Memorias del Congreso Nacional de Control Automático
ISSN: 2594-2492

Finite-Time Leaderless Consensus of Euler-Lagrange Agents without Velocity Measurements: the Bounded Input Case ⋆

Emmanuel Cruz-Zavala ∗, **Emmanuel Nuño** ∗, **Jaime A. Moreno** ∗∗

∗ *Departamento de Ciencias Computacionales, CUCEI, Universidad de Guadalajara. Guadalajara, Jalisco, Mexico.*
∗∗ *Instituto de Ingeniería, Universidad Nacional Autónoma de México (UNAM), Ciudad de Mexico, Mexico.*

Abstract: One of the basic synchronization strategies of multiple agents is the leaderless consensus where, independently of the initial conditions, the agents agree at a certain constant coordinate. In this paper, we propose a novel controller that is capable of solving the leaderless consensus problem in finite-time in networks of fully-actuated Euler-Lagrange (EL) systems without employing velocity measurements and with bounded inputs. The controller is another EL-system with its own dissipation of energy that is back-propagated to the plant and the plant-controller interconnection is the force of a nonlinear spring.

Keywords: Finite-Time, Consensus, Robots.

1. INTRODUCTION

When controlling a network of multiple agents, different *coordinated* behaviors can be achieved, namely: synchronization, flocking, formation control and consensus (Cao and Ren, 2011; Hatanaka et al., 2015). Being consensus the basic behavior for which all the agents state agree on a common constant value using a decentralized interaction among these agents (Ren, 2008, 2009; Chung and Slotine, 2009; Nuño et al., 2011). In the leaderless consensus problem there exists a common coordinate value where all agents agree. Since the fundamental works (Jadbabaie et al., 2003; Olfati-Saber and Murray, 2004), motivated by the several practical applications in engineering, the study of consensus and synchronization of multiple agents has increased in the recent years (Abdessameud et al., 2017; Nuño and Ortega, 2018; Nuño, 2017; Klotz et al., 2018).

In this paper, we consider networks composed of N *fully-actuated and conservative* Euler-Lagrange (EL) agents, with n-Degrees-of-Freedom (DoF). Each ith-agent is described by

$$\frac{d}{dt}\nabla_{\dot{\mathbf{q}}_i}\mathcal{L}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i) - \nabla_{\mathbf{q}_i}\mathcal{L}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i) = \boldsymbol{\tau}_i, \qquad (1)$$

where $\mathcal{L}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)$ is the Lagrangian function that is defined as $\mathcal{L}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i) := \mathcal{K}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i) - \mathcal{U}_i(\mathbf{q}_i)$, with $\mathcal{K}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i) := \frac{1}{2}\dot{\mathbf{q}}_i^\top \mathbf{M}_i(\mathbf{q}_i)\dot{\mathbf{q}}_i$ the kinetic energy and $\mathcal{U}_i(\mathbf{q}_i)$ the potential energy. $\mathbf{q}_i, \dot{\mathbf{q}}_i \in \mathbb{R}^n$ are the generalized position and velocity, respectively, $\mathbf{M}_i(\mathbf{q}_i) \in \mathbb{R}^{n\times n}$ is the generalized inertia matrix, which is symmetric positive definite, and $\boldsymbol{\tau}_i \in \mathbb{R}^n$ is the control input.

Most of the previous works on the consensus of EL-systems require velocities to be measurable. However, many of the commercially available devices are not equipped with velocity sensors and those with velocity sensors are often prone to noise and additional velocity filters should be incorporated (Arteaga and Kelly, 2004). There are few proposed schemes that do not rely on velocity measurements, e.g., (Ren, 2010; Abdessameud and Tayebi, 2013) and, more recently, (Nuño and Ortega, 2018) and (Nuño, 2017). However, all of them can only ensure that the leaderless consensus control objective can be ensured when time tends to infinity, i.e., asymptotically or exponentially.

Compared to asymptotic or exponential, Finite Time (FT) convergence has better appealing features such as better robustness and disturbance rejection properties, faster transients and higher-precision performance are guaranteed when the uncertainty (or disturbance) bounds are available (Venkataraman and Gulati, 1993; Bhat and Bernstein, 2000; Orlov, 2005; Galicki, 2015). Different FT-control strategies for multi agent first order systems can be found in (Cortés, 2006; Davila and Pisano, 2016). The FT-control of multiple second order systems is studied in (Ge et al., 2016; Liu et al., 2017). Using different, continuous and discontinuous, control techniques all these works have established their results assuming that velocities are available. A notable exception is (Zhao et al., 2015), which does not rely on such requirement and, instead, it proposes to use a velocity observer to solve the leader-follower consensus problem in FT. Nevertheless, in order to set the observer gains, the work of Zhao et al. (2015) relies on the *a priori* knowledge of the largest *in-degree* of the interconnection graph and of the bound of the velocities. Hence, such result is not *distributed* and assumes that velocities are bounded and such bound is known.

⋆ This work has been partially supported by the Mexican CONACyT grant CB-282807.

Here we are interested in designing a controller that can ensure consensus of multiple EL-agents in *finite-time*, when there are *input constraints* and when *velocities are not available* for measurement. In this scenario we consider that only position measurements are available and that the multi-agent network interconnection can be modeled using a *connected* and *undirected* graph. In particular, we propose a controller that solves the following problem.

(LC) Leaderless Consensus Problem. The network has to reach a consensus position in finite-time. That is, *there exists* a constant $\mathbf{q}_\star \in \mathbb{R}^n$ such that, for all $i \in \bar{N} := \{1, ..., N\}$,

$$\lim_{t\to T_i(\mathbf{q}_i(0),\dot{\mathbf{q}}_i(0))} \mathbf{q}_i(t) = \mathbf{q}_\star, \qquad \lim_{t\to T(\mathbf{q}(0),\dot{\mathbf{q}}(0))} |\dot{\mathbf{q}}_i(t)| = 0, \tag{2}$$

where $T_i(\mathbf{q}_i(0), \dot{\mathbf{q}}_i(0)) < \infty$ is the settling-time. ◁

2. PRELIMINARIES

2.1 Notation

Throughout the paper, the following *notation* is employed. $\mathbb{R} := (-\infty, \infty)$, $\mathbb{R}_{>0} := (0, \infty)$, $\mathbb{R}_{\geq 0} := [0, \infty)$, $\mathbb{N} := \{1, 2, 3, \dots\}$ and $\bar{n} := \{1, 2, ..., n\}$ for $n \in \mathbb{N}$. $\mathbf{I}_n \in \mathbb{R}^{n\times n}$ denotes the $n \times n$ identity matrix and $\mathbf{1}_n \in \mathbb{R}^n$ defines the vector of n elements equals to one. For all $x \in \mathbb{R}$, $|x|$ is its absolute value. For $\mathbf{x} \in \mathbb{R}^m$, for any $m \in \mathbb{N}$, $\|\mathbf{x}\|$ stands for its Euclidean norm. For any $\delta \in \mathbb{R}_{>0}$, $B_\delta := \{\mathbf{x} \in \mathbb{R}^m : \|\mathbf{x}\| < \delta\}$ and $S_\delta^{m-1} := \{\mathbf{x} \in \mathbb{R}^m : \|\mathbf{x}\| = \delta\}$ are an open ball and an $m-1$ sphere, centered at the origin with radius δ, respectively. A function $\mathbf{f}(t) : \mathbb{R}_{\geq 0} \mapsto \mathbb{R}^m$ is said to be of class $\mathcal{C}^k$, for $k \in \mathbb{N}$, if its derivatives $\dot{\mathbf{f}}, \ddot{\mathbf{f}}, ..., \mathbf{f}^{(k)}$ exist and are continuous. For any $\mathbf{x} \in \mathbb{R}^m$, $\nabla_\mathbf{x} := [\partial_{x_1}, \dots, \partial_{x_m}]^\top$ stands for the gradient operator of a scalar function and $\nabla^2_\mathbf{x} := [\partial_{x_i}\partial_{x_j}]$ is the Hessian operator where $\partial_{x_i} := \frac{\partial}{\partial x_i}$ and $i, j \in \bar{m}$. $\lambda_m\{\mathbf{A}\}$ and $\lambda_M\{\mathbf{A}\}$ are the minimum and the maximum eigenvalues of the symmetric matrix $\mathbf{A} \in \mathbb{R}^{m\times m}$.

