathcal{C}_l^+) \tag{39}$$

with

$$\mathcal{K}_i = \mathcal{R}^{-1}\mathcal{B}_l^T\vartheta_i\mathcal{C}_r^T(\mathcal{C}_r\vartheta_i\mathcal{C}_r^T)^{-1} + \mathcal{S}_i^{1/2}\phi(\mathcal{C}_r\vartheta_i\mathcal{C}_r^T)^{-1/2} \tag{40}$$

$$\mathcal{S}_i = \mathcal{R}^{-1} - \mathcal{R}^{-1}\mathcal{B}_l^T[\vartheta_i - \vartheta_i\mathcal{C}_r^T(\mathcal{C}_r^T\vartheta_i\mathcal{C}_r^T)^{-1}\mathcal{C}_r\vartheta_i]\mathcal{B}_l\mathcal{R}^{-1} \tag{41}$$

$$\vartheta_i = (\mathcal{B}_l\mathcal{R}^{-1}\mathcal{B}_l^T - \mathcal{D}_i)^{-1} > 0 \tag{42}$$

where

$$\mathcal{D}_i = \begin{bmatrix} \mathbb{A}_i^TX + X\mathbb{A}_i + \mathbb{P}^T\mathbb{P} & X\Gamma \\ (*) & -\gamma^2 I_n \end{bmatrix}$$

$\mathcal{B} = \begin{bmatrix} -I \\ 0 \end{bmatrix}$, $\mathcal{C} = [\, \mathbb{A}_2 \;\; 0 \,]$ and $X = \begin{bmatrix} X_1 & X_1 \\ X_1 & X_2 \end{bmatrix}$. ϕ is an arbitrary matrix such that $||\phi|| < 1$ and $\mathcal{R} > 0$. Matrices $\mathcal{C}_l$, $\mathcal{C}_r$, $\mathcal{B}_l$ and $\mathcal{B}_r$ are any full rank matrices such that $\mathcal{C} = \mathcal{C}_l\mathcal{C}_r$ and $\mathcal{B} = \mathcal{B}_l\mathcal{B}_r$.

Proof. Consider the following Lyapunov function candidate

$$V(\varphi(t)) = \varphi(t)^TX\varphi(t) > 0 \tag{43}$$

with $X = \begin{bmatrix} X_1 & X_1 \\ X_1 & X_2 \end{bmatrix} > 0$ and by the Schur complement $X_2 > X_1$. Its derivative along the trajectory of (35) is given by

$$\dot{V}(\varphi(t)) = \sum_{i=1}^{r} \mu_i(\hat{x}(t))(\varphi(t)^T((\mathbb{A}_i - \mathbb{Y}_i\mathbb{A}_2)^TX + X(\mathbb{A}_i - \mathbb{Y}_i\mathbb{A}_2))\varphi(t) + \omega(t)^T\Gamma^TX\varphi(t) + \varphi(t)^TX\Gamma\omega(t)) < 0 \tag{44}$$

Then, we need to ensure asymptotic convergence of $e(t)$ toward zero if $\omega(t) = 0$ and simultaneously guaranteeing a bounded $\mathcal{L}_2$ if $\omega(t) \neq 0$. This problem is reduced to

$$\dot{V}(\varphi(t)) + e(t)^Te(t) - \gamma^2\omega(t)^T\omega(t) < 0 \tag{45}$$

After some calculation by using the convex sum property of the scheduling functions, equation (45) is satisfied if the following LMI hold

$$\begin{bmatrix} (\mathbb{A}_i - \mathbb{Y}_i\mathbb{A}_2)^TX + X(\mathbb{A}_i - \mathbb{Y}_i\mathbb{A}_2) + \mathbb{P}^T\mathbb{P} & X\Gamma \\ (*) & -\gamma^2 I \end{bmatrix} < 0 \tag{46}$$

which can be written as

$$\mathcal{B}\mathcal{X}_i\mathcal{C} + (\mathcal{B}\mathcal{X}_i\mathcal{C})^T + \mathcal{D}_i < 0 \tag{47}$$

where $\mathcal{B} = \begin{bmatrix} -I \\ 0 \end{bmatrix}$, $\mathcal{C} = [\, \mathbb{A}_2 \;\; 0 \,]$, $\mathcal{X}_i = X\mathbb{Y}_i$ and $\mathcal{D}_i$ is defined in Theorem 1. According to the solvability conditions of Lemma 1, the equation (47) is reduced to:

$$\mathcal{C}^{T\perp}\mathcal{D}_i\mathcal{C}^{T\perp T} < 0 \tag{48}$$

with $C^{T\perp} = \begin{bmatrix} [\, N_3^{T\perp} \;\; 0 \,] & 0 \\ 0 & I_{q_0+q_1} \end{bmatrix}$. By using the definition of $\mathcal{D}_i$ and W, the inequality (48) becomes (37). If condition (37) is satisfied, the matrix $\mathbb{Y}_i$ is obtained as (39)-(42). □

