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Finite-Time Stabilization of the Prey-Predator Model ⋆

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Abstract:
This paper deals with the solution of achieving finite-time stabilization for a Leslie-Gower prey-predator system through a bounded control input. Simulation results show the effectiveness of the proposed control methodology.

1. INTRODUCTION

An interesting system exhibiting oscillations and chaotic behavior is the prey-predator model Collings (1997); Li and Xiao (2007); Jiang and Song (2013), which because of its complex dynamic characteristics results in a challenging system to be controlled Gakkhar and Singh (2012). This model has been used to study biological phenomena and the equilibrium of the species. The earliest ratio-dependent model was given by Leslie. In this model, the predator is also assumed to be growing logistically with a carrying capacity that depends on the availability of a variable resource (prey). This formulation is based on the assumption that a reduction in a predator population has a reciprocal relationship with the per capita availability of its preferred food. This interesting formulation for the predator dynamics has been discussed by Leslie and Gower in Leslie and Gower (1960) and by Pielou in Pielou (1969).

From a control view point, it is desirable to reach an equilibrium point for the system, particularly in finite time and by a bounded control input, as considered in this work.

1.1 Notations

Let $\mathbb{R}$ denote the set of real numbers. Let S be an $m \times n$ matrix. By S^{T}, we denote the transpose matrix of S. Let $x \in \mathbb{R}^n$. By $||x||$ we denote the euclidian norm of $x := (x_1, \ldots, x_n)$, i.e., $||x|| := (x_1^2, \ldots x_n^2)^{\frac{1}{2}}$. The norm of an $n \times n$ matrix S is defined by $||S|| := \max\limits_{1 \le j \le n} \sum_{i=1}^{m} |s_{ij}|$.

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2. THE PREY-PREDATOR MODEL

Consider the nonlinear control system

$$\begin{aligned} \dot{x}_1 &= x_1(1 - x_1) - \frac{x_1 x_2}{x_1^2 + \alpha} \\ \dot{x}_2 &= \gamma\left(1 - \frac{x_2}{\beta x_1}\right) x_2 + u, \quad |u| \le u_1 \end{aligned} \tag{1}$$

defined in set $D := \{(x_1, x_2) \in \mathbb{R}^2 : x_1 \ge 0, x_2 \ge 0\}$ with initial condition (x_1^0, x_2^0) and $x_1^0 > 0$, $x_2^0 > 0$, with u being a control input for achieving stabilization of the system at the equilibrium point. Let $(\xi, \eta) \in D$ the equilibrium point of the system (1) with $u = 0$. In system (1), $x_1 = X/K$, $x_2 = mY/r\,K^2$, $t = r\,T$, $\alpha = a/K^2$, $\beta = m\,n/K\,r$ and $\gamma = s/r$, where X and Y represent the prey and predator population, respectively. The parameter r is the intrinsic growth of prey species with carrying capacity K, T is a scaled time variable, m denotes the per capita consumption rate of the predator, parameter a denotes the number of prey required to make maximum rate just half, while s is the growth rate of the logistically growing population Y, and finally n is the magnitude of food quality of prey for reproduction in the predator population Singh (2016); Gakkhar and Singh (2012). All the parameters are assumed to be positive.

The statement of the problem we consider is the following: find a *bounded* positional $u = u(x)$ with $|u(x)| \le u_1$ and such that the trajectory $x(t) = (x_1(t), x_2(t))$ starting at the initial point $x_0 := (x_1^0, x_2^0)$ and belonging to a certain neighborhood of the point $\bar{x} := (\xi, \eta)$ terminates at $\bar{x}$ at *finite* time $T(x, \bar{x})$. This problem is called *the synthesis problem.*