For any $x \in \mathbb{R}$ and any $p > 0$, we define the signed power function $\lceil x\rfloor^p : \mathbb{R} \mapsto \mathbb{R}$ as a strictly increasing odd (continuous) function given by $\lceil x\rfloor^p := |x|^p \mathrm{sign}(x)$, where $\mathrm{sign}(x)$ is the standard *sign* function and it has the following properties:

P1: For each $p \in [0, 1)$, $\lceil x\rfloor^p$ is differentiable for all $x \neq 0$, $\lceil x\rfloor^p \in \mathcal{C}^1$ if $p \in [1, 2]$ and $\lceil x\rfloor^p \in \mathcal{C}^2$ if $p \in (2, \infty)$. ◁

P2: For each $p \in (0, 1]$, there exists $\delta \in \mathbb{R}_{>0}$ such that

$$|x|^{p+1} \geq \begin{cases} \delta^{p-1}|x|^2 & \text{if } |x| < \delta, \\ \delta^p |x| & \text{if } |x| \geq \delta. \end{cases} \tag{3}$$

◁

A (p, δ)-saturation function $\mathrm{sat}_\delta(\lceil x\rfloor^p) : \mathbb{R} \mapsto \mathbb{R}$, $p, \delta \in \mathbb{R}_{>0}$, is a strictly increasing odd function defined by

$$\mathrm{sat}_\delta(\lceil x\rfloor^p) := \begin{cases} \lceil x\rfloor^p & \text{if } |x| < \delta, \\ \delta^p \mathrm{sign}(x) & \text{if } |x| \geq \delta. \end{cases} \tag{4}$$

The saturation function satisfies the following property:

P3: For all $x \in \mathbb{R}$ and $p, \delta \in \mathbb{R}_{>0}$, $\lceil \mathrm{sat}_\delta(x)\rfloor^p = \mathrm{sat}_\delta(\lceil x\rfloor^p)$ and $\int_0^x \mathrm{sat}_\delta(\lceil z\rfloor^p)dz = s(x) \in \mathcal{C}^1$, where

$$s(x) := \begin{cases} \dfrac{1}{p+1}|x|^{p+1} & \text{if } |x| < \delta, \\ \delta^p|x| - \dfrac{p}{p+1}\delta^{p+1} & \text{if } |x| \geq \delta. \end{cases} \tag{5}$$

Note that $\delta^p|x| - \frac{p}{p+1}\delta^{p+1} \geq \frac{1}{p+1}\delta^p|x|$ for all $|x| \geq \delta$. ◁

When the signed power and (p, δ)-saturation functions are vector-valued, we consider them to be applied element-wise, i.e., for any $\mathbf{x} \in \mathbb{R}^m$ we have, respectively, $\lceil \mathbf{x}\rfloor^p := [\lceil x_1\rfloor^p, ..., \lceil x_m\rfloor^p]^\top$ and

$$\mathrm{sat}_{\boldsymbol{\delta}}(\lceil \mathbf{x}\rfloor^p) := [\mathrm{sat}_{\delta_1}(\lceil x_1\rfloor^p), ..., \mathrm{sat}_{\delta_m}(\lceil x_m\rfloor^p)]^\top.$$

Let us now define the following scalar function

$$\mathcal{S}_i(\mathbf{y}_i, p_\mathcal{U}, \mathbf{K}_{pi}, \boldsymbol{\delta}_i) := \sum_{k\in\bar{n}} \mathcal{S}_{ik}(y_{ik}, p_\mathcal{U}, k_{pik}, \delta_{ik}), \tag{6}$$

where $\mathbf{y}_i \in \mathbb{R}^n$, $\delta_{ik} > 0$, $p_\mathcal{U} \in (0, 1]$, $k_{pik} > 0$ and $\mathbf{K}_{pi} := \mathrm{diag}\{k_{pik}\} \in \mathbb{R}^{n\times n}$. Further,

$$\mathcal{S}_{ik} := \begin{cases} \dfrac{k_{pik}}{p_\mathcal{U}+1}|y_{ik}|^{p_\mathcal{U}+1} & \text{if } |y_{ik}| < \delta_{ik}, \\ k_{pik}\delta_{ik}^{p_\mathcal{U}}\left(|y_{ik}| - \dfrac{p_\mathcal{U}}{p_\mathcal{U}+1}\delta_{ik}\right) & \text{if } |y_{ik}| \geq \delta_{ik}, \end{cases}$$

Function (6) satisfies the following properties:

P4: $\mathcal{S}_i(\mathbf{0}, p_\mathcal{U}, \mathbf{K}_{pi}, \boldsymbol{\delta}_i) = 0$ and $\mathcal{S}_i(\mathbf{y}_i, p_\mathcal{U}, \mathbf{K}_{pi}, \boldsymbol{\delta}_i)$ admits an $(\mathbf{r}, r_{\mathbf{y}_i}(p_\mathcal{U}+1))$-homogeneous approximation, for all $\mathbf{y}_i \in B_{\underline{\delta}_i}$, where $\underline{\delta}_i := \min_{k\in\bar{n}}\{\delta_{ik}\}$ and $r_{\mathbf{y}_i}$ is the weight associated to the $\mathbf{y}_i$ coordinate. ◁

P5: $\nabla_{\mathbf{y}_i}\mathcal{S}_i(\mathbf{y}_i, p_\mathcal{U}, \mathbf{K}_{pi}, \boldsymbol{\delta}_i) = \mathbf{K}_{pi}\mathrm{sat}_{\boldsymbol{\delta}_i}(\lceil \mathbf{y}_i\rfloor^{p_\mathcal{U}})$ and $\dot{\mathcal{S}}_i = \dot{\mathbf{y}}_i^\top \mathbf{K}_{pi}\mathrm{sat}_{\boldsymbol{\delta}_i}(\lceil \mathbf{y}_i\rfloor^{p_\mathcal{U}})$. ◁

P6: $\mathcal{S}_i$ is positive definite and radially unbounded, w.r.t. $\mathbf{y}_i$, and it has an isolated global minimum at $\mathbf{y}_i = \mathbf{0}$. ◁

2.2 Homogeneity and Finite-Time Stability

Consider a dynamical system described by

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}), \quad \mathbf{x}(0) = \mathbf{x}_0 \tag{7}$$

where $\mathbf{x} \in \mathbb{R}^m$ is the state vector, $\mathbf{f} : \mathbb{R}^m \mapsto \mathbb{R}^m$ is the associated continuous vector field and $m \in \mathbb{N}$. Assume that the origin is an equilibrium point, i.e. $\mathbf{f}(\mathbf{0}) = \mathbf{0}$.