5. NUMERICAL EXAMPLE

In order to illustrate our results, let us consider a numerical example described by the quasi-LPV system (2) with the matrices

$$A(x_1(t)) = \begin{bmatrix} -2x_1(t) & 2 & 1 \\ 0 & -3 & 2 \\ 1 & 0 & -x_1(t) \end{bmatrix},\ B(x_1(t)) = \begin{bmatrix} 0 \\ 2x_1(t) \\ 2 \end{bmatrix}$$

and $C = [\,0\ 1\ 0\,]$, where $x_1(t) \in [1.3, 5.2]$. Such that the scheduling functions $\mu_i(x_1(t))$ are

$$\mu_1(x_1(t)) = \frac{5.2 - x_1(t)}{3.9}$$

$$\mu_2(x_1(t)) = \frac{x_1(t) - 1.3}{3.9}$$

We obtain the observer gains by solving the LMI's of Theorem 1 and choosing the matrix $E = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 2 & 2 \end{bmatrix}$, $\phi = 0.1 \times \text{ones}_{6,4}$ $\mathcal{Z} = 0$ and $\mathcal{R} = I_6 \times 0.01$. The observer gains are not showed due to lack of space. The attenuation level obtained by solving Theorem 1 is $\gamma = 0.3001$ which guarantee a good attenuation of $\omega(t)$. The initial condition for the system are $x(0) = [45, 80]^T$ for the GDO are $\zeta(0)_{GDO} = [0.4, 1.85, -1.1]^T$, $v(0)_{GDO} = [1, 1, 1]^T$. To evaluate the performance of the observers an uncertainty $\Delta A(t)$ is added in the system matrix A_i, then we obtain the following matrix $A_i + \Delta A(t)$ where

$$\Delta A(t) = \delta(t) \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0.2 & 0 & 0.5 \end{bmatrix}.$$

The results of the simulation are depicted in Fig. 1-6. Fig. 1 shows the input variable $u(t)$. Fig. 2 shows the uncertainty factor $\delta(t)$. Fig. 3-5 show the system states an their estimations by GDO. Fig. 6 shows the variation of the scheduling functions depending on $\hat{x}(t)$.

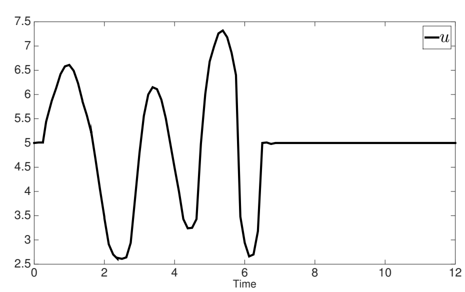

Fig. 1. Input variable $u(t)$

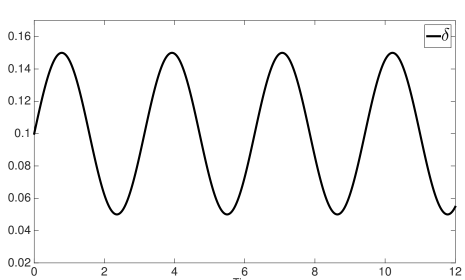

Fig. 2. Uncertainty factor $\delta(t)$

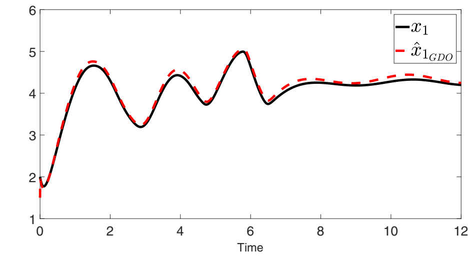

Fig. 3. Estimation of x_1.

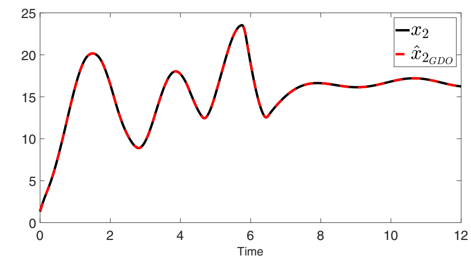

Fig. 4. Estimation of x_2.