2.1 System (1) translated to the origin

We first rewrite system (1). By translating the equilibrium point (ξ, η) to the origin, we have

$$\dot{y} = Ay + bu + g(y), \tag{2}$$

where

$$A := \begin{pmatrix} \frac{2\xi(1-\xi)^2}{\beta} - \xi & -\frac{1-\xi}{\beta} \\ \beta\gamma & -\gamma \end{pmatrix}, \, b := \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{3}$$

$$g_1(y_1, y_2) := -\frac{d_4^2 y_1^4 + d_3 y_1^3 + d_2 y_1^2 + d_1 y_2 y_1}{d_4^2 \left(2y_1\xi + y_1^2 + \alpha + \xi^2\right)},$$

$$g_2(y_1, y_2) := -\frac{\gamma \left(y_2 - \beta y_1\right)^2}{\beta \left(\xi + y_1\right)}.$$

$$g(y) := \begin{pmatrix} g_1(y_1, y_2) \\ g_2(y_2, y_2) \end{pmatrix} \tag{4}$$

and

$$d_1 := \left(\alpha - \xi^2\right)\left(\alpha + \xi^2\right),$$
$$d_2 := \left(\alpha + \xi^2\right)\left(-w\xi + \alpha^2 + \alpha(5\xi - 3)\xi + \xi^3\right),$$
$$d_3 := \left(\alpha + \xi^2\right)\left(3\alpha\xi - \alpha + \xi^3 + \xi^2\right), d_4 := \alpha + \xi^2.$$

We assume that the parameters α and β are positive. Consequently, the function g appearing in (4) and the system (2) is well-defined in the region:

$$D_1 := \{(y_1, y_2) \in \mathbb{R}^2 : y_1 + \xi > 0, \, y_2 + \eta > 0\}. \tag{5}$$

The linear part of (2) is completely controllable if and only if $\text{rank}(b, Ab) = 2$, i.e., if and only if

$$\frac{\gamma \left(\beta - 2(\xi - 1)^2\xi\right)}{\beta} \neq 0. \tag{6}$$

Note that if $\xi = 1$ the linear part of (2) is not completely controllable. In the sequel, we will study the control system in a certain neighborhood of the origin V_0 such that $\xi \neq 1$ and the inequality

$$||g(y)|| \leq C_1||y|| \tag{7}$$

is satisfied, for some $C_1 > 0$.

2.2 Local feedback stabilization of system (1)

The feedback stabilization of the system (1) was considered in Singh (2016). A positional control $u(x)$ of the form

$$u = k_1(x_1 - \xi) + k_2(x_2 - \eta) \tag{8}$$

was proposed. In terms of our notations and taking into account that (ξ, η) is an equilibrium point of (1), we obtain that constants k_1 and k_2 should satisfy the following inequalities:

$$k_2 + \gamma + \xi - 1 > 0,$$
$$(\xi - 1)\left(k_2\beta + k_1\right) > 0.$$

Note that the feedback stabilization of the linear part of (2) via control (8) is not possible if $\xi - 1 = 0$.

3. FINITE TIME STABILIZATION

It seems to be that Kamenkov (1953) was the first who used the term *finite-time stability* (FTS). Further developments in FTS were made by a number of researchers: Weiss and Infante (1967), Lasalle and Lefschetz (1961), Dorato (1967), Dorato (2006) and references therein. See also Bath (1995) and Poznyak et al. (2011).

In this work, we employ the theorem appearing in (Korobov , 1979, Page 552), where the synthesis of bounded controls in the first approximation of a certain general nonlinear system is treated. Our work differs from Korobov (1979) mainly because we construct a specific control that depends on the equilibrium point $\bar{x}$, which in turn depends on the parameters of the system α, β and γ. We also describe a certain ellipse "centered" at the equilibrium point $\bar{x}$, so from every inner point x_0 of this ellipse it is possible to arrive to $\bar{x}$ at finite time $T(x, \bar{x})$. Another important novelty is the fact that in the construction of the bounded control $u(x)$ we use the method proposed in Choque1 et al. (2004). See also Choque2 et al. (2004) and Choque3 (2008). The controls appearing in Choque1 et al. (2004) depend on a parameter (as (14)) that in turn enables having a family of controls that could solve the synthesis problem.