Definition 1. (Bhat and Bernstein, 2000, 2005). The origin of (7) is Finite-Time Stable (FTS) if it is Lyapunov stable and there exists a locally bounded function $T : B_\delta \mapsto \mathbb{R}_{\geq 0}$ (called the settling-time function) such that for each $\mathbf{x}_0 \in B_\delta \setminus \{\mathbf{0}\}$, any solution $\mathbf{x}(t, \mathbf{x}_0)$ of (7) is defined on $t \in [0, T(\mathbf{x}_0))$ and $\mathbf{x}(t, \mathbf{x}_0) = \mathbf{0}$ for all $t \geq T(\mathbf{x}_0)$. If $B_\delta = \mathbb{R}^m$, $\mathbf{x} = \mathbf{0}$ is globally FTS. ◇

Finite-time stability can be determined with homogeneity notions.

Definition 2. (Bacciotti and Rosier, 2005). Let $r_i > 0$, $i \in \bar{m}$, be the weights of the elements x_i of $\mathbf{x} \in \mathbb{R}^m$ and define the vector of weights as $\mathbf{r} := [r_1, ..., r_m]^\top \in \mathbb{R}^m$. Let $\Delta^\mathbf{r}_\epsilon$ be the dilation operator such that $\Delta^\mathbf{r}_\epsilon \mathbf{x} := [\epsilon^{r_1}x_1, ..., \epsilon^{r_m}x_m]^\top$. A function $V : \mathbb{R}^m \mapsto \mathbb{R}$ (resp. a vector field $\mathbf{f} : \mathbb{R}^m \mapsto \mathbb{R}^m$) is said to be $\mathbf{r}$-homogeneous of degree $l \in \mathbb{R}$, or $(\mathbf{r}, l)$-homogeneous for short, if for all $\epsilon \in \mathbb{R}_{>0}$ and for all $\mathbf{x} \in \mathbb{R}^m$ the equality $V(\Delta^\mathbf{r}_\epsilon \mathbf{x}) = \epsilon^l V(\mathbf{x})$ (resp., $\mathbf{f}(\Delta^\mathbf{r}_\epsilon \mathbf{x}) = \epsilon^l \Delta^\mathbf{r}_\epsilon \mathbf{f}(\mathbf{x})$) holds. System (7) is called $(\mathbf{r}, l)$-homogeneous if the vector field $\mathbf{f}$ is $(\mathbf{r}, l)$-homogeneous. ◇

We highlight the fact that nonlinear systems are, in general, non-homogeneous. However, as it occurs in the linearization approach, homogeneous approximations (h-approximations for short) are used to study the stability of its equilibria (Orlov, 2009; Bacciotti and Rosier, 2005; Andrieu et al., 2008; Zavala-Río and Fantoni, 2014). The relation between stability and h-approximations is stated in the following lemma.

Lemma 1. (Bacciotti and Rosier, 2005). Consider system (7) with $\mathbf{f}(\mathbf{x}) = \mathbf{f}_H(\mathbf{x}) + \mathbf{f}_{NH}(\mathbf{x})$. Suppose that[1] $\mathbf{f}_H(\mathbf{x})$ is an $(\mathbf{r}, l)$-homogeneous continuous vector field such that $\mathbf{f}_H(\mathbf{0}) = \mathbf{0}$ is a locally Asymptotically Stable (AS) equilibrium point of $\dot{\mathbf{x}} = \mathbf{f}_H(\mathbf{x})$. Assume that $\mathbf{f}_{NH}(\mathbf{x})$ is a continuous vector field such that $\mathbf{f}_{NH}(\mathbf{0}) = \mathbf{0}$ and the following *vanishing condition* $\lim_{\epsilon \to 0} \epsilon^{-(l+r_i)} f_{NH_i}(\Delta_\epsilon^{\mathbf{r}}\mathbf{x}) = 0$ holds uniformly with respect to (w.r.t.) $\mathbf{x} \in S_\delta^{m-1}$ for $\delta > 0$ and all $i \in \bar{m}$. Then, the origin of (7) is locally AS. Furthermore, if $l = 0$ and all $r_i = 1$, the origin is locally Exponentially Stable (ES); and if $l < 0$, the origin is locally FTS. ◇

The next lemma is a direct consequence of Lemma 1, see (Hong et al., 2002; Zavala-Río and Fantoni, 2014; Zavala-Río and Zamora-Gómez, 2017; Orlov, 2009) for other equivalent versions.

Lemma 2. Suppose that $\dot{\mathbf{x}} = \mathbf{f}_H(\mathbf{x})$ is a h-approximation of (7) and the vector field $\mathbf{f}_H(\mathbf{x})$ is $(\mathbf{r}, l)$-homogeneous and $\mathbf{x} = \mathbf{0}$ is AS. Further, let $\mathbf{x} = \mathbf{0}$ of system (7) be GAS. Then, the origin of (7) is GAS and locally ES, if $l = 0$ and all $r_i = 1$; and the origin is globally FTS if $l < 0$. ◇

2.3 Interconnection Topology

We use graphs to represent the communication topology among the agents. In particular, we employ the graph Laplacian matrix $\mathbf{L} := \{L_{ij}\} \in \mathbb{R}^{N\times N}$ that is defined as $L_{ii} = \sum_{j\in\mathcal{N}_i} a_{ij}$ and $L_{ij} = -a_{ij}$, where $a_{ij} > 0$ if $j \in \mathcal{N}_i$ and $a_{ij} = 0$ otherwise. The set $\mathcal{N}_i$ contains all the neighbors of the ith-node. For an undirected graph $a_{ij} = a_{ji}$. Further, by construction, $\mathbf{L}$ has a zero row sum. For an undirected and connected graph, $\mathbf{L} = \mathbf{L}^\top > 0$, has a single zero-eigenvalue, with the associated eigenvector $\mathbf{1}_N$, and all of the other eigenvalues are strictly positive. Thus, rank$(\mathbf{L}) = N - 1$. Therefore, exists $\alpha \in \mathbb{R}$ such that $\ker(\mathbf{L}) = \alpha\mathbf{1}_N$.

Throughout the paper, we assume that there are not time-delays in the information exchanged between EL-agents. Also, these agents exchange information according to the following assumption.

A1. The EL-agents interconnection graph is *undirected, static* and *connected.* ◇

For any $\mathbf{x} \in \mathbb{R}^N$, **A1** ensures that

$$\frac{1}{2}\mathbf{x}^\top\mathbf{L}\mathbf{x} = \frac{1}{4}\sum_{i\in\bar{N}}\sum_{j\in\mathcal{N}_i} a_{ij}|x_i - x_j|^2. \quad (8)$$

The following lemma establishes the uniqueness of solutions of nonlinear interconnection potentials. For sake of brevity, the proof is omitted here but it can be found in (Cruz-Zavala et al., 2018).