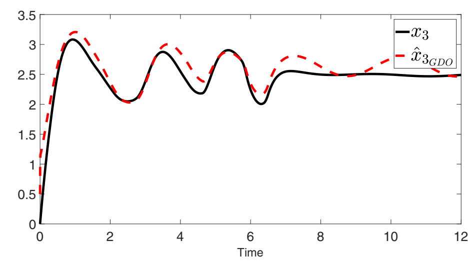

Fig. 5. Estimation of x_3.

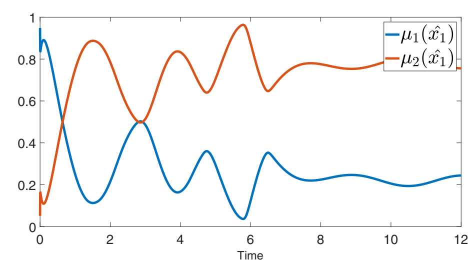

Fig. 6. Scheduling functions.

6. CONCLUSION

In this paper a new approach for the estimation of the state variables for quasi-LPV systems with unmeasured scheduled functions is proposed. The conditions for the existence of the GDO are provided and its stability is proved. The design is given in terms of LMI's. This observer constitutes a generalization of the existing POs and PIOs, which are only particular cases. A numerical example is presented to show the performances and robustness to parametric uncertainties and unknown scheduling functions.

REFERENCES

Balas, G.J. (2002). Linear, parameter-varying control and its application to a turbofan engine. *International Journal of Robust and Nonlinear Control*, 12(9), 763–796. doi:10.1002/rnc.704.

Bergsten, P., Palm, R., and Driankov, D. (2001). Fuzzy observers. *10th IEEE International Conference on*

Fuzzy Systems. (Cat. No.01CH37297), 2(June 2017), 700–703. doi:10.1109/FUZZ.2001.1009051.

Bezzaoucha, S., Marx, B., Maquin, D., and Ragot, J. (2013). Nonlinear joint state and parameter estimation: Application to a wastewater treatment plant. *Control Engineering Practice*, 21(10), 1377–1385. doi:10.1016/j.conengprac.2013.06.009.

Bianchi, F.D., Mantz, R.J., and Christiansen, C.F. (2005). Gain scheduling control of variable-speed wind energy conversion systems using quasi-LPV models. *Control Engineering Practice*, 13(2), 247–255. doi:10.1016/j.conengprac.2004.03.006.

Chadli, M., Davoodi, M., and Meskin, N. (2017). Distributed state estimation, fault detection and isolation filter design for heterogeneous multi-agent linear parameter-varying systems. *IET Control Theory & Applications*, 11(2), 254–262. doi:10.1049/iet-cta.2016.0912.

De Caigny, J., Camino, J.F., and Swevers, J. (2011). Interpolation-based modeling of MIMO LPV systems. *IEEE Transactions on Control Systems Technology*, 19(1), 46–63. doi:10.1109/TCST.2010.2078509.

Gao, N., Darouach, M., Voos, H., and Alma, M. (2016). New unified H_∞ dynamic observer design for linear systems with unknown inputs. *Automatica*, 65, 43–52. doi:10.1016/j.automatica.2015.10.052.

Ichalal, D., Mammar, S., and Ragot, J. (2016). Auxiliary dynamics for observer design of nonlinear TS systems with unmeasurable premise variables. *IFAC-PapersOnLine*, 49(5), 1–6. doi:10.1016/j.ifacol.2016.07.080.

Ichalal, D., Marx, B., Ragot, J., and Maquin, D. (2010). State estimation of TakagiSugeno systems with unmeasurable premise variables. *IET Control Theory & Applications*, 4(5), 897. doi:10.1049/iet-cta.2009.0054.

Ichalal, D., Marx, B., Ragot, J., and Maquin, D. (2009). Simultaneous state and unknown inputs estimation with PI and PMI observers for Takagi Sugeno model with unmeasurable premise variables. *2009 17th Mediterranean Conference on Control and Automation*, 353–358. doi:10.1109/MED.2009.5164566.

Lendek, Z., Babu??ka, R., and Schutter, B.D. (2009). Stability of cascaded fuzzy systems and observers. *IEEE Transactions on Fuzzy Systems*, 17(3), 641–653. doi:10.1109/TFUZZ.2008.924353.