In the sequel, we assume that $\xi - 1 \neq 0$. Let

$$F := \begin{pmatrix} \frac{\beta}{\xi - 1} & 0 \\ \frac{\left(2(\xi-1)^2 - \beta\right)\xi}{\xi - 1} & 1 \end{pmatrix}. \tag{9}$$

Clearly, we see that $\det F \neq 0$.

Remark 3.1. The matrix F can be written as $F = \begin{pmatrix} c^\intercal \\ c^\intercal A, \end{pmatrix}$ where c is a vector satisfying $(c, b) = 0$ and $(c, Ab) = 1$.

Furthermore, we use the transformation

$$z = Fy \tag{10}$$

to rewrite equation (2) in the canonical form

$$\dot{z} = A_0 z + bw + Fg(F^{-1}z) \quad |w| \leq w_1, \tag{11}$$

where

$$p = (p_1, p_2)^\intercal \tag{12}$$

with $p_1 := -\frac{\gamma\left(\eta - 2(\xi-1)^2\xi^2\right)}{\eta}$ and $p_2 := \frac{\xi\left(2(\xi-1)^2\xi - \eta\right)}{\eta} - \gamma$ and $A_0 := \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$. The new control w has the following form:

$$w := p^\intercal z + u. \tag{13}$$

with the restriction $w \leq w_1$ where

$$w_1 := u_1 - u_2 \sum_{j=1}^{2} |p_j|. \tag{14}$$

We assume that $u_2 < \frac{u_1}{\sum_{j=1}^{2} |p_j|}$. As in Korobov (1979), we require that system (11) is considered in the neighborhood

$$Q := \{z : |z_j| \leq u_2\}. \tag{15}$$

Our next step is to construct a positional control $w(z)$ such that $|w| \leq w_1$ and that the trajectory of any initial point $z_0 := (z_1^0, z_2^0)$ belonging to a certain neighborhood of the origin arrives to the origin at finite time $T(z_0)$. To this end, we will use V.I. Korobov's method, which consists of a Lyapunov type function $\theta(z)$, which is the only positive solution of the following equation:

$$2a_0\theta = (K(\theta)z, z), \tag{16}$$

where

$$K(\theta) := \frac{1}{4+a_1}\begin{pmatrix} \frac{a_1}{\theta^3} & -\frac{2}{\theta^2} \\ -\frac{2}{\theta^2} & -\frac{1}{\theta} \end{pmatrix} \tag{17}$$

is a positive matrix for $\theta > 0$. The number a_1 is a negative number such that the matrices K and $\frac{1}{\theta}K - \frac{d}{d\theta}K$ are both positive definite matrices. In terms of the parameter a_1, this condition is equivalent to the following inequality:

$$a_1 < -\frac{9}{2}. \tag{18}$$

The number a_0 satisfies the inequality

$$a_0 \leq \frac{w_1^2}{2a_1(a_1+3)}. \tag{19}$$

In the frame of Korobov's method, the positional control $w(z)$ has the form

$$w(z) := \frac{a_1 z_1}{\theta^2(z_1, z_2)} - \frac{3z_2}{\theta(z_1, z_2)} \tag{20}$$

Recall that in Choque1 et al. (2004), for the linear system $\dot{z} = A_0 z + bw$, a family of bounded positional controls was proposed which exactly stabilized this system at time $T(z_0) = \theta_0$, where θ_0 is the root of equation (16) for z_0.

Let us now rewrite the matrices K and $\frac{1}{\theta}K - \frac{d}{d\theta}K$ in a more convenient form.

Let $D(\theta) := \begin{pmatrix} \theta^{-\frac{3}{2}} & 0 \\ 0 & \theta^{-\frac{1}{2}} \end{pmatrix}$. Thus, the matrices $K = K(\theta)$ and $\frac{1}{\theta}K - \frac{d}{d\theta}K$ can be written as follows:

$$D(\theta)K_1D(\theta) = K, \quad \frac{1}{\theta}D(\theta)K_2D(\theta) = \frac{1}{\theta}K - \frac{d}{d\theta}K, \tag{21}$$

where

$$K_1 := \frac{1}{4+a_1}\begin{pmatrix} a_1 & -2 \\ -2 & -1 \end{pmatrix}, \quad K_2 := \frac{1}{4+a_1}\begin{pmatrix} 4a_1 & -6 \\ -6 & -2 \end{pmatrix}. \tag{22}$$

In the sequel, we assume that θ satisfies the inequality $\theta \leq 1$.