Lemma 3. Consider function $\mathcal{W}(\mathbf{x}, p_\mathcal{W}, \mathbf{P})$, defined as

$$\mathcal{W} := c_1 \sum_{i\in\bar{N}}\sum_{j\in\mathcal{N}_i} a_{ij}(\mathbf{x}_i - \mathbf{x}_j)^\top \mathbf{P}\lceil \mathbf{x}_i - \mathbf{x}_j \rfloor^{p_\mathcal{W}}, \quad (9)$$

where $\mathbf{x}_i \in \mathbb{R}^n$, $\mathbf{x} := \mathrm{col}(\mathbf{x}_i) \in \mathbb{R}^{Nn}$, $c_1 := \frac{1}{2(p_\mathcal{W}+1)}$, $\mathbf{P}$ is a positive definite diagonal matrix and $p_\mathcal{W} \in (0, 1]$. Then if **A1** holds, $\mathcal{W}$ is positive definite and radially unbounded w.r.t. $\mathbf{x}_i - \mathbf{x}_j$. Moreover, there exists $\mathbf{x}_\star \in \mathbb{R}^n$ such that $\mathcal{W}$ has a unique global minimum at $\mathbf{x} = \mathbf{1}_N \otimes \mathbf{x}_\star$. ◇

2.4 Euler-Lagrange Robot Model

The EL-equations of motion of each agent can be written in compact form as

$$\mathbf{M}_i(\mathbf{q}_i)\ddot{\mathbf{q}}_i + \mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i + \nabla_{\mathbf{q}_i}\mathcal{U}_i(\mathbf{q}_i) = \boldsymbol{\tau}_i \quad (10)$$

where $\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) \in \mathbb{R}^{n\times n}$ is the Coriolis and centrifugal forces matrix, defined via the Christoffel symbols of the first kind.

In this work we restrict to EL-systems (10) that satisfy the following assumptions:

A2: There exist strictly positive constants m_{1i} and m_{2i}, such that $m_{1i} \le \|\mathbf{M}_i(\mathbf{q}_i)\| \le m_{2i}$, $\forall \mathbf{q}_i \in \mathbb{R}^n$. ◁

A3: The potential energy $\mathcal{U}_i(\mathbf{q}_i) \in \mathcal{C}^2$ is bounded from below. Furthermore, for all $\mathbf{q}_i \in \mathbb{R}^n$, there exists $k_{gi} > 0$, such that $\sup_{\mathbf{q}_i\in\mathbb{R}^n} \|\nabla_{\mathbf{q}_i}\mathcal{U}_i(\mathbf{q}_i)\| \le k_{gi}$. ◁

Last assumption implies that there exist constants $k_{gik} > 0$ such that $\sup_{\mathbf{q}_i\in\mathbb{R}^n} |\nabla_{q_{ik}}\mathcal{U}_i(\mathbf{q}_i)| \le k_{gik}$, for all $k \in \bar{n}$, where $\nabla_{q_{ik}}\mathcal{U}_i(\mathbf{q}_i)$ is the kth-element of $\nabla_{\mathbf{q}_i}\mathcal{U}_i(\mathbf{q}_i)$.

Model (10) has the following fundamental property (Kelly et al., 2005):

P7: Matrix $\dot{\mathbf{M}}_i(\mathbf{q}_i) - 2\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)$ is skew-symmetric and there exists $L_{ci} > 0$ such that $\|\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i\| \le L_{ci}\|\dot{\mathbf{q}}_i\|^2$. ◁

In this paper we suppose that the control input $\boldsymbol{\tau}_i$ is bounded. Such a case arises when τ_{ik}, the kth-element of $\boldsymbol{\tau}_i$, is subjected to the restriction $|\tau_{ik}| \le \bar{\tau}_{ik}$ where $\bar{\tau}_{ik} > 0$. We further assume that

A4: The bound $\bar{\tau}_{ik}$ is known and it is such that the control can cancel-out the system potential energy, i.e., $\bar{\tau}_{ik} > k_{gik}$. ◁

3. SOLUTION TO THE FT CONSENSUS PROBLEM WITH BOUNDED INPUTS

Theorem 1. Suppose that Assumptions **A1—A4** hold. Set the controller

$$\boldsymbol{\tau}_i = \nabla_{\mathbf{q}_i}\mathcal{U}_i(\mathbf{q}_i) + \mathbf{K}_{pi}\mathrm{sat}_{\boldsymbol{\delta}_i}\left(\lceil \boldsymbol{\theta}_i - \mathbf{q}_i \rfloor^{p_\mathcal{U}}\right), \quad (11)$$

[1] The vector field $\mathbf{f}_H(\mathbf{x})$ is known as the h-approximation of $\mathbf{f}(\mathbf{x})$. Similarly, an $\mathbf{r}$-homogeneous function $V_H : \mathbb{R}^m \mapsto \mathbb{R}$ is said to be h-approximation of $V : \mathbb{R}^m \mapsto \mathbb{R}$ if there exists $V_{NH} : \mathbb{R}^m \mapsto \mathbb{R}$ such that $V = V_H + V_{NH}$ and $\lim_{\epsilon\mapsto 0} \epsilon^{-l} V_{NH}(\Delta_\epsilon^{\mathbf{r}}\mathbf{x}) = 0$ uniformly w.r.t. $\mathbf{x} \in S_\delta^{m-1}$, for $\delta > 0$ Andrieu et al. (2008); Sepulchre and Aeyels (1996).

with the $\boldsymbol{\theta}_i$ dynamics is given by

$$\begin{aligned}\ddot{\boldsymbol{\theta}}_i =& -\mathbf{K}_{di}\lceil\dot{\boldsymbol{\theta}}_i\rfloor^{p_{\mathcal{F}}} - \mathbf{K}_{pi}\mathrm{sat}_{\boldsymbol{\delta}_i}\left(\lceil\boldsymbol{\theta}_i - \mathbf{q}_i\rfloor^{p_{\mathcal{U}}}\right)\\ &- \mathbf{P}\sum_{j\in\mathcal{N}_i} a_{ij}\lceil\boldsymbol{\theta}_i - \boldsymbol{\theta}_j\rfloor^{p_{\mathcal{U}}},\end{aligned} \tag{12}$$

where $\mathbf{K}_{pi}, \mathbf{K}_{di}, \mathbf{P}$ are diagonal positive definite matrices and

$$p_{\mathcal{U}} := \frac{2r_2 - r_1}{r_1}, \qquad p_{\mathcal{F}} := \frac{2r_2 - r_1}{r_2}.$$

Setting k_{pik} and δ_{ik} such that

$$k_{pik}\delta_{ik}^{p_{\mathcal{U}}} < \bar{\tau}_{ik} - k_{gik}, \tag{13}$$

ensures that the controller forces are bounded and the actuators are not saturated and that there exists $\mathbf{q}_\star \in \mathbb{R}^n$ such that: i) if $2r_2 > r_1 > r_2 > 0$ then the **(LC)** Problem is globally solved in FT; and ii) if $r_1 = r_2 = 1$ then the **(LC)** Problem is globally solved asymptotically and locally solved exponentially. ◇

Proof. First note that, from **A3** and (4), it holds that $|\tau_{ik}| \le k_{gik} + k_{pik}\delta_{ik}^{p_{\mathcal{U}}}$. Therefore, from **A4** and the fact that (13) holds, the controller torques do not saturate the actuators.