López-Estrada, F.R., Ponsart, J.C., Astorga-Zaragoza, C.M., Camas-Anzueto, J.L., and Theilliol, D. (2015). Robust sensor fault estimation for descriptor-LPV systems with unmeasurable gain scheduling functions: Application to an anaerobic bioreactor. *International Journal of Applied Mathematics and Computer Science*, 25(2), 233–244. doi:10.1515/amcs-2015-0018.

López-Estrada, F.R., Ponsart, J.C., Theilliol, D., Zhang, Y., and Astorga-Zaragoza, C.M. (2016). LPV Model-Based Tracking Control and Robust Sensor Fault Diagnosis for a Quadrotor UAV. *Journal of Intelligent & Robotic Systems*, 84(1-4), 163–177. doi:10.1007/s10846-015-0295-y.

Marquez, H.J. (2003). A frequency domain approach to state estimation. *Journal of the Franklin Institute*, 340(2), 147–157. doi:10.1016/S0016-0032(03)00017-6.

Millerioux, G., Rosier, L., Bloch, G., and Daafouz, J. (2004). Bounded State Reconstruction Error for LPV Systems With Estimated Parameters. *IEEE Transactions on Automatic Control*, 49(8), 1385–1389. doi:10.1109/TAC.2004.832669.

Nagy-Kiss, A.M. and Schutz, G. (2013). Estimation and diagnosis using multi-models with application to a wastewater treatment plant. *Journal of Process Control*, 23(10), 1528–1544. doi:10.1016/j.jprocont.2013.09.027.

Nagy-Kiss, A.M., Schutz, G., and Ragot, J. (2015). Parameter estimation for uncertain systems based on fault diagnosis using Takagi-Sugeno model. *ISA Transactions*, 56, 65–74. doi:10.1016/j.isatra.2014.11.022.

Ortiz-Torres, G., López-Estrada, F.R., Reyes-Reyes, J., García-Beltrán, C.D., and Theilliol, D. (2016). An Actuator Fault Detection and Isolation method design for Planar Vertical Take-off and Landing Unmanned Aerial Vehicle modelled as a qLPV system. *4th IFAC Conference on Intelligent Control and Automation SciencesICONS 2016*, 49(5), 272–277. doi:10.1016/j.ifacol.2016.07.125.

Osorio-Gordillo, G.L., Darouach, M., Astorga-Zaragoza, C.M., and Boutat-Baddas, L. (2016). New dynamical observers design for linear descriptor systems. *IET Control Theory & Applications*, 10(17), 2223–2232. doi:10.1049/iet-cta.2016.0431.

Osorio-Gordillo, G.L., Darouach, M., and Astorga-Zaragoza, C.M. (2015). H_∞ dynamical observers design for linear descriptor systems. Application to state and unknown input estimation. *European Journal of Control*, 26, 35–43. doi:10.1016/j.ejcon.2015.07.001.

Park, J.K., Shin, D.R., and Chung, T.M. (2002). Dynamic observers for linear time-invariant systems. *Automatica*, 38(6), 1083–1087. doi:10.1016/S0005-1098(01)00293-X.

Pérez-Estrada, A.J., Osorio-Gordillo, G.L., Darouach, M., and Olivares-Peregrino, V.H. (2018). Generalized dynamic observer design for quasi-LPV systems. *at - Automatisierungstechnik*, 66(3), 225–233. doi:10.1515/auto-2017-0060.

Rodrigues, M., Theilliol, D., Aberkane, S., and Sauter, D. (2007). Fault Tolerant Control Design For Polytopic LPV Systems. *International Journal of Applied Mathematics and Computer Science*, 17(1), 27–37. doi:10.2478/v10006-007-0004-5.

Shamma, J.S. and Athans, M. (1991). Guaranteed properties of gain scheduled control for linear parameter-varying plants. *Automatica*, 27(3), 559–564. doi:10.1016/0005-1098(91)90116-J.

Shamma, J.S. and Cloutier, J.R. (1993). Gain-scheduled missile autopilot design using linear parameter varying transformations. *Journal of Guidance, Control, and Dynamics*, 16(2), 256–263. doi:10.2514/3.20997.

Skelton, R.E., Iwasaki, T., and Grigoriadis, D.E. (1997). *A Unified Algebraic Approach To Control Design*. Series in Systems and Control. Taylor & Francis.

Theilliol, D. and Aberkane, S. (2011). *Design of LPV observers with unmeasurable gain scheduling variable under sensor faults*, volume 18. IFAC. doi:10.3182/20110828-6-IT-1002.03039.