Lemma 3.2. Let $\lambda_{\min,K_2}$ be the minimal eigenvalue of the matrix K_2 and C_1 the constant appearing in (7). Thus, the following is valid:

$$\frac{(Kz, Fg(F^{-1}z))}{((\frac{1}{\theta}K - \frac{d}{d\theta}K)z, z)} \leq \theta\frac{C_1||K_1||}{\lambda_{\min,K_2}}. \tag{23}$$

Proof. Denote

$$q := D(\theta)z. \tag{24}$$

By using (22) and (24), we then have

$$\begin{aligned}
&\frac{(Kz, Fg(F^{-1}z))}{((\frac{1}{\theta}K - \frac{d}{d\theta}K)z, z)} = \frac{(DK_1Dz, Fg(F^{-1}z))}{\frac{1}{\theta}(DK_2Dz, z)} \\
&= \frac{(K_1q, DFg(F^{-1}D^{-1}q))}{\frac{1}{\theta}(K_2q, q)} \\
&\leq \theta C_1\frac{||K_1||||q||||D||||F||||F^{-1}||||D^{-1}||||q||}{\lambda_{\min,K_2}||q||^2} \\
&= \theta\frac{C_1||K_1||}{\lambda_{\min,K_2}}.
\end{aligned}$$

■

The following result gives an estimation of the derivative of the controllability function θ with respect to the system (11).

Theorem 3.3. The following inequality is valid

$$\dot{\theta} \leq -1 + \theta\frac{C_1||K_1||}{\lambda_{\min,K_2}}. \tag{25}$$

Proof. Let $a := (a_1, a_2)^\intercal$. We take the derivative of equality (16) with respect to system (11), we have

$$\begin{aligned}
\dot{\theta} =&\frac{((KA_0 + A_0^\intercal K + ab^\intercal K + Kba^\intercal)z, z)}{((\frac{1}{\theta}K - \frac{d}{d\theta}K)z, z)} \qquad (26)\\
&+ 2\frac{(Kz, Fg(F^{-1}z))}{((\frac{1}{\theta}K - \frac{d}{d\theta}K)z, z)} \\
=& -1 + 2\frac{(Kz, Fg(F^{-1}z))}{((\frac{1}{\theta}K - \frac{d}{d\theta}K)z, z)}. \qquad (27)
\end{aligned}$$

The right-hand side of (26) is equal to -1 because of (Choque1 et al. , 2004, Equation (2.9)). Finally, inequality (25) readily follows from (27) and Lemma 3.2.

Remark 3.4. Let $\widehat{\theta} > 0$, $C_2 > 0$ such that for $\theta \leq \widehat{\theta}$

$$-1 + \theta\frac{C_1||K_1||}{\lambda_{\min,K_2}} \leq -C_2. \tag{28}$$

By taking into account (25) and (28), we have that

$$\dot{\theta} \leq -C_2. \tag{29}$$

By integrating (29) on the trajectory $z = z(t)$, we attain $\theta(z(t)) - \theta_0 \leq -C_2 t$. By using (Korobov , 1979, Page 552), we have that $z(T) = 0$, which implies $\theta(z(T)) = 0$. Thus, we obtain the following inequality:

$$T(z) \leq \frac{\theta_0}{C_2}. \tag{30}$$

Now we present the main result of our work.