Let us now define $\tilde{\mathbf{q}}_i := \mathbf{q}_i - \mathbf{q}_\star$ and $\tilde{\boldsymbol{\theta}}_i := \boldsymbol{\theta}_i - \mathbf{q}_i$, then the closed-loop system (10), (11) and (12) is

$$\begin{aligned}\dot{\tilde{\mathbf{q}}}_i =& \dot{\mathbf{q}}_i,\\ \ddot{\mathbf{q}}_i =& -\mathbf{M}_i^{-1}(\tilde{\mathbf{q}}_i + \mathbf{q}_\star)\mathbf{C}_i(\tilde{\mathbf{q}}_i + \mathbf{q}_\star, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i\\ &+ \mathbf{M}_i^{-1}(\tilde{\mathbf{q}}_i + \mathbf{q}_\star)\mathbf{K}_{pi}\mathrm{sat}_{\boldsymbol{\delta}_i}\left(\lceil\tilde{\boldsymbol{\theta}}_i\rfloor^{p_{\mathcal{U}}}\right),\\ \dot{\tilde{\boldsymbol{\theta}}}_i =& \dot{\boldsymbol{\theta}}_i - \dot{\mathbf{q}}_i,\\ \ddot{\boldsymbol{\theta}}_i =& -\mathbf{K}_{di}\lceil\dot{\boldsymbol{\theta}}_i\rfloor^{p_{\mathcal{F}}} - \mathbf{K}_{pi}\mathrm{sat}_{\boldsymbol{\delta}_i}\left(\lceil\tilde{\boldsymbol{\theta}}_i\rfloor^{p_{\mathcal{U}}}\right)\\ &- \mathbf{P}\sum_{j\in\mathcal{N}_i} a_{ij}\lceil\boldsymbol{\theta}_i - \boldsymbol{\theta}_j\rfloor^{p_{\mathcal{U}}}.\end{aligned} \tag{14}$$

Consider the following Lyapunov candidate function

$$\begin{aligned}V =& \frac{1}{2}\sum_{i\in\bar{N}}\left[\dot{\mathbf{q}}_i^\top\mathbf{M}_i^{-1}(\mathbf{q}_i)\dot{\mathbf{q}}_i + \|\dot{\boldsymbol{\theta}}_i\|^2 + 2\mathcal{S}_i(\tilde{\boldsymbol{\theta}}_i, p_{\mathcal{U}}, \mathbf{K}_{pi}, \boldsymbol{\delta}_i)\right]\\ &+ \frac{1}{2(p_{\mathcal{U}}+1)}\sum_{i\in\bar{N}}\sum_{j\in\mathcal{N}_i} a_{ij}(\boldsymbol{\theta}_i - \boldsymbol{\theta}_j)^\top\mathbf{P}\lceil\boldsymbol{\theta}_i - \boldsymbol{\theta}_j\rfloor^{p_{\mathcal{U}}},\end{aligned}$$

where $\mathcal{S}_i$ is defined in (6).

Invoking Lemma 3 and using **P6**, we conclude that V is positive definite and radially unbounded w.r.t. $\dot{\mathbf{q}}_i$, $\dot{\boldsymbol{\theta}}_i$, $\tilde{\boldsymbol{\theta}}_i$ and $\|\boldsymbol{\theta}_i - \boldsymbol{\theta}_j\|$.

Using **P5**, **P7** and invoking, again, Lemma 3, we have that

$$\dot{V} = -\sum_{i\in\bar{N}}\dot{\boldsymbol{\theta}}_i^\top\mathbf{K}_{di}\lceil\dot{\boldsymbol{\theta}}_i\rfloor^{p_{\mathcal{F}}} = -\sum_{i\in\bar{N}}\sum_{k\in\bar{n}} k_{dik}|\dot{\theta}_{ik}|^{p_{\mathcal{U}}+1} \le 0.$$

The Krasovskii-LaSalle's Invariance Theorem implies that

$$(\tilde{\mathbf{q}}_i, \dot{\mathbf{q}}_i, \tilde{\boldsymbol{\theta}}_i, \dot{\boldsymbol{\theta}}_i) = (\mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{0}) \tag{15}$$

is a GAS equilibrium of (14).

In order to prove that such equilibrium is GFTS we will invoke Lemma 2. Therefore, we need to prove that the closed-loop system (14) admits a h-approximation of negative degree. In order to establish this fact, let us first rewrite (14) as

$$\begin{aligned}\dot{\tilde{\mathbf{q}}}_i =& \dot{\mathbf{q}}_i,\\ \ddot{\mathbf{q}}_i =& \mathbf{M}_i^{-1}(\mathbf{q}_\star)\mathbf{K}_{pi}\lceil\tilde{\boldsymbol{\theta}}_i\rfloor^{p_{\mathcal{U}}} + \mathbf{f}_{NHi}(\tilde{\mathbf{q}}_i, \dot{\mathbf{q}}_i, \tilde{\boldsymbol{\theta}}_i)\\ \dot{\tilde{\boldsymbol{\theta}}}_i =& \dot{\boldsymbol{\theta}}_i - \dot{\mathbf{q}}_i,\\ \ddot{\boldsymbol{\theta}}_i =& -\mathbf{K}_{di}\lceil\dot{\boldsymbol{\theta}}_i\rfloor^{p_{\mathcal{F}}} - \mathbf{K}_{pi}\lceil\tilde{\boldsymbol{\theta}}_i\rfloor^{p_{\mathcal{U}}}\\ &- \mathbf{P}\sum_{j\in\mathcal{N}_i} a_{ij}\lceil\boldsymbol{\theta}_i - \boldsymbol{\theta}_j\rfloor^{p_{\mathcal{U}}} + \mathbf{g}_{NHi}(\tilde{\boldsymbol{\theta}}_i),\end{aligned} \tag{16}$$

where

$$\begin{aligned}\mathbf{f}_{NH} :=& \left[\mathbf{M}_i^{-1}(\tilde{\mathbf{q}}_i + \mathbf{q}_\star) - \mathbf{M}_i^{-1}(\mathbf{q}_\star)\right]\mathbf{K}_{pi}\lceil\tilde{\boldsymbol{\theta}}_i\rfloor^{p_{\mathcal{U}}}\\ &- \mathbf{M}_i^{-1}(\tilde{\mathbf{q}}_i + \mathbf{q}_\star)\mathbf{C}_i(\tilde{\mathbf{q}}_i + \mathbf{q}_\star, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i\\ &+ \mathbf{M}_i^{-1}(\tilde{\mathbf{q}}_i + \mathbf{q}_\star)\mathbf{K}_{pi}\left(\mathrm{sat}_{\boldsymbol{\delta}_i}\left(\lceil\tilde{\boldsymbol{\theta}}_i\rfloor^{p_{\mathcal{U}}}\right) - \lceil\tilde{\boldsymbol{\theta}}_i\rfloor^{p_{\mathcal{U}}}\right),\\ \mathbf{g}_{NH} :=& -\mathbf{K}_{pi}\left(\mathrm{sat}_{\boldsymbol{\delta}_i}\left(\lceil\tilde{\boldsymbol{\theta}}_i\rfloor^{p_{\mathcal{U}}}\right) - \lceil\tilde{\boldsymbol{\theta}}_i\rfloor^{p_{\mathcal{U}}}\right).\end{aligned}$$

Clearly $\mathbf{f}_{NHi}(\mathbf{0}, \mathbf{0}, \mathbf{0}) = \mathbf{0}$ and $\mathbf{g}_{NHi}(\mathbf{0}) = \mathbf{0}$.

In what follows we assign the vectors $\mathbf{r}_1 = r_1\mathbf{1}_n$, $\mathbf{r}_2 = r_2\mathbf{1}_n$, $\mathbf{r}_3 = \mathbf{r}_1$ and $\mathbf{r}_4 = \mathbf{r}_2$ to be the homogeneity weights of the coordinates $\tilde{\mathbf{q}}_i, \dot{\mathbf{q}}_i, \tilde{\boldsymbol{\theta}}_i$ and $\dot{\boldsymbol{\theta}}_i$, respectively. We further define $\mathbf{r} := [\mathbf{r}_1^\top, \mathbf{r}_2^\top, \mathbf{r}_1^\top, \mathbf{r}_2^\top]^\top$.

Now, on one hand, it can be easily established that the reduced closed-loop

$$\Sigma_H\begin{cases}\dot{\tilde{\mathbf{q}}}_i = \dot{\mathbf{q}}_i,\\ \ddot{\mathbf{q}}_i = \mathbf{M}_i^{-1}(\mathbf{q}_\star)\mathbf{K}_{pi}\lceil\tilde{\boldsymbol{\theta}}_i\rfloor^{p_{\mathcal{U}}}\\ \dot{\tilde{\boldsymbol{\theta}}}_i = \dot{\boldsymbol{\theta}}_i - \dot{\mathbf{q}}_i,\\ \ddot{\boldsymbol{\theta}}_i = -\mathbf{K}_{di}\lceil\dot{\boldsymbol{\theta}}_i\rfloor^{p_{\mathcal{F}}} - \mathbf{K}_{pi}\lceil\tilde{\boldsymbol{\theta}}_i\rfloor^{p_{\mathcal{U}}}\\ \qquad - \mathbf{P}\sum_{j\in\mathcal{N}_i} a_{ij}\lceil\boldsymbol{\theta}_i - \boldsymbol{\theta}_j\rfloor^{p_{\mathcal{U}}},\end{cases}$$

is $(\mathbf{r}, r_2 - r_1)$-homogeneous and, on the other hand, the equilibrium point (15) is GAS. This last can be established using the funtion

$$\begin{aligned}V_H =& \frac{1}{2}\sum_{i\in\bar{N}}\left[\dot{\mathbf{q}}_i^\top\mathbf{M}_i^{-1}(\mathbf{q}_\star)\dot{\mathbf{q}}_i + \|\dot{\boldsymbol{\theta}}_i\|^2 + c_2\tilde{\boldsymbol{\theta}}_i^\top\mathbf{K}_{pi}\lceil\tilde{\boldsymbol{\theta}}_i\rfloor^{p_{\mathcal{U}}}\right]\\ &+ \frac{1}{2(p_{\mathcal{U}}+1)}\sum_{i\in\bar{N}}\sum_{j\in\mathcal{N}_i} a_{ij}(\boldsymbol{\theta}_i - \boldsymbol{\theta}_j)^\top\mathbf{P}\lceil\boldsymbol{\theta}_i - \boldsymbol{\theta}_j\rfloor^{p_{\mathcal{U}}},\end{aligned}$$

where $c_2 := \frac{2}{p_{\mathcal{U}}+1}$.

It only rests to prove that $\mathbf{f}_{NH}$ and that $\mathbf{g}_{NH}$ vanish when ϵ tends to zero.

Now, since $\|\mathbf{K}_{pi}\lceil\epsilon^{r_1}\tilde{\boldsymbol{\theta}}_i\rfloor^{p_{\mathcal{U}}}\| = \epsilon^{2r_2 - r_1}\|\mathbf{K}_{pi}\lceil\tilde{\boldsymbol{\theta}}_i\rfloor^{p_{\mathcal{U}}}\|$, then

$$\begin{aligned}&\lim_{\epsilon\to 0}\frac{\left\|\left[\mathbf{M}_i^{-1}(\epsilon^{r_1}\tilde{\mathbf{q}}_i + \mathbf{q}_\star) - \mathbf{M}_i^{-1}(\mathbf{q}_\star)\right]\mathbf{K}_{pi}\lceil\epsilon^{r_1}\tilde{\boldsymbol{\theta}}_i\rfloor^{p_{\mathcal{U}}}\right\|}{\epsilon^{2r_2 - r_1}}\\ &\le \|\mathbf{K}_{pi}\lceil\tilde{\boldsymbol{\theta}}_i\rfloor^{p_{\mathcal{U}}}\|\lim_{\epsilon\to 0}\|\mathbf{M}_i^{-1}(\epsilon^{r_1}\tilde{\mathbf{q}}_i + \mathbf{q}_\star) - \mathbf{M}^{-1}(\mathbf{q}_\star)\| = 0,\end{aligned}$$

Similarly, from **A2** and **P10**, there exists $\alpha_i > 0$ such that

$$\begin{aligned}&\lim_{\epsilon\to 0}\frac{\|\mathbf{M}_i^{-1}(\epsilon^{r_1}\tilde{\mathbf{q}}_i + \mathbf{q}_\star)\mathbf{C}_i(\epsilon^{r_1}\tilde{\mathbf{q}}_i + \mathbf{q}_\star, \epsilon^{r_2}\dot{\mathbf{q}}_i)\epsilon^{r_2}\dot{\mathbf{q}}_i\|}{\epsilon^{2r_2 - r_1}}\\ &\qquad \le \alpha_i\|\dot{\mathbf{q}}_i\|^2\lim_{\epsilon\to 0}\epsilon^{r_1} = 0.\end{aligned}$$

Finally, the last term of $\mathbf{f}_{NH}$ satisfies

$$\lim_{\epsilon\to 0}\frac{\|\mathbf{M}_i^{-1}(\epsilon^{r_1}\tilde{\mathbf{q}}_i + \mathbf{q}_\star)\mathbf{K}_{pi}\left(\mathrm{sat}_{\boldsymbol{\delta}_i}\left(\lceil\epsilon^{r_1}\tilde{\boldsymbol{\theta}}_i\rfloor^{p_{\mathcal{U}}}\right) - \lceil\epsilon^{r_1}\tilde{\boldsymbol{\theta}}_i\rfloor^{p_{\mathcal{U}}}\right)\|}{\epsilon^{2r_2 - r_1}} = 0,$$

when $\epsilon^{r_1}\tilde{\boldsymbol{\theta}}_i \in B_{\underline{\delta}_i}$, because in such a case $\mathrm{sat}_{\boldsymbol{\delta}_i}\left(\lceil \epsilon^{r_1}\tilde{\boldsymbol{\theta}}_i \rfloor^{p\mathcal{U}}\right) = \lceil \epsilon^{r_1}\tilde{\boldsymbol{\theta}}_i \rfloor^{p\mathcal{U}}$ and for all bounded $\tilde{\boldsymbol{\theta}}_i \in \mathbb{R}^n$, $\epsilon^{r_1}\|\tilde{\boldsymbol{\theta}}_i\| \to 0$ as $\epsilon \to 0$. $\mathbf{g}_{NH}$ also vanishes due to the same facts.