Theorem 3.5. Let $c := (\frac{\eta}{(\xi-1)\xi}, 0)^\intercal$, $a := (\,a_1,\, a_2\,)^\intercal$ with $a_2 = -3$ and p be defined as in (12). Let $\bar{x} := (\xi, \eta)$ be the equilibrium point of (1). Furthermore, let $(k_{j,\ell})_{j,\ell=1}^2 := K_1$, and the parameter a_0 satisfies (19). Let $\theta(x - \bar{x})$ be the unique positive solution of

$$\mathcal{E}(x, \theta, \bar{x}) = 0 \tag{31}$$

with

$$\mathcal{E}(x,\theta,\bar{x}) := 2a_0\theta^4 - \sum_{j,\ell=1}^{2} k_{j,\ell}\theta^{j+\ell-2}(c, A^{j-1}(x-\bar{x}))(c, A^{\ell-1}(x-\bar{x})). \quad (32)$$

In addition, let

$$u(x,\bar{x}) = \sum_{j=1}^{2} a_j\theta^{j-3}(x-\bar{x})(c, A^{j-1}(x-\bar{x})) - \sum_{j=1}^{2} p_j(x-\bar{x})(c, A^{j-1}(x-\bar{x})). \quad (33)$$

Suppose that (x_1^0, x_2^0) belongs to the region

$$D_2 := \{\mathcal{E}(x,\widehat{\theta},\bar{x}) \geq 0\} \cap \{x_1 > 0, x_2 > 0\}. \quad (34)$$

Thus, the control (33) satisfies the condition $|u(x)| \leq u_1$ and solves the synthesis problem.

c) The time of motion from $x_0 = (x_1^0, x_2^0)$ to the origin satisfies the following inequality

$$T(x_0,\bar{x}) \leq \frac{\theta_0}{C_2}. \quad (35)$$

Proof. The proof of this theorem readily follows from Theorem 3.3 and Remark 3.4. ■

Remark 3.6. For fixed θ, equation $\mathcal{E}(x,\theta,\bar{x}) = 0$ represents an ellipse. Since the control (33) stabilizes the the system (1), its trajectory of the will not leave the ellipse (31) calculated at $\theta = \theta_0$. In turn, θ_0 is the solution of (31) for $x = x_0$.
The large of the neighborhood of the equilibrium point where we can apply the proposed position control (33) can be determined by the fact that the ellipse (31) shall be included in the domain $D_1 \cap D_2$; see (5) and (34).

Remark 3.7. Note that for all initial points belonging to the region $D_1 \cap D_2$, we guarantee that there is a family of positional controls $u(x,\bar{x})$ determined by the parameter a_1 (18).

Remark 3.8. We emphasize that the trajectory $x(t)$ under the influence of control $u(x,\bar{x})$ approaches the equilibrium point $\bar{x}$ for $t \to T = T(x_0,\bar{x})$. For the $t > T$, the trajectory stays at the equilibrium point $\bar{x}$.

4. GRAPH OF THE TRAJECTORY AND CONTROL

To plot the graph of the trajectory $x(t)$ from a given initial point (x_1^0, x_2^0), the control $u(x(t))$ and the controllability function $\theta(x(t))$, we add a differential equation on θ:

$$\begin{aligned} \dot{x}_1 &= x_1(1-x_1) - \frac{x_1x_2}{x_1^2+\alpha}, \\ \dot{x}_2 &= \gamma\left(1 - \frac{x_2}{\beta x_1}x_2\right) + u(x_1,x_2), \\ \dot{\theta} &= -1 + 2\psi(x,\theta,\bar{x}), \end{aligned} \quad (36)$$

with initial conditions $x_1(0) = x_1^0$, $x_2(0) = x_2^0$ and $\theta(0) = \theta_0$. Here θ_0 is the root equation (32). Moreover,

$$\psi(x,\theta,\bar{x}) := \frac{(D(\theta)K_1D(\theta)(x-\bar{x}), Fg(F^{-1}(x-\bar{x})))}{\frac{1}{\theta}(D(\theta)K_2D(\theta)(x-\bar{x}), (x-\bar{x}))}. \quad (37)$$

Let us remark that the initial point (x_1^0, x_2^0) should belong to the neighborhood $\mathcal{E}(x,\widehat{\theta},\bar{x}) \geq 0$, for a fixed $\widehat{\theta}$.