Hence, since Σ_H is a $(\mathbf{r}, r_2 - r_1)$−homogeneous for all $\tilde{\mathbf{q}}_i, \dot{\mathbf{q}}_i, \tilde{\boldsymbol{\theta}}_i, \dot{\boldsymbol{\theta}}_i \in \mathbb{R}^n$, Σ_H will be a homogenous approximation of the closed-loop (14), as required. This completes the proof. ◁

In order to improve performance, the controller can also incorporate linear terms, this result is stated in the following proposition. Its proof is omitted for sake of space.

Proposition 3. Theorem 1 holds replacing (11) by

$$\boldsymbol{\tau}_i = \nabla_{\mathbf{q}_i}\mathcal{U}_i(\mathbf{q}_i) + \mathbf{K}_{pi}\mathrm{sat}_{\boldsymbol{\delta}_i}\left(\lceil\tilde{\boldsymbol{\theta}}_i\rfloor^{p\mathcal{U}}\right) + \mathbf{K}_{pi}\mathrm{sat}_{\boldsymbol{\delta}_i}\left(\tilde{\boldsymbol{\theta}}_i\right),$$

and (12) by

$$\ddot{\boldsymbol{\theta}}_i = -\mathbf{K}_{di}\lceil\dot{\boldsymbol{\theta}}_i\rfloor^{p\mathcal{F}} - \mathbf{K}_{pi}\mathrm{sat}_{\boldsymbol{\delta}_i}\left(\lceil\tilde{\boldsymbol{\theta}}_i\rfloor^{p\mathcal{U}}\right) - \mathbf{K}_{pi}\mathrm{sat}_{\boldsymbol{\delta}_i}\left(\tilde{\boldsymbol{\theta}}_i\right) - \mathbf{P}\sum_{j\in\mathcal{N}_i} a_{ij}\lceil\boldsymbol{\theta}_i - \boldsymbol{\theta}_j\rfloor^{p\mathcal{U}},$$

provided that $k_{pik}(\delta_{ik}^{p\mathcal{U}} + \delta_{ik}) < \bar{\tau}_{ik} - k_{gik}$. ◇

4. SIMULATIONS

This section provides a simulation comparison with different control schemes for a network of ten 2-DoF nonlinear manipulators with revolute joints. The dynamics are borrowed from (Nuño, 2017).

The ten-agent network is composed of three different groups of robot manipulators and the manipulators at each group have the same physical description. The physical parameters, for each group, are: $m_1 = 4$kg, $m_2 = 2$kg and $l_1 = l_2 i = 0.4$m, for Agents 1, 2 and 3; $m_1 = 2.5$kg, $m_2 = 3$kg, $l_1 = 0.3$m and $l_2 = 0.5$m for Agents 4, 5 and 6; $m_1 = 3$kg, $m_2 = 2.5$kg, $l_1 = 0.5$m and $l_2 = 0.2$m for Agents 7, 8, 9 and 10.

The network interconnection Laplacian matrix is given by $\mathbf{L} = 0.1\mathbf{L}_g$, where

$$\mathbf{L}_g = \begin{bmatrix} 14 & 0 & -3 & 0 & 0 & 0 & 0 & -4 & 0 & -7 \\ 0 & 14 & 0 & -8 & 0 & 0 & 0 & 0 & 0 & -6 \\ -3 & 0 & 7 & 0 & 0 & 0 & 0 & -4 & 0 & 0 \\ 0 & -8 & 0 & 10 & 0 & 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & 8 & 0 & -5 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 & 0 & -4 & -2 & 0 \\ 0 & 0 & 0 & 0 & -5 & 0 & 14 & 0 & 0 & -9 \\ -4 & 0 & -4 & 0 & 0 & -4 & 0 & 12 & 0 & 0 \\ 0 & 0 & 0 & -2 & -3 & -2 & 0 & 0 & 7 & 0 \\ -7 & -6 & 0 & 0 & 0 & 0 & -9 & 0 & 0 & 22 \end{bmatrix}$$

The initial conditions are set as $\dot{\boldsymbol{\theta}}(0) = \dot{\mathbf{q}}(0) = \mathbf{0}$ and $\boldsymbol{\theta}(0) = \mathbf{q}(0)$ where

$$\mathbf{q}^\top(0) = [-2, 6, -7, 3, -5, -1, 0, 1, -6, 2, 1, 0, -4, 5, -3, 4, -2, 7, -1, 1]. \quad (17)$$

Three different controllers are simulated in this section. The structure of the first and second schemes is the one resulting from Theorem 1. The third controller results from Proposition 3. The values of the homogeneous weights are: $r_1 = r_2 = 1$ for Controller 1; $r_1 = 1.3$ and $r_2 = 1$ for Controller 2; and $r_1 = 1.3$ and $r_2 = 1$ for Controller 3. These weights ensure that Controllers 2 and 3 induce finite-time convergence to the consensus position.

The gains are the same for the three schemes and they have been set as: $\mathbf{K}_{pi} = 5\mathbf{I}$, $\mathbf{P} = 8\mathbf{I}$ and $\mathbf{P} = 20\mathbf{I}$ for all the ten robots. It is assumed that $\bar{\tau}_{ik} = 4$ for all the robots and for the two degrees of freedom. Therefore, three different values of δ_{ik} are obtained for the three controllers, namely 0.75, 0.6 and 0.25, respectively.

Fig. 1 shows the position performance of the three different controllers. It can be concluded that the FT schemes have a more damped behavior.

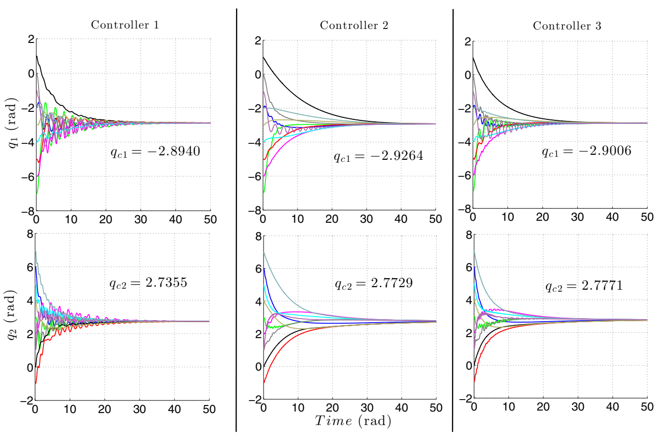

Fig. 1. Leaderless consensus comparison with input saturation and with the initial positions given in (17).

In order to measure the performance of the three different schemes, the following Root-Mean-Square error quality measure is employed

$$y_{RMS} := \sqrt{\frac{1}{T}\int_0^T \|(\mathbf{L}_\ell \otimes \mathbf{I}_n)\tilde{\mathbf{q}}(\sigma)\|^2 d\sigma}. \quad (18)$$

Such value measures how disperse are the agents of the network.