4.1 Example 1

Let $\alpha = \frac{1}{5}$, $\beta = \frac{1513}{1200}$, $\gamma = \frac{1}{50}$ and $a_1 = -6$. The equilibrium point is equal to $\bar{x} = (\frac{3}{20}, \frac{1513}{8000})$. Let $u_1 = 1$. The positional control has the form $u(x,\bar{x}) = \frac{\mathrm{num}(x,\bar{x})}{\mathrm{den}(x,\bar{x})}$ where

$$\begin{aligned} \mathrm{num}(x,\bar{x}) &:= \left(-523418\theta^2 + 20826600\theta + 190104000\right)x_1 \\ &- 3\left(\theta^2(13600x_2 - 28743) + 5340\theta(4000x_2 - 737)\right. \\ &\left.+9505200\right), \quad \mathrm{den}(x,\bar{x}) := 213600000\theta^2 \end{aligned}$$

with $\theta = \theta(x - \bar{x})$.

The function $\psi(x,\theta,\bar{x})$ is given by

$$\psi(x,\theta,\bar{x}) = \frac{\frac{\theta(a_1+a_2+a_3)a_4}{-\frac{1780}{3}\left(x_1-\frac{3}{20}\right)-89} - \frac{1346570}{3}\left(x_1 - \frac{3}{20}\right)a_5a_6}{b_1}$$

with

$$\begin{aligned} a_1 &:= -\frac{123407713822900}{3}\left(x_1-\frac{3}{20}\right)^5 \\ &+ \frac{365192907781007}{9}\left(x_1-\frac{3}{20}\right)^4 - \frac{62742241}{60} \\ &\times\left(20648000\left(-\frac{13}{400}\left(x_1-\frac{3}{20}\right)+x_2-\frac{1513}{8000}\right)\right. \\ &\left.-14296369\right)\left(x_1-\frac{3}{20}\right)^3, \\ a_2 &:= \frac{62742241\left(a_{21}-a_{22}+415944793\right)\left(x_1-\frac{3}{20}\right)^2}{3600}, \\ a_{21} &:= 1152000000\left(-\frac{13}{400}\left(x_1-\frac{3}{20}\right)+x_2-\frac{1513}{8000}\right)^2, \\ a_{22} &:= 2493264000\left(-\frac{13}{400}\left(x_1-\frac{3}{20}\right)+x_2-\frac{1513}{8000}\right), \\ a_{31} &:= 288000\left(-\frac{13}{400}\left(x_1-\frac{3}{20}\right)+x_2-\frac{1513}{8000}\right) \\ a_3 &:= \frac{62742241}{3}\left(-\frac{13}{400}\left(x_1-\frac{3}{20}\right)+x_2-\frac{1513}{8000}\right) \\ &\quad (a_{31}-760109)\left(x_1-\frac{3}{20}\right)+a_{31}, \\ a_{32} &:= 4467247559200\left(-\frac{13}{400}\left(x_1-\frac{3}{20}\right)+x_2-\frac{1513}{8000}\right)^2, \\ a_4 &:= \theta\left(-\frac{13}{400}\left(x_1-\frac{3}{20}\right)+x_2-\frac{1513}{8000}\right) - \frac{89}{30}\left(x_1-\frac{3}{20}\right) \end{aligned}$$

$$
\begin{aligned}
a_5 :=& -14099380\left(x_1-\frac{3}{20}\right)^3+5639752\left(x_1-\frac{3}{20}\right)^2\\
&+\frac{23763}{20}\left(a_{51}+979\right)\left(x_1-\frac{3}{20}\right)-11247820\\
&\times\left(-\frac{13}{400}\left(x_1-\frac{3}{20}\right)+x_2-\frac{1513}{8000}\right)\\
a_{51} :=& 8000\left(-\frac{13}{400}\left(x_1-\frac{3}{20}\right)+x_2-\frac{1513}{8000}\right),\\
a_6 :=& \theta\left(-\frac{13}{400}\left(x_1-\frac{3}{20}\right)+x_2-\frac{1513}{8000}\right)-\frac{89}{20}\left(x_1-\frac{3}{20}\right)\\
b_1 :=& 1346570\theta^3\left(b_{11}+704969\right)\\
&\times\left(b_{12}b_{13}-\frac{89}{60}\left(x_1-\frac{3}{20}\right)\left(\frac{b_{14}}{\theta^3}-\frac{89\left(x_1-\frac{3}{20}\right)}{5\theta^4}\right)\right),\\
b_{11} :=& 3168400\left(x_1-\frac{3}{20}\right)^2+950520\left(x_1-\frac{3}{20}\right),\\
b_{12} :=& -\frac{13}{400}\left(x_1-\frac{3}{20}\right)+x_2-\frac{1513}{8000},\\
b_{13} :=& \frac{-\frac{13}{400}\left(x_1-\frac{3}{20}\right)+x_2-\frac{1513}{8000}}{\theta^2}-\frac{89\left(x_1-\frac{3}{20}\right)}{20\theta^3},\\
b_{14} :=& 3\left(-\frac{13}{400}\left(x_1-\frac{3}{20}\right)+x_2-\frac{1513}{8000}\right).
\end{aligned}
$$