The numerical values of (18) for the three controllers are: 3.6766; 4.9207 and 3.3671, respectively. It should be underscored that the linear scheme, Controller 1, outperforms the simplest FT scheme, Controller 2. However, the FT scheme Controller 3 has exhibit better performance.

We have further added noise in the measurements of $\mathbf{q}_i$ and analyzed the quality measure y_{RMS}. Such noise is the output of a normal Gaussian distributed random signal with mean equal to 0.02 and variance equal to 0.001. The numerical values are: 3.7075; 4.9589 and 3.4333, respectively. Again, Controller 3 has exhibit better robustness w.r.t. this added noise.

5. CONCLUSIONS

In this paper we present a novel controller that provides a solution to the finite-time leaderless consensus problem in networks of multiple EL-systems without making use of velocity measurements and with bounded control inputs. The proposed decentralized controller has a dynamic behavior ruled by the EL-equations of motion. Global FT stability is concluded by means of homogeneity notions. Simulations show the effectiveness of our proposal.

REFERENCES

Abdessameud, A. and Tayebi, A. (2013). On consensus algorithms design for double integrator dynamics. *Automatica*, 49(1), 253–260.

Abdessameud, A., Tayebi, A., and Polushin, I.G. (2017). Leader-follower synchronization of Euler-Lagrange systems with time-varying leader trajectory and constrained discrete-time communication. *IEEE Transactions on Automatic Control*, 62(5), 2539–2545.

Andrieu, V., Praly, L., and Astolfi, A. (2008). Homogeneous approximation, recursive observer design and output feedback. *SIAM J. Control Optim.*, 47(4), 1814–1850.

Arteaga, M. and Kelly, R. (2004). Robot control without velocity measurements: New theory and experimental results. *IEEE Transactions on Robotics and Automation*, 20(2), 297–308.

Bacciotti, A. and Rosier, L. (2005). *Lyapunov functions and stability in control theory*. Springer-Verlag, New York, 2nd edition.

Bhat, S. and Bernstein, D. (2000). Finite-time stability of continuous autonomous systems. *SIAM Journal on Control and Optimization*, 38, 751–766.

Bhat, S. and Bernstein, D. (2005). Geometric homogeneity with applications to finite-time stability. *Mathematics of Control, Signals, and Systems*, 17(2), 101–127.

Cao, Y. and Ren, W. (2011). *Distributed Coordination of Multi-agent Networks: Emergent Problems, Models, and Issues*. Springer-Verlag.

Chung, S. and Slotine, J. (2009). Cooperative robot control and concurrent synchronization of Lagrangian systems. *IEEE Trans. on Robotics*, 25(3), 686–700.

Cortés, J. (2006). Finite-time convergent gradient flows with applications to network consensus. *Automatica*, 42(11), 1993–2000.

Cruz-Zavala, E., Moreno, J., and Nuño, E. (2018). Leaderless and leader-follower consensus of euler-lagrange agents: Finite-time convergence without velocity measurements. In *IEEE Conference on Decision and Control (Submitted)*.

Davila, J. and Pisano, A. (2016). Fixed-time consensus for a class of nonlinear uncertain multi-agent systems. In *Proc. of the American Control Conference (ACC)*.

Galicki, M. (2015). Finite-time control of robotic manipulators. *Automatica*, 51(1), 49–54.

Ge, M.F., Guan, Z.H., Yang, C., Li, T., and Wang, Y.W. (2016). Time-varying formation tracking of multiple manipulators via distributed finite-time control. *Neurocomputing*, 202, 20–26.

Hatanaka, T., Chopra, N., Fujita, M., and Spong, M. (2015). *Passivity-Based Control and Estimation in Networked Robotics*. Communications and Control Engineering. Springer.

Hong, Y., Xu, Y., and Huang, J. (2002). Finite-time control for robot manipulators. *Systems & Control Letters*, 46, 243–253.

Jadbabaie, A., Lin, J., and Morse, A. (2003). Coordination of groups of mobile autonomous agents using nearest neighbor rules. *IEEE Transactions on Automatic Control*, 48(6), 988–1001.

Kelly, R., Santibáñez, V., and Loría, A. (2005). *Control of Robot Manipulators in Joint Space*. Springer-Verlag, London, U.K.

Klotz, J., Obuz, S., Kan, Z., and Dixon, W. (2018). Synchronization of uncertain euler-lagrange systems with uncertain time-varying communication delays. *IEEE Transactions on Cybernetics*, 48(2), 807–817.

Liu, Y., Zhao, Y., Shi, Z., and Wei, D. (2017). Specified-time containment control of multi-agent systems over directed topologies. *IET Control Theory & Applications*, 11(4), 576–585.

Nuño, E. (2017). Consensus of Euler-Lagrange systems using only position measurements. *IEEE Transactions on Control of Network Systems. DOI:10.1109/TCNS.2016.2620806*.

Nuño, E. and Ortega, R. (2018). Achieving consensus of Euler-Lagrange agents with interconnecting delays and without velocity measurements via passivity-based control. *IEEE Transactions on Control Systems Technology*, 26(1), 222–232.

Nuño, E., Ortega, R., Basañez, L., and Hill, D. (2011). Synchronization of networks of nonidentical Euler-Lagrange systems with uncertain parameters and communication delays. *IEEE Transactions on Automatic Control*, 56(4), 935–941.

Olfati-Saber, R. and Murray, R. (2004). Consensus problems in networks of agents with switching topology and time-delays. *IEEE Transactions on Automatic Control*, 49(9), 1520–1533.

Orlov, Y. (2005). Finite time stability and robust control synthesis of uncertain switched systems. *SIAM Journal on Control and Optimization*, 43(4), 1253–1271.

Orlov, Y. (2009). *Discontinuos Systems: Lyapunov analysis and robust synthesis under uncertainty conditions*. Springer.

Ren, W. (2008). On consensus algorithms for double-integrator dynamics. *IEEE Transactions on Automatic Control*, 53(6), 1503–1509.

Ren, W. (2009). Distributed leaderless consensus algorithms for networked Euler-Lagrange systems. *Int. Jour. of Control*, 82(11), 2137–2149.

Ren, W. (2010). Distributed cooperative attitude synchronization and tracking for multiple rigid bodies. *IEEE Transactions on Control Systems Technology*, 18(2), 383–392.

Sepulchre, R. and Aeyels, D. (1996). Homogeneous Lyapunov functions and necessary conditions for stabilization. *Math. Control. Signals Syst.*, 9, 34–58.

Venkataraman, S. and Gulati, S. (1993). Terminal slider control of robot systems. *Journal of Intelligent and Robotic Systems*, 5, 31–55.

Zavala-Río, A. and Fantoni, I. (2014). Global finite-time stability characterized through a local notion of homogeneity. *IEEE Trans. Autom. Control*, 59(2), 471–477.

Zavala-Río, A. and Zamora-Gómez, G. (2017). Local homogeneity-based global continuous control for mechanical systems with constrained inputs: finite-time and exponential stabilisation. *International Journal of Control*, 90(5), 1037–1051.

Zhao, Y., Duan, Z., and Wen, G. (2015). Distributed finite-time tracking of multiple Euler-Lagrange systems without velocity measurements. *International Journal of Robust and Nonlinear Control*, 25(11), 1688–1703.