With the initial conditions $x_1^0 = 0.143258426$ and $x_2^0 = 0.19890589$, the graph in Fig. 1 shows the trajectories of $x_1(t)$ and $x_2(t)$. Fig. 2 shows the controllability function

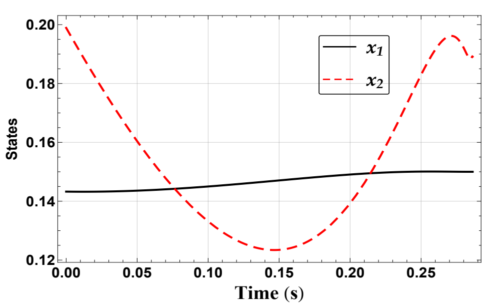

Fig. 1. Trajectories of $x_1(t)$ and $x_2(t)$

$\theta(x(t)-\bar{x})$ on the trajectory $x(t)$. The graph of the

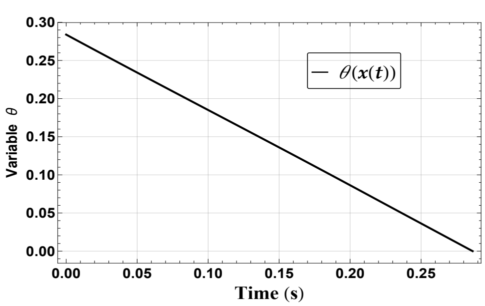

Fig. 2. The controllability function $\theta(x(t)-\bar{x})$

position control $u(x(t)-\bar{x})$ on the trajectory $x(t)$ is as shown in Fig. 3.

By using Wolfram Mathematica, we have calculated that the time of arriving from x_0 to $\bar{x}$ is $T(x_0,\bar{x}) = 0.283881$

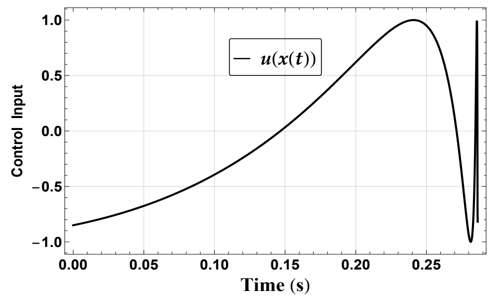

Fig. 3. The positional control $\theta(x(t)-\bar{x})$

and that $|x_1(T)-\xi| \leq 1.05618 * 10^{-11}$ and $|x_2(T)-\eta| \leq 7.15817 * 10^{-6}$.

To the best of the authors' knowledge, no control methodologies have been applied to this system, which considers two main features: achieving finite-time convergence with a bounded control input

5. CONCLUSION

We have presented a family of explicit bounded controls which stabilizes the predator-prey system (1) in finite time. An ellipse depending on the parameters of the system (1) and a number $\widehat{\theta}$ is given. The translation of any initial point $x_0 = (x_1^0, x_2^0)$ to the equilibrium point $\bar{x}$ is guaranteed if x_0 belongs to this ellipse and satisfies the conditions $x_1^0 > 0$ and $x_2^0 > 0$

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